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Is Pairs Trading Performance Sensitive to the Methodologies?: A Comparison

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O retorno de portfolios de pares autofinanciados é comparado nos Estados Unidos, no Brasil e nos principais mercados Europeus utilizando-se duas metodologias diferentes de seleção de pares: Para todas as três bases de dados, foram utilizados dados diários para comparar a performance do pairs trading baseada no método de seleção via a minimização da soma dos desvios ao quadrado (método da distância) e na seleção baseada em testes de cointegração (método da cointegração) para identificar ativos adequados à estratégia. Nos mercados europeus, os portfólios formados pelo método da cointegração exibiram retorno médio anual de até 13,73%, com correlação próxima de zero com o mercado. Também encontramos que estes pares performaram melhor fora da amostra do que os pares formados através do método da distância, obtendo um retorno líquido e Sharpe Ratio superiores. Para o Brasil, entre 1996 e 2012, os portfólios gerados por pares cointegrados exibiram um retorno médio anual de até 23%, performando melhor fora da amostra, com retorno médio anual e Sharpe ratio superiores ao método da distância. Para os Estados Unidos, por outro lado, os portfólios formados pelo método da distância obtiveram resultados superiores, com retorno médio anual de até 10,47% superior aos 5,57% obtido pelo método da cointegração.

Keywords: Arbitragem Estatística; Cointegracão; Soma dos Desvios ao Quadrado; Pares de ativos

The profitability of self-financing pairs portfolio trading strategy is compared in the American, Brazilian and main European stock markets with two different pairs selection methodologies: For all three databases we use daily data to compare the performance of pairs trading based on the selection of pairs through minimizing the sum of squared deviation (distance method) and the selection based on cointegration tests (cointegration method) for identifying stocks suited for pairs trading strategies. In the european markets, the portfolios formed through the cointegration tests exhibits average annual excess return of up to 13,73%, with close to zero correlation with the market. We also find that the pairs formed through cointegration tests perform better out of sample than the pairs formed through the minimization of the sum of squared deviations, with higher net return and Sharpe Ratio. For Brazil, between 1996 and 2012 we use daily data to compare the perfomance of both pairs trading methodologies. The portfolios based on the cointegration method exhibits average annual excess return of up to 23%, with close to zero correlation with the market. They also perform better out of sample with higher net return, higher sharpe ratio and slightly lower volatility. For the United States, on the other hand, the portfolios formed through the distance method have a superior performance, with excess returns of up to 10,47% superior than the 5,57% return obtained by the cointegration method.

Keywords: Statistical Arbitrage; Pairs Trading; Cointegration; Sum Square Deviation;

JEL Classification: C58, G11, G14, G15. Area 8: Microeconomia, Metodos Quantitativos e Financas

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1. Introduction

The computational advances of the past decades have stimulated the development of trading via computer programs and the rise of algorithmic trading. These systems are designed to search for patterns in financial markets, detect deviation of market prices from these patterns, and profit from detected anomalies. Algorithmic trading is now responsible for more than 70 percent of the trading volume in the the US markets (Hendershott *et al.* 2011). On the other hand, events like the Flash Crash of May 6 2010, when the Dow Jones Industrial Average dropped 600 points in less than 5 minutes, revealed the lack of knowledge about the consequences and robustness of algorithms used in practice (Nuti *et al.* 2011).

Advances in data storage and access have also opened interesting research possibilities. The availability of "big" data sets allows a more robust detection of statistical arbitrage opportunities within and across markets, allowing a more comprehensive evaluation of the effectiveness of trading algorithms. This paper proposes the use of large datasets from stock markets in Europe and Brazil to test two popular pairs trading algorithms out-of-sample, namely: the sum of squared deviations approach of Gatev *et al.* (2006), and the cointegration approach suggested by Alexander and Dimitriu (2002). The markets selected allow the analysis of both strategies under different market conditions in an attempt to uncover differences between trading algorithms.

Pairs trading is an algorithmic trading strategy designed to exploit short-term deviations from an existing long-run equilibrium between two stocks. However, different methods have been proposed in the literature to identify pairs to be traded (see, for example, Gatev *et al.* 2006, Elliott *et al.* 2005, Vidyamurthy 2004, Alexander and Dimitriu 2005, Caldeira and Moura 2013). The motivation for trading pairs has its roots in works that preach the existence of long term relation between stocks. If there exists indeed a long term equilibrium, deviation from this relation are expected to revert. Since future observations of a mean-reverting time series can potentially be forecasted using historical data, this literature challenges the notion that stock prices cannot be predicted (see, for example Lo and MacKinlay 1988, 1997, Guidolin *et al.* 2009). Active asset allocation strategies based on mean-reverting portfolios, which generally fall under the umbrella of statistical arbitrage, have been used by investment banks and hedge funds for several years (Gatev *et al.* 2006). The word statistical in context of an investment approach is an indication of the speculative character of investment strategy. It is based on the assumption that the patterns observed in the past are going to be repeated in the future. This is in opposition to the fundamental investment strategy that both explores and predicts the behaviour of economic forces that influence the share prices. Pairs trading is possibly the simplest statistical arbitrage strategy, since it consists of a portfolio of only two assets. In this approach, we are not interested about trends for particular assets but with a common trend among a pair of stocks, which defines a long-run equilibrium between them. The idea behind pairs trading is that when prices of two shares move together there could be short term deviations to be arbitraged. Thus, this trading strategy consists in detecting pairs of stocks that historically move together, waiting for the spread between them to widen, longing the underpriced stock and shorting the overpriced one to profit when prices revert back to their long-run equilibrium.

Thus, pairs trading is a purely statistical approach designed to exploit equity market inefficiencies defined as the deviation from a long-term equilibrium across stock prices observed in the past. As argued by Do and Faff (2010), pairs trading falls under the big umbrella of the long-short investing approach. According to Avellaneda and Lee (2010) the term statistical arbitrage includes investment strategies that have certain characteristics in common: (i) trading signals follow a systematic rule, in opposition to fundamentals based strategies; (ii) strategies seek to be market-neutral, in the sense that they are not exposed to broad market risk, i.e, they have a zero beta; (iii) the mechanism used to obtain abnormal returns is based on statistical analysis. The success of pairs trading, especially statistical arbitrage strategies, depends heavily on the modeling and forecasting of the spread time series although fundamental insights can aid in the pre-selection step. Pairs trading needs not be market neutral although some say it is a particular implementation of market neutral investing (Jacobs *et al.* 1999).

Pairs trading strategies speculate on future convergence of spread between similar securities. Similarity concerns industry, sector, market capitalization, and other common exposures that might imply a comovement between stocks. However a profitable strategy might also be constructed with stocks covering different sectors based purely on statistical properties of the time series. Gatev *et al.* (2006) test a simple non-parametric pairs trading algorithm on the US market between 1963 and 2002, finding average annualized returns of up to 11% for portfolios of pairs. They suggested that the abnormal returns to pairs strategies were a compensation to arbitrageurs for enforcing the law of one price. Another popular algorithm to select pairs is based on the presence of a cointegration relation between stock prices (see Vidyamurthy 2004, for textbook treatment of the subject). The use of the cointegration technique to asset allocation was pioneered by Lucas (1997) and Alexander (1999) and in the previous decade it was increasingly applied in financial econometrics (see, among others, Alexander and Dimitriu 2002, Bessler and Yang 2003, Yang *et al.* 2004, Caldeira and Moura 2013, Galenko *et al.* 2012, Gatarek *et al.* 2014).

Many studies attempted to test the profitability of pairs trading strategies. However, all them focus on either a single market studies or on a single methodology to select pairs. Do and Faff (2012) examined the impact of trading costs on pairs trading profitability in the U.S. equity market and documented that after 2002 pairs trading strategies were largely unprofitable. Bowen *et al.* (2010) back-test a pairs trading algorithm using intraday data over a twelve month period in 2007, and conclude that returns are highly sensitive to the speed of execution. Moreover, accounting for transaction costs and enforcing a "wait one period" restriction, excess returns are complete eliminated. Broussard and Vaihekoski (2012) tested the profitability of pairs trading under different weighting structures and trade initiation conditions using data from the Finnish stock market. Although the proposed strategy is profitable, the authors note that returns have declined in recent years possible due to increased competition among hedge funds, and/or a reduction in the importance of an underlying common factor that drives the returns in a pairs trading strategy.

The datasets used in these analysis can be divided into two groups: First, the data comes from developed countries which have plenty of historical financial information available, as is the case of the United States. The articles by Gatev *et al.* (2006), Engelberg *et al.* (2009), Huck (2010), Do and Faff (2010), Bowen *et al.* (2010), Do and Faff (2012) are examples that use data from the United States. The second group includes datasets from developing countries. These studies analyse shorter time periods and a smaller number of assets in the database. Yuksel *et al.* (2010) analyse pairs trading in Turkey, Broussard and Vaihekoski (2012) in Finland and Perlin (2009), Caldeira and Moura (2013) in Brazil.

Although many papers have been written about pairs trading, the literature lacks a comprehensive study of the performance of different methodology across developed and emerging markets. Moreover, most studies use different trading periods, different criteria to select assets to be included in the sample and different formation period, rendering a cross study comparison impossible. Since pairs trading performance is influenced by the methodology chosen, it is important to compare them under different circumstances to understand if there is a overall winner, or if some strategies are better suited to specific market conditions. We, on the other hand, use the database of an emerging country Brazil, a developed monetary union, the Euro Area, and an important world financial center, the USA, along with equal parameters for each methodology in order to compare them and test to see which is more suitable to which environment. The cointegration method has shown results statistically superior Sharpe Ratio then the distance method for both markets studied for all 3 portfolios formed.

The article is organized as follows. In the next section we described both pairs trading methodologies analyzed and some. Section 3 presents some implementation details common to both methods, as well as the evaluation strategy. Section 4 describes the three large data sets used and discusses the results of our comparison. Section 5 evaluates the performance of both methodologies and the last section concludes with some final remarks.

2. Pairs trading methodologies

Broadly defined, there are three different approaches to pairs trading: the distance approach, the stochastic approach and the cointegration approach. These methods all vary with regard to how the spread of the stock pairs is defined. This paper compares two most popular methods of selecting pairs of stocks between practitioners and researchers: the distance method proposed by Gatev *et al.* (2006) and the cointegration approach used in Lucas (1997), Alexander and Dimitriu (2005), Do *et al.* (2006), and Caldeira and Moura (2013). The two approaches used in this paper will be discussed in the next subsection.

2.1. *The distance approach*

The distance approach is proposed by Gatev *et al.* (2006) and is used among others by Andrade *et al.* (2005), Engelberg *et al.* (2009), Do and Faff (2010), Bowen *et al.* (2010), and Broussard and Vaihekoski (2012). By this approach the co-movement in the pair is measured by the *distance*, which is defined as the sum of squared deviations (SSD) between the two normalized price series. Normalized price series are defined to start from one, and then evolve using the return series. The normalized price series for a stock is given by its cumulative total returns index over the moving formation period of 252 days. Formally, we compute

$$\tilde{P}_{ti} = \prod_{\tau=1}^{t} (1 + r_{\tau i}) \tag{1}$$

where $\tilde{P}_{ti}$ is the normalized price of stock i at time t, $r_{i\tau}$ is the dividend-adjusted return of stock i at time τ, and τ is the index for all trading days between $t - 252$ and t. The normalized series begin the observation period with a value equal to one, and increases or decreases each day given its return. For each stock i, we find the stock j that minimizes the sum of square deviations between the two normalized price series. The distance is thus defined as

$$\Delta_t^{ij} = \sum_{t=1}^{252} \left(\tilde{P}_{ti} - \tilde{P}_{tj}\right)^2 \tag{2}$$

where Δ_t^{ij} is the distance between the normalized prices of stock i and j over the formation period. This means that pairs are formed by exhaustive matching in normalized price space, where price is the daily closing price adjusted for dividends and splits. We rank all possible pairs by distance, identify the combinations with the highest measure of co-movement and monitor these pairs for the duration of the trading period. Similar to Gatev *et al.* (2006), we set the periodicity of pair updates to 20 days (approximately 1 month)

In order to select a pair for a given stock, we search on the database for an asset whose normalized price has the smallest squared distance to the normalized price of the chosen stock up to time t. A long-short position is opened when the distance exceeds a pre-specified threshold[1] based on a standard deviation metric. Following Gatev *et al.* (2006), the signal to start trading occurs when the distance between the normalized price diverges by more than two standard deviations. An open long-short position is closed either upon convergence in normalized prices, if a superior matching

[1]The threshold can be constructed in a variety of ways, but the most common method is to select some proportion of the historical standard deviation of the spread:

$$q = \delta\sigma_{\mathsf{spread}}$$

Gatev *et al.* (2006), Andrade *et al.* (2005) and Do and Faff (2010)) set $\delta = 2$, whereas Bowen *et al.* (2010) and Broussard and Vaihekoski (2012) experiment with a range of values. It is also possible to let q be a variable by defining δ as a rolling parameter with window size n; this may allow us to better capture the profit potential of periods with higher volatility in the spread.

partner is identified, or at the end of the trading period. The latter imposes a restriction on the investment horizon and works as an automatic risk control mechanism.

The distance approach is a model free approach and non-parametrically exploits a statistical relationship among two stocks prices. From a practical point of view, the distance method is easy to implement and independent of economic models, which avoids misspecification problems. On the other hand, non-parametric strategies have lower prediction ability compared to well-specified parametric models. The fundamental assumption of this approach is that pair spreads exhibit mean-reversion. Accordingly, a price-level divergence is an indication of disequilibrium and price distance is the measure of mispricing.

2.2. *The cointegration approach*

The use of the cointegration technique to asset allocation was pioneered by Lucas (1997) and Alexander (1999) and in the previous decade it was increasingly applied in financial econometrics (see, among others, Alexander and Dimitriu 2002, Bessler and Yang 2003, Yang *et al.* 2004). Cointegration is an extremely powerful technique, which allows dynamic modelling of non-stationary time-series sharing a common stochastic trend. The fundamental observation that justifies the application of the concept of cointegration to the analysis of stock prices is that a system involving non-stationary stock prices in levels can display a common stochastic trend (see **?**). When compared to the concept of correlation, the main advantage of cointegration is that it enables the use of the information contained in the levels of financial variables.

Similar to the previous trading strategy, the main concern of the cointegration approach is the mean reversion of the spread. However, instead of defining the spread as the distance between standardized prices of a pair of stocks, the spread is defined with respect to the long-run equilibrium of a cointegrated system; that is, the long-run mean of the linear combination of two time series (Vidyamurthy 2004). Deviations from the equilibrium should revert to the long-run mean, implying that one or both time series should adjust in order to restore the equilibrium.

Using cointegration as a theoretical basis, the spread is generated based on the actual error term of the long-run relation:

$$\log(P_{it}) - \gamma \log(P_{jt}) = \mu + \varepsilon_t \tag{3}$$

where γ is the cointegration coefficient, the constant term μ captures a possible premium in stock i versus stock j, and ε_t is the estimated error term. Thus, it is not needed to predict P_t^i and P_t^j, but only their difference $\log\left(P_t^i\right) - \log(P_t^j)$. If we assume that $\left\{\log\left(P_t^i\right), \log(P_t^j)\right\}$ in (3) is a non-stationary VAR(p) process, and there exists a value γ such that $\log\left(P_t^i\right) - \gamma\log(P_t^j)$ is stationary, we will have a cointegrated pair.

For detected cointegrating relations, the algorithm creates trading signals based on predefined investment decision rules. In order to implement the strategy we need to determine when to open and when to close a position. First, we calculate the spread between the shares. The spread is calculated as

$$\varepsilon_t = \log\left(P_t^i\right) - \gamma \log\left(P_t^j\right) - \mu, \tag{4}$$

where ε_t is the value of the spread at time t. Accordingly, we compute the dimensionless z-score defined as

$$z_t = \frac{\varepsilon_t - \mu_\varepsilon}{\sigma_\varepsilon}, \tag{5}$$

the z-score measures the distance to the long-term mean in units of long-term standard deviation.

After selecting the most appropriate pairs, the same trading strategy used under the distance approach is executed using the z-score series instead. This method is based on Vidyamurthy (2004), Avellaneda and Lee (2010), and Caldeira and Moura (2013). It is an attempt to parametrize the long-term relationship between two assets and explore price-deviations from their historical relationship using cointegration. Even if two time series are non-stationary, cointegration implies the possibility that a linear combination of both series could be stationary. If this is indeed the case,The existence of both series move “closely together as if they were connected to each other.

The quality of estimation of the correction error model depends on the econometric technique applied. The first method for testing cointegration by Engle and Granger (1991) is a two step procedure in which the first step, stationarity test of the residuals errors, renders results sensitive to the ordering of the variables, and such mispecification error is carried to the second step, the error correction model estimation. The way found to reduce this error is to use two cointegration tests. Besides the Engle and Granger (1991) we also used the Johansen (1988) test, and use only the pairs that are considered cointegrated by both tests. Nonetheless Engle and Granger (1991) well-known limitations (small sample problems, maximum of one cointegrating vector, treating the variables assymetricaly) are not an issue in this work, due to our samples having 252 observations, only two variables are included in the estimation procedure, and it is only possible to find one cointegrating vector.

3. Implementation Details

In this work we follow the methodology by Gatev *et al.* (2006), Broussard and Vaihekoski (2012) to implement the distance method and the methodology used by Caldeira and Moura (2013), Vidyamurthy (2004) in the implementation of cointegration methodology. The formation period for the pairs is 12 months long, and the trading period comprises the following 6 months. The pairs of assets are selected by minimizing the sum of squared deviations in the portfolios formed from the distance method and ranked beginning from the smallest sum of squared deviations. In portfolios formed from the cointegration method, the pairs are selected if they are found cointegrated with both tests, Engle and Granger (1991), Johansen (1988), and later ranked by their Sharpe index within the sample as in Gatev *et al.* (2006), Caldeira and Moura (2013).

Next, portfolios are formed with 5, 10, and 20 pairs with the lowest sum of squared deviations and the best Sharpe ratios within the sample, and will be used in the trading period in the 6 months following the formation of pairs. At the end of each period of trading all positions are closed. A new 12 month period for the pairs formation is created and ends on the last observation of the previous trading period, when all cointegration tests and pairing are redone. The assets to be used must be traded in the 12 month formation period, but not necessarily they will be listed during the 6 month trading period.

In order to generate trading signals, it is necessary to calculate the distance between the asset prices in the pair, measured by the spread $\varepsilon_t = P_t^l - \gamma P_t^s$, where ε_t is the spread value at time t.

From the spread, the distance measure is given by the formula $z_t = \frac{\varepsilon_t - \mu_\varepsilon}{\sigma_\varepsilon}$. The goal is to identify when z_t departs from the long term average, given by the error correction model, measured in terms of standard deviation. Initially, the position opens when $|z_t| > 2$ and closes when $z_t = 0$.

Let P_t^l be the long asset price and P_t^s the price of the asset sold short, then the net return in t of par i is given by:

$$r_{it}^{raw} = ln\left[\frac{P_t^l}{P_{t-1}^l}\right] - \gamma\left[\frac{P_t^s}{P_{t-1}^s}\right] + 2ln\left(\frac{1-C}{1+C}\right) \tag{6}$$

This formula can be explained intuitively. Suppose we buy stock ξ at price P_{t-1}^{ξ} at time $t-1$ and sell it at time at price P_t^{ξ}. Including transaction costs, the cost of buying is $P_{t-1}^{\xi}(1+C)$ and the profit of selling is $P_t^{\xi}(1-C)$. This corresponds to the decomposed net return: $ln\left[\frac{P_t^{\xi}(1-C)}{P_{t-1}^{\xi}(1+C)}\right] = ln\left[\frac{P_t^{\xi}}{P_{t-1}^{\xi}}\right] + ln\left[\frac{(1-C)}{(1+C)}\right] = r_t^{\xi} + ln\left[\frac{(1-C)}{(1+C)}\right]$

This equation already includes transaction costs in its second term. To calculate the net return of a portfolio with N pairs, we do the weighted average net returns of each pair, with the weight defined by the percentage of the amount invested in each pair with respect to the value of the portfolio in time t. Let p be a portfolio with N pairs, where w_i is the weight for each pair i. Thus, the net return of the portfolio in t is $R_t^p = \sum_{i=1}^{N} w_{it} R_{it}$. As explained in the Caldeira and Moura (2013), the calculation of compound return (log returns) of a portfolio of assets is, for small values, close to the weighted average of the continuously compounded returns for each asset i.e, $R_t^p \cong \sum_{i=1}^{N} w_{it} r_{it}$. However, to calculate the return accurately, log-returns are transformed back to simple return, with the monthly compound rate of return, r_{it} given by:

$$r_{it} = ln(1 + R_{it}) = ln\left(\frac{P_t}{P_{t-1}}\right), \tag{7}$$

to transform back we just multiply by e to remove the logarithm and obtain the net return R_{it}. $e^{r_{it}} = 1 + R_{it} \Longrightarrow R_{it} = e^{r_{it}} - 1$.

From this net return of the portfolio equation, we used the weighting scheme of returns as in Gatev *et al.* (2006), Broussard and Vaihekoski (2012). The scheme used is the weighting of the returns to the capital previously committed (committed capital scheme), in which an amount of capital is distributed evenly across the entire universe of pairs for the period. Even if the pair does not open or if it closes before the trading period finishes, capital remains committed to that pair. This scheme divides the payoff in pairs for all pairs that were selected for the period of trading. This method considers the opportunity cost of hedge funds when they commit resources on a pair that ends up not being used during trading. We are conservative and assume a rate of return of zero for capital in pairs that are not open, as in Broussard and Vaihekoski (2012), and unlike Gatev *et al.* (2006), which assumes a risk-free rate of return.

The change in the weights of the pairs within the portfolio follows the method of equal weights (Equally weighted approach), defined as in Broussard and Vaihekoski (2012). That is, the sum of returns of each pair is divided by the number of pairs that were selected for the period of trading, in the committed capital scheme. In practice, the use of stop-loss is critical to minimize losses. However, most academic works on pairs trading don't use them. Exceptions are Nath (2006), Caldeira and Moura (2013), and in this work we follow the method of Caldeira and Moura (2013) and the stop-loss is triggered and the position in the pair is closed when losses reach 10% and we also include a stop gain of 20% with other values being tested.[1]

Transaction costs considered follow Dunis et al. (2010), Caldeira and Moura (2013) and total 0.4% for each change of position in the pair (opening and closing): 0.1% brokerage in total for each action (buying and selling), totaling 0.2% for each pair in brokerage costs. Slippage of 0.05% for each stock in the pair, and 0.1% for the lease of the asset to be sold short (divided in 0.1% for opening and 0.1% when closing the position). The performance of the pairs portfolios is measured from 4 statistics:

[1]We considered values of 5%, 7%, 10%, 15%, 20% and no trigger for the stop-loss or stop-gain, finding that the lower/higher the stop-loss/gain better the strategy performance being the the highest sharpe ratio when no trigger was used either for stop-loss or stop-gain.

$$\text{Cumulative Returns: } R^A = 252 \times \left(\frac{1}{T} \sum_{t=1}^{T} R_t \right)$$

$$\text{Variance of Returns: } \hat{\sigma}^A = \sqrt{252} \times \left(\frac{1}{T} \sum_{t=1}^{T} (R_t - \hat{\mu})^2 \right)$$

$$\text{Sharpe Index: } SR = \frac{\hat{\mu}}{\hat{\sigma}}, \quad \text{where} \quad \hat{\mu} = \frac{1}{T} \sum_{t=1}^{T} w_{it} R_{wit}$$

$$\text{Maximum Drawdown: } MDD = \sup_{t\epsilon[0,T]} \left[\sup_{t\epsilon[0,t]} R_s - R_t \right]$$

4. Database and empirical results

Previous pairs trading studies aimed at testing a specific methodologies for a given stock market. However, the availability of "big" financial datasets across the globe allows the researcher to expand the analysis to markets with different characteristics, allowing a more robust evaluation of the strategies. Our data comprises three highly liquid financial markets described bellow: Brazilian, American and Euro Area Stock Markets

BR Dataset

The database used in this study consists of all daily closing prices for stocks that have daily trading during the 12 month formation period and are listed in the Bolsa de Valores de So Paulo (Bovespa). The data were obtained from Economtica for the period between 1995 and 2012, and are adjusted for dividends and splits, in order to avoid false trading signals. It is usual for some studies in pairs trading strategies that the stocks paired are required to belong to the same industry or sector, as in Do and Faff (2010), Gatev *et al.* (2006). Here, due to a limited universe of assets, we do not adopt such restriction, and as long as the pairs satisfy the cointegration criterion or belong to the top 20 pairs that have the smallest squared deviations, they can be traded. We do not use the risk free rate as the return on the pairs that are not being traded in order to keep the analysis as conservative as possible.

USA Dataset

The North American database was obtained at CRSP and contains the highest most liquid stocks for every 10 year period, totalling 4471 stocks. The period analyzed goes from 1962 to 2012 comprising a total of 12.586 observations. We did not limit the universe with which each stock could pair up.

EU Dataset

The EU dataset contains daily data from the 1,000 most liquid stocks from 1973 to 2012. The data was obtained from Datastream comprising 10,435 observations. All countries in the euro area were considered for the sample, however not all countries have stocks in the 1000 most liquid for the sample period. Our period has stocks from companies based in Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Portugal and Spain.

4.1. *Estimation and Results for the Brazilian Database*

Table 2 shows the results for the selected pairs during the second semester of 2012. Although 302 pairs were considered cointegrated by the Engle & Granger and the Johansen Trace test, only the top 20 were selected for trading based on the best Sharpe ratio in-sample. Since we did not include any restrictions for the possible pairs a given stock may form, its interesting to see which pairs may

Table 1. Descriptive statistics of the datasets.

	Brazilian data	US data	European data
Panel A: datasets			
Start date	Jan-1995	Jan-1962	Jan-1973
End date	Dec-2012	Dec-2012	Dec-2012
Number of stoks	450	4.471	1.000
Number of observations in the sample	4.087	12.586	10.435
Number of days of training periods	100	100	100
Average number of days of training period	247	252	261
Average number of days of trading period	123	126	130
Average number of cointegrated pairs per period	190	1.000	500
Panel B: Descriptive statistics of returns			
Mean (in %)	0.14	0.0274	0.0135
Standard deviation (in %)	1.47	2.3843	0.67
Minimum (in %)	-39.95	-29.972	-39.709
Maximum (in %)	40.2	30.697	49.84
Skewness	1.7941	0.6927	0.1430
Kurtosis	17.65	14.779	15.73

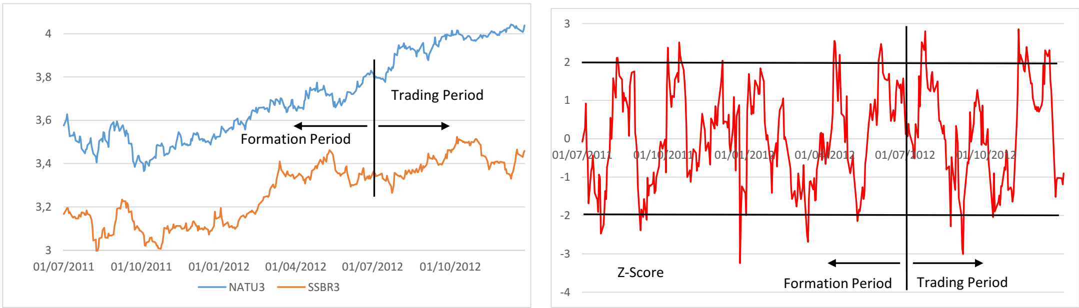
be formed between companies not in the same sector. However the majority of pairs selected are from companies that are in some way or another related, for instance, the pair formed between BR Properties (BRPR3) and Iguatemi (IGTA3), both companies in he real estate sector, or between Amil (AMIL3) and Odontoprev (ODPV3), both belonging to the health sector.

Table 2. Descriptive Statistics of the Top 20 Cointegrated Pairs the 2° semester of 2012.

	Stock 1	Stock 2	Sharpe in-sample	Net Return	EG Stat	JH Trace	Half Life
Pairs	'NATU3'	'SSBR3'	3,27	32,68	-3,82	22,48	8,11
	'AMIL3'	'BEEF3'	2,99	-2,53	-4,48	21,31	8,47
	'ELET3'	'GSHP3'	2,95	-7,57	-4,23	22,77	8,95
	'ABCB4'	'BRSR6'	2,86	58,54	-4,50	20,47	9,21
	'EZTC3'	'RENT3'	2,75	32,08	-3,88	17,76	10,24
	'AMIL3'	'CCRO3'	2,73	-10,32	-5,18	22,08	8,20
	'AMIL3'	'GRND3'	2,70	15,13	-4,34	19,27	9,32
	'BRIN3'	'IGTA3'	2,66	11,04	-4,04	13,85	15,12
	'BRML3'	'BRPR3'	2,60	16,51	-6,18	35,16	5,34
	'AMIL3'	'ODPV3'	2,58	-9,67	-4,06	18,82	10,55
	'LREN3'	'STBP11'	2,57	8,03	-5,11	27,95	6,90
	'AMIL3'	'ECOR3'	2,55	-12,80	-4,97	25,30	7,32
	'IGTA3'	'PINE4'	2,51	5,13	-3,92	15,49	11,63
	'BRPR3'	'SSBR3'	2,50	26,42	-5,46	21,09	8,63
	'ENBR3'	'SANB11'	2,49	8,61	-3,84	19,07	10,66
	'ECOR3'	'BEEF3'	2,44	9,27	-4,14	15,68	11,38
	'GETI3'	'EQTL3'	2,40	8,54	-6,61	42,01	4,47
	'GETI3'	'BEMA3'	2,30	6,61	-3,83	19,55	9,41
	'DTEX3'	'BEEF3'	2,30	8,28	-4,12	17,30	10,34
	'BRPR3'	'IGTA3'	2,28	7,90	-3,85	19,12	9,57

All pairs selected obtained positive Sharpe Ratios in-sample, but that is not necessarily true for the performance out of sample. From the total of 20 pairs selected, to form the portfolio, 5 had negative net returns out of sample but the other pairs more than compensated for the bad performance of those few ones, with the portfolio totalling an average net return of 10,59% during the second semester of 2012. Most of the pairs have a half life inferior to 10, reassuring the existance of a long run equibrium with a sufficiently fast error correction, a very necessary characteristic for our strategy.

Figures 1(a) and 1(b) show an example of the historical prices between two cointegrated stocks, NATU3 and SSBR3, during the formation period and the subsequent trading period. The z-score is a measure of distance from the long run equilibrium in terms of standard deviations and it crosses the 2 standard deviation threshold more than 6 times during the trading period, indicating that the spread widens and shortens very frequently in a given 6 month period.



(a) Historical Price of NATU3 and SSBR3 between july 2011 and december 2012.

(b) Z-score between NATU3 and SSBR3 between july 2011 and december 2012.

Figure 1. An example for the brazilian stock market.

The portfolios created had between 5 to 20 pairs selected be it through the cointegration or the distance method. Table 3 first half shows the summary statistics for the portfolios created through both methodologies. As is expected, the average number of pairs opened increases with the number of pairs in the portfolio. Nonetheless, all other statistics remain very similar, except for the average price deviation for opening pairs, which also increases with the number of pairs in the portfolio. Given that every extra cointegrated pair or maatched pairs through SSD included in the portfolio has a smaller sharpe ratio and more likely to diverge by more due to its stocks 'weaker' comovement, it is probable that those pairs tend to diverge more from their long run equilibrium, explaining the higher average price deviation. Each cointegrated pair had on average between 2.97 and 2.3 round trip trades depending on the size of the portfolio, which means that each pair was traded on average more than 2 complete times each 6 months period.

Since the prices used for the distance method are normalized, the average standard deviations are not directly comparable between the pairs selection method in Table 3. However, the average number of pairs opened is comparable, and it shows that the distance method is more 'trigger happy' than the cointegration method. This selection method starts, for the 20 pairs portfolio, a 1200 pairs more than the cointegration method, possibly resulting in over trading and incurring in excessive transaction costs. Also, these pairs remain open on average around 25 trading days, while in the cointegration method the pairs remain open on average 16,8 days, indicating that the distance method not only opens more pairs in a given period, it also stays longer in a open position. Each pair was traded on average between 3.5 and 4 times each 6 months for the distance method, meaning that in a period of an average of 120 trading days, there was room for an average of opening a pair up to 4 times and closing it also 4 times.

The excess returns are for the whole trading out of sample, between july 1996 and december 2012 already discounted for transaction costs and slippage effects. Some periods exhibit less than 20 cointegrated pairs, specialy the years between 1996 and 2002, consequently when applicable, we used the maximun number available of pairs to form a portfolio.The average annualized return for the cointegrated pairs dimishes with the inclusion of more pairs, from 23% with 5 pairs to 18,37% when a portfolio consists of 20 pairs. Nonetheless, the average annualized volatility also dimishes, from 11,20% with a portfolio of 5 pairs to 7,9% when with 20 pairs in a portfolio. Consequently, the Sharpe Ratio for the whole sample is higher for the top 20 pairs (2,11) due to the lower volatility, which more than compensate the lower average annualized return (18,37%). The cumulative return between 1996 and 2012, range from 1364% for a portfolio of 20 pairs to 2499% for a portfolio with 5 pairs. The correlation with the market is slightly negative and not significantly different from zero, exactly what we want and would expect from a market neutral strategy. The share of days with negative returns is between 36% and 38% and very inferior to the 48% share of days with negative returns in the Ibovespa. The maximum drawdown is also included in the summary reaching 21,68% for the all cointegrated pairs portfolios. This is a simple measure thay indicates the largest cumulative loss after a given maximun of a cumulative positive rallying of the returns, signaling how fast the leverage can increase, much smaller than the 65% drawdown of the Ibovespa.

The summary statistics for the pairs selected through the distance method are presented on table 3. The volatility for the portfolios created through this method is over 10% for all 3 portfolios, higher than for the cointegration method portfolios, which range between 11,2% and 7,9%, but the annualized average return are all inferior to the cointegrated pairs portfolios, ranging between 3,2% and 6,4%, a little less than a third when compared with the cointegration method. The consequence is that the Sharpe Ratios are very small, not statistically different from the Ibovespa Sharpe ratio of 0,58 for the whole sample. Even though the strategy is market neutral with a close to zero Spearman rho for all 3 portfolios, the maximum drawdown is mostly higher than the ones that occur in the cointegration method. Not only, the share of days with negative returns is over 48% for all porfolios, similar to the Ibovespa performance, with all these statistics indicating that this strategy is not superior than buying and holding the market index.

Table 3. Excess returns of unrestricted pairs trading strategies: brazilian dataset

Note: This table reports a summary statistics of the excess returns on portfolios of pairs between January 1995 and December 2012 (216 observations). Pairs are formed over a 12-month period according to a minimum-distance criterion and then traded over the subsequent 6-month period. Pairs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis.

Mehodology	Distance approach			Cointegration approach		
Pairs Portfolio	Top 5	Top 10	Top 20	Top 5	Top 10	Top 20
Total number of pairs opened	592	1289	2757	506	897	1577
Total number of 6 month trading periods	34	34	34	34	34	34
Average price deviation for opening pairs	0.114	0.140	0.172	0.590	0.654	0.695
Average no of pairs opened each 6 mo period	17.41	37.91	81.08	14.88	26.38	46.38
Average number of pairs traded in months when at least one pair opened	3.588	3.906	4.177	3.265	3.159	3.092
Average number of round-trip trades per pair	3.482	3.791	4.054	2.977	2.638	2.319
Standard deviation of round-trips per pair	2.099	2.108	3.208	1.843	1.942	1.937
Average time pairs are open in days	27.60	25.36	23.78	16.54	16.82	16.94
Median time pairs are open in days	13.50	14.00	14.00	12.00	12.00	13.00
Average time pairs are open in months	1.314	1.208	1.132	0.788	0.801	0.807
Standard deviation of time open per pair in days	33.18	29.57	26.22	13.63	14.36	14.08
Standard deviation of time open per pair in months	1.580	1.408	1.249	0.649	0.684	0.671
Share of negative excess returns	0.485	0.484	0.498	0.368	0.377	0.378
Average Annualized Return (in %)	4.924	6.460	3.200	23.00	19.77	18.37
Average Annualized Volatility (in %)	13.01	11.52	10.70	11.20	9.24	7.90
Total Sample Sharpe Ratio	0.370	0.544	0.294	1.850	1.950	2.110
Largest Daily Return (in %)	4.240	4.230	4.020	9.600	9.600	9.600
Lowest Daily Return (in %)	-6.380	-4.200	-3.750	-4.430	-3.920	-3.840
Cumulative Profit (in %)	90.00	148.00	51.88	2,499.00	1,642.00	1,364.00
Spearman Correlation Rho with Ibovespa	0.051	0.062	0.095	-0.009	-0.013	-0.014
CAPM beta (β-market)	0.029	0.027	0.040	-0.004	-0.003	-0.004
Annual Skewness	0.821	1.291	1.248	1.010	1.130	1.740
Annual Kurtosis	3.056	4.427	4.407	3.730	5.060	8.910
Maximun Drawdown (in %)	31.78	18.65	22.44	21.68	21.68	21.68

Table 4 shows the results found for the American data set for the whole sample period between january 1962 and december 2012. The average number of pairs opened, just like in the other databases, also increases with the number of pairs in the portfolio. However, both strategies perform in a very similar way in terms of the number of pairs that open and close each trading period. The distance method does not starts a significant number of pairs more than the cointegration method, with the portfolios of 5, 10 and 20 pairs starting, respectivelly 17, 32, and 74 pairs on average for the distance method. Slightly more than the 16, 31 and 64 pairs on average for the cointegration method with the same portfolio size. The average number of round trips per pair was between 3 and 4 for both strategies, with all portfolios having a similar number o round trips. The average time pairs remain open in the both method is very similar, betwenn 7 and 10, with the standard deviation slight higher for the distance method portfolios. The descriptive results for the United

States show a tendency towards similarity betwenn both strategies in terms of round trips, median time pairs are open and they even have similarshare of negative excess returns.

The average annualized excess return for both strategies dimishes with the inclusion of more pairs, from 10.47% with 5 pairs to 9.55% when a portfolio consists of 20 pairs in the distance method. Nonetheless, the average annualized volatility also dimishes, from 5.54% with a portfolio of 5 pairs to 3.13% when with 20 pairs in a portfolio. Consequently, the Sharpe Ratio for the whole sample is higher for the top 20 pairs (2.9) due to the lower volatility, which more than compensate the lower average annualized return (9,55%). The cumulative return between 1962 and 2012, range from 9184% for a portfolio of 20 pairs to 13331% for a portfolio with 5 pairs. The correlation with the market is slightly positive and not significantly different from zero, exactly what we want and would expect from a market neutral strategy. The share of days with negative returns is between 35% and 44The maximum drawdown is also included in the summary reaching 21.14% for the 5 pairs portfolios chosen by the distance method. This is a simple measure thay indicates the largest cumulative loss after a given maximun of a cumulative positive rallying of the returns, signaling how fast the leverage can increase.

The summary statistics for the pairs selected through the cointegration method are presented on table 4 also. The volatility for the portfolios created through this method is betwenn 5,4% and 10,36% for all 3 portfolios, higher than for the distance method portfolios, and the annualized average return are all inferior to the distance method pairs portfolios, ranging between3.7% and 5.5%. The consequence is that the Sharpe Ratios are also low. The cointegration method strategy is market neutral with a market beta and a Spearman rho close to zero for all 3 portfolios and the maximum drawdown is mostly higher than the ones that occur in the cointegration method. The share of days with negative returns ranges between 26% and 34% for all cointegratied porfolios and are mostly similar to the distance method portfolios. In summary, the distance method performs better than the cointegration method for the United States, with most performance metrics being superior and indicating a clear advantage.

Table 5 shows the results found for the european data set for the whole sample period between january 1973 and december 2012. The average number of pairs opened, just like in the brazilian database, also increases with the number of pairs in the portfolio. However, the difference between both strategies is smaller. The distance method does not starts a significant number of pairs more than the cointegration method, with the portfolios of 5, 10 and 20 pairs starting, respectivelly 27, 54, and 105 pairs on average for the distance method. Slightly more than the 24, 49 and 95 pairs on average for the cointegration method with the same portfolio size. The average number of round trips per pair was slightly over 5 for the distance method and slightly under 5 for the cointegration method, still indicating that the former tends to open more pairs then the latter. The average time pairs remain open in the distance method portfolio is around 10 days, almost double than the 5.3 to 5.9 average days the cointegration pairs remain open, and the standard deviation of the time open in days is around 20 days, while for the cointegration method is a little over 6 days, with such result pointing towards the tendency for the distance approach to be more "trigger happy" and at the same remain longer on a given pair.

The average annualized excess return for the cointegrated pairs dimishes with the inclusion of more pairs, from 13.73% with 5 pairs to 11.19% when a portfolio consists of 20 pairs. Nonetheless, the average annualized volatility also dimishes, from 8.41% with a portfolio of 5 pairs to 6.15% when with 20 pairs in a portfolio. Consequently, the Sharpe Ratio for the whole sample is higher for the top 20 pairs (2.3) due to the lower volatility, which more than compensate the lower average annualized return (11.19%). The cumulative return between 1973 and 2012, range from 6479% for a portfolio of 20 pairs to 14,600% for a portfolio with 5 pairs. The correlation with the market is slightly positive and not significantly different from zero, exactly what we want and would expect from a market neutral strategy. The share of days with negative returns is between 26% and 38% and similar to the 29,9% share of days with negative returns in the MSCI Europe excluding UK and Switzerland Index. The maximum drawdown is also included in the summary reaching 37,95% for the 5 pairs cointegrated portfolios. This is a simple measure thay indicates the largest cumulative

Table 4. Excess returns of unrestricted pairs trading strategies: USA dataset

Nota: This table reports a summary statistics of the monthly excess returns on portfolios of pairs between January 1962 and December 2012 (600 observations). Pairs are formed over a 12-month period according to a minimum-distance criterion and then traded over the subsequent 6-month period. airs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis.

Methodology	Distance approach			Cointegration approach		
Pairs Portfolio	Top 5	Top 10	Top 20	Top 5	Top 10	Top 20
Total number of pairs opened	1784	3233	7425	1624	3190	6479
Total number of 6 month trading periods	100	100	100	100	100	100
Average price deviation for opening pairs	0.0215	0.0279	0.0304	0.414	0.4180	0.4172
Average no of pairs opened each 6 mo period	17.84	32.33	74.25	16.24	31.9	64.79
Average number of pairs traded in months when at least one pair opened	3.5752	3.2362	3.7144	3.248	3.19	3.2444
Average number of round-trip trades per pair	3.5680	3.233	3.7125	3.2480	3.19	3.2395
Standard deviation of round-trips per pair	2.6607	2.4728	2.2231	1.9125	1.8765	1.8966
Average time pairs are open in days	11.3879	12.7145	21.3436	9.322	9.5558	9.5367
Median time pairs are open in days	7	8	10	8	8	8
Average time pairs are open in months	0.5423	0.6055	1.0164	0.4439	0.455	0.4541
Standard deviation of time open per pair in days	10.6119	10.9617	28.1507	6.1963	6.4086	6.48
Standard deviation of time open per pair in months	0.5053	0.5220	1.3405	0.2951	0.3052	0.3086
Share of negative excess returns	0.3552	0.4138	0.4398	0.3264	0.4027	0.4348
Average Annualized Return (in %)	10.4778	9.772	9.55	5.5728	3.7912	4.2045
Average Annualized Volatility (in %)	5.5407	4.1152	3.1358	10.3695	7.4568	5.4070
Total Sample Sharpe Ratio	1.798	2.2662	2.9097	0.5231	0.4991	0.7618
Largest Daily Return (in %)	4.170	1.90	2.71	6.78	3.71	2.58
Lowest Daily Return (in %)	-3.03	-3.22	-1.61	-6.46	-4.47	-2.97
Cumulative Profit (in %)	13,331.00	9,923.00	9,188.00	1,047.00	458.00	627.00
Spearman Correlation Rho with MSCI	0.0163	0.0211	0.013	0.0123	0.0163	0.0143
CAPM beta (β-market)	0.0184	0.01	0.0104	0.0271	0.0222	0.0174
Annual Skewness	1.0844	0.3914	0.3033	1.0311	0.7708	1.6479
Annual Kurtosis	4.2085	2.2037	2.2352	4.3151	2.9412	8.9737
Maximun Drawdown (in %)	21.14	19.48	14.99	26.55	34.76	28.85

loss after a given maximun of a cumulative positive rallying of the returns, signaling how fast the leverage can increase, much smaller than the 65,85% drawdown of the MSCI Index.

The summary statistics for the pairs selected through the distance method are presented on table 5 also. The volatility for the portfolios created through this method is betwenn 3,4% and 5,2% for all 3 portfolios, lower than for the cointegration method portfolios, which range between 4,5% and 8,4%, but the annualized average return are all inferior to the cointegrated pairs portfolios, ranging between 6,3% and 7,8%. The consequence is that the Sharpe Ratios are are increasing with the portfolio's size. The distance method strategy is market neutral with a market beta and a Spearman rho close to zero for all 3 portfolios and the maximum drawdown is mostly lower than the ones that occur in the cointegration method. The share of days with negative returns ranges between 36% and 41% for all porfolios higher than the MSCI Index and the cointegration method.

5. Pairs trading performance evaluation

5.1. *Bootstrap for assessing pairs trading performance*

In order to evaluate the performance of the strategies, we compare it to a naive strategy, i.e., we create bootstrapped return series in which the signal to start the strategy of pairs trading is inserted,and the performance of such a strategy is monitored and compared to the performance of the original series of returns. We follow the method used by Gatev *et al.* (2006), Caldeira and Moura (2013), in which the bootstrap initiates at the time at which the signal is sent to begin trading pairs. In each bootstrap, the original series is replaced by two series of random assets similar to the assets earlier, similarity being defined as returns in the previous month belonging to

Table 5. Excess returns of unrestricted pairs trading strategies: european dataset

Nota: This table reports a summary statistics of the monthly excess returns on portfolios of pairs between January 1973 and December 2012 (468 observations). Pairs are formed over a 12-month period according to a minimum-distance criterion and then traded over the subsequent 6-month period. airs are opened when prices diverge by two standard deviations. CAPM-estimates are from the OLS regression analysis.

Methodology	Distance approach			Cointegration approach		
Pairs Portfolio	Top 5	Top 10	Top 20	Top 5	Top 10	Top 20
Total number of pairs opened	2108	4264	8246	1930	3851	7427
Total number of 6 month trading periods	77	77	77	77	77	77
Average price deviation for opening pairs	0.017	0.019	0.021	0.299	0.341	0.390
Average no of pairs opened each 6 mo period	27.02	54.66	105.71	24.74	49.37	95.21
Average number of pairs traded in months when at least one pair opened	5.475	5.538	5.355	5.039	5.021	4.896
Average number of round-trip trades per pair	5.405	5.467	5.286	4.949	4.937	4.761
Standard deviation of round-trips per pair	3.400	3.527	3.587	2.838	2.777	2.832
Average time pairs are open in days	10.08	9.776	10.24	5.351	5.593	5.907
Median time pairs are open in days	3.00	3.00	3.00	3.00	3.00	3.00
Average time pairs are open in months	0.48	0.466	0.488	0.255	0.266	0.281
Standard deviation of time open per pair in days	20.30	19.22	19.54	5.96	6.22	6.48
Standard deviation of time open per pair in months	0.967	0.915	0.931	0.284	0.297	0.309
Share of negative excess returns	0.364	0.405	0.410	0.262	0.334	0.385
Average Annualized Return (in %)	6.391	7.591	7.814	13.73	12.65	11.19
Average Annualized Volatility (in %)	5.261	4.229	3.414	8.410	6.156	4.59
Total Sample Sharpe Ratio	1.178	1.731	2.204	1.530	1.936	2.300
Largest Daily Return (in %)	4.170	2.340	1.960	6.810	3.880	2.220
Lowest Daily Return (in %)	-5.420	-2.710	-1.890	-7.390	-3.690	-3.150
Cumulative Profit (in %)	1,018.00	1,683.00	1,860.00	14,602.00	10,629.00	6,479.00
Spearman Correlation Rho with MSCI	0.019	0.013	0.013	0.009	0.006	0.029
CAPM beta (β-market)	0.004	0.003	0.002	0.001	0.003	0.004
Annual Skewness	1.586	1.437	1.399	1.220	0.659	0.540
Annual Kurtosis	7.769	6.118	5.256	5.150	2.750	2.320
Maximun Drawdown (in %)	21.54	13.47	30.48	37.95	27.20	28.85

the same decile. Thus, the difference in performance of the original assets and simulated give an indication of performance. The net return of the naive strategy is given by:

$$R_t^{naive} = \sum_{i=1}^{N} w_{it} r_{it} + 2N\, ln\left(\frac{1-C}{1+C}\right) \tag{8}$$

The results were calculated in every 6 month trading period and are withheld due to space constraints and can be obtained by contacting the author. We bootstrap each period 2500 for each of the pairs selection methodology and for each portfolio size, and found that both strategies obtain statisticallly significant positive performance when compared to a naive trader for both countries. In other words, the pairs trading strategies based on the selection of pairs through cointegration and through the distance method have a superior performance when compared to the random selection of pairs of stocks to be traded. The average returns on the random pairs is slightly negative, possibly due to the inclusion of transaction costs, and the standard deviations are large compared to the pairs trading portfolio's standard deviations.

5.2. *Hypothesis testing for the difference between the Sharpe Ratios*

Given that the objective of this paper is to compare the performance of two pairs selection methods, we must use a metric in order to assess if any of the strategies has a superior performance. In order to test the statistical significance of the difference between the Sharpe ratios of both strategies we use the methodology proposed in Ledoit and Wolf (2008) and obtain the p-values of the stationary bootstrap of Politis and Romano (1994) with B = 1000 bootstrap resamples and block length b=

5.

The whole sample result for the difference between Sharpe ratios through the methodology proposed by Ledoit and Wolf (2008) indicates that for Brazil, the cointegration method is superior during the whole sample period to the distance method. However, for the subperiods the results are not as robust, with most subperiods results, available upon request, indicating that the cointegration method does not deliver a statistically significant higher Sharpe Ratio which hints at the fact that some subperiods might be driving the full sample results, or that the performance is slightly superior in each period, but not statistically higher due to sample limitations, since the size of most subperiods sample is 120 compared to the whole period that comprises 4087 observations. For europe the results also indicate that for the whole sample period the cointegration strategy is superior when using a portfolio consisting of 10 and 20 pairs. However for the 5 pairs portfolio the p-value of the statistic calculated is 0.107, and the cointegration strategy cannot be considered superior. Also, for the 6 month subperiods the cointegration strategy in most subsample periods is not superior, with the results most likely being driving by some subperiods. These findings hint that the cointegration strategy may be superior to the distance method in some periods, while the distance method may be superior in other subperiods, but on average the cointegration method delivers a higher Sharpe Ratio. For the USA the results are the other way around. The distance method Sharpe Ratio is statistically superior than on the cointegration method, for all 3 portfolio sizes. However, again as in Brazil and for Europe, the results in the subperiods are mixed, with the distance method not being superior in all 6 month subperiods (tables available upon request).

Table 6. P-value of the test statistic pair wise between the portfolios for the Sharpe Ratio for Brazil, total sample

P-Value	Top 5 pairs	Top 10 pairs	Top 20 pairs
	Robust Sharpe Ratio Difference Test		
All Sample ______	0.000	0.000	0.000

Table 7. P-value of the test statistic pair wise between the portfolios for the Sharpe Ratio for USA, total sample

P-Value	Top 5 pairs	Top 10 pairs	Top 20 pairs
	Robust Sharpe Ratio Difference Test		
All Sample ______	0.000	0.000	0.000

Table 8. P-value of the test statistic pair wise between the portfolios for the Sharpe Ratio for Europe, total sample

P-Value	Top 5 pairs	Top 10 pairs	Top 20 pairs
	Robust Sharpe Ratio Difference Test		
All Sample ______	0.107	0.010	0.001

6. Conclusion

In this paper we compared two methodologies for the strategy called pairs trading. The distance method presented in Gatev *et al.* (2006) and the cointegration method used by Caldeira and Moura (2013), for the american stock market between 1962 and 2012, for the brazilian stock market between 1996 and 2012 and for the european market between 1973 and 2012. We create portfolios comprising 5, 10 and 20 pairs for each method, and bootstrap the results in order to compare the their performance. The pairs were ranked by their in sample sharpe in the cointegration method and by the smallest to the highest SSD for the distance method in order to form the portfolios. The signal to open the position out of sample was given whenever the distance between the stocks on a given pair crossed the 2 standard deviation threshold. Both methodologies had a good performance when compared to a naive trader that randomly selection pairs to trade on a given period. For Brazil, the cointegration method had a cumulative return between 1996 and 2012 of up to 2499%, while the distance method had up to 148% of cumulative return. When compared to each other, the cointegration method had a clear, statistically significant higher average annualized return, with a superior Sharpe Ratio, and, most of the time, a statistically significant inferior volatility. Both strategies can be considered market neutral, with a close to zero spearman correlation with the market.

For Europe, while the results were not so clear cut, they also pointed towards the cointegration method being superior, delivering up to 14.602% of cumulative returns against 1.860% for the distance method. The Sharpe Ratio was also considered superior for the whole sample period, although in some subsamples both strategies were very similiar. Both strategies had an excess returns superior than a naive trader. For the United States, the results were very different, indicating that the distance method is superior, delivering up to 13.331% of cumulative return, more than the 1.047% of the cointegration method. Considering that this strategy is self-financed, since the cash obtained by shortening a stock is used to buy the long stock in the pair, these results are encouraging and indicate a clear path for more research regarding the drivers of such difference in performance, the optimality of the trading thresholds and the stability of the cointegration parameters.

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