A Tale of Three Gaps: Unemployment, Capacity Utilization and Output*

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Abstract

The paper assesses the relationship between unemployment, industrial capacity utilization and inflation in Brazil by means of disaggregated Phillips curves. The key aspect of the analysis is to consider separately the dynamics of tradable and nontradable sectors, and emphasize the importance of employment and capacity utilization to explain disaggregated inflation dynamics. Using quarterly Brazilian data for 1999Q2-2012Q4, we estimate the NAIRU, NAICU and the output gap incorporating economic structure in the Kalman filter. The results suggest that the unemployment gap is the relevant demand variable to explain inflation of nontradable goods, while the capacity utilization gap is important for inflation of tradable goods. There is evidence of substantial reduction in the NAIRU in recent years, and that it has been above the unemployment rate since mid-2010 – its value was close to 6.3% by the end of 2012. The results also suggest a dichotomy in the Brazilian economy – while the industrial sector shows poor performance and difficulties to react, the labor market is heated, generating pressures on the output gap. Our study also emphasizes possible biases produced by aggregate estimations in a dichotomous environment.

Keywords: NAIRU, NAICU, Industrial Capacity Utilization, Unemployment, Output Gap, Monetary Policy.

JEL Classification: E3, E32, E52, J01, J64

Área 4 - Macroeconomia, Economia Monetária e Finanças

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"It was the best of times, it was the worst of times, it was the age of wisdom, it was the age of foolishness, it was the epoch of belief, it was the epoch of incredulity, it was the season of Light, it was the season of Darkness, it was the spring of hope, it was the winter of despair, we had everything before us, we had nothing before us, we were all going direct to Heaven, we were all going direct the other way".

Charles Dickens, A Tale of Two Cities

1 Introduction

Brazil has been experiencing an odd economic instance since 2005. Employment seems to be skyrocketing and almost immune to many shocks that hit the Brazilian economy during the period, including the second great recession of 2007-2009. Over the sample, employment seems not to be correlated at all with production or capacity utilization. Since 2010, in particular, the correlation between employment and production seems to be negative. Indeed, capacity utilization and GDP growth has been decreasing, whereas employment has remained in its steadily increasing path. Stylized facts are shown in Section 2. On top of that, economists have been facing a hard time to find a positive correlation between employment and inflation, which should be a trivial task in most countries. Indeed, some economists have reported difficulties in estimating significant coefficients for the employment gap (or unemployment) in empirical aggregate Phillips curves (e.g. Delfim Netto (2013), Mendonca et al. (2012) and Minella et al. (2003)).

Hence, a puzzle arises in reconciling this dichotomous behavior between employment, capacity utilization and production, and explaining the lack of correlations between employment/production and employment/inflation.

In order to put some light into solving this puzzle, we look into disaggregate measures to find that strong idiosyncrasies in two important production sectors, with opposite directions, are at play. In particular, we explore the relation between employment, capacity utilization, output, and inflation in the sector of non-traded goods, whose production is intensive in labor, and the (industrial) sector of traded goods, whose production is intensive in capital.

We do not seek to explain the mechanisms behind this dichotomy. We take for granted the interpretation called "two blades of a scissor", coined by Pastore et al. (2012) and Pastore (2012). Their analysis is based on a model with two sectors for the Brazilian economy. The explanation for the slowdown in the industry is that labor market pressures on wages caused by the dynamism of the service sector, coupled with the drop in productivity, increases the unit cost of labor. The industrial sector, assumed to be nearly price taker, is unable to pass on increased costs to prices and have reduced profit margins.

We rather explore its consequences. We use an empirical semi-structural approach to model Phillips curves in both sectors, and infer the non-accelerating inflation rate of unemployment (NAIRU) and non-accelerating inflation rate of capacity utilization (NAICU) by means of Kalman filtering. In this dichotomous context, it is difficult to define what are relevant demand variables to be used in empirical aggregate Phillips curves. Current consensus is that Phillips curves should be total (theoretical) or partially (semi-structural) funded in micro-economic theory. One of the strong assumptions behind most theoretical models is that all sectors are homogeneous in the use of production factors. Under weaker assumptions, however, one cannot theoretically justify functional forms in which only the aggregate output gap or unemployment gap affects aggregate inflation.

1 Phillips curves are refinements of contemporary empirical relationship shown in Phillips (1958), where the change rate of nominal wages has a negative correlation with the unemployment rate.

2 For the derivation of the simplified functional form, in which the aggregate inflation rate is affected by a single gap measure, it is necessary to take several strong assumptions, e.g. inflation rates of all sectors have the same inertial behavior, productivity shocks are the same in all sectors, there is no wage rigidity, and there are no frictions in the labor market. For example, by assuming specific capital, Woodford (2005) shows that the Phillips curve should have both the output gap and the investment gap. In order to include a more realistic labor market, by embedding search
Nevertheless, the popularity of theoretical models with homogeneous firms contributed to the use of only the output gap in most empirical exercises\textsuperscript{3}, even in Brazil (e.g. Bogdanski et al. (2000), Alves and Muinhos (2003), Tombini and Alves (2006) and Correa and Minella (2010)).\textsuperscript{4} In Brazil, the use of employment in empirical Phillips curves is still uncommon.\textsuperscript{5} However, considering separately how labor and capital affect inflation seems crucial to understanding the transmission mechanisms of monetary policy and responses to shocks, in a context characterized by a weak performance in the industrial sector couple with large employment rates.

We find that the correlation puzzle only arises on aggregate measures. Individually looking at both sectors, we find that employment is an important variable to explain inflation in the sector of non-traded goods, while it is the inflation rate of traded goods that is highly correlated with output and capacity utilization.

We also find that the NAIRU has systematically decreased in Brazil, but not as much as the actual unemployment rate. While the its central path was close to 11-12% by 2002, it has decreased to around 6.3% in late 2012. Additionally, our results also suggest that the unemployment rate has been below the NAIRU since mid-2010. In the industrial sector, our results suggest an increase in NAICU over the sample. By the end of 2012, it has been above the actual industrial capacity utilization rate. We highlight, however, that our central estimates of NAIRU and NAICU carry a high level of uncertainty, as in any empirical exercise of this nature.

Finally, the estimated three gaps (unemployment gap, capacity utilization gap and the output gap) highlight the role of the labor market as a source of pressure on economic activity and inflation, and emphasize the dichotomy experienced by the Brazilian economy. We infer two sources of pressure acting in opposite directions on the output gap in the last two years. On the one hand, our results suggest that the industrial sector is more sluggish (negative capacity utilization gap) in recent years than what a simple HP filtering suggests, pressing the economic activity down. On the other hand, the labor market is stronger (negative unemployment gap) than what a simple HP filtering suggests, pressing the economic activity up.

In this paper, we also find evidence that the use of HP filtered gaps might have contributed to the correlation puzzle. Indeed, as different productive sectors use factors in different intensities, the dynamics of sectorial inflation rates might behave very differently in response to shocks. Therefore, using a single Phillips curve to explain the aggregate inflation rate as a function of the aggregate output gap, or the unemployment gap, generates specification and/or omitted variable bias. Moreover, gap variables obtained by HP filtering (Hodrick and Prescott (1997)), or other filtering method that does not embed economic structure, might have serious measurement error problems. And it is a known in the econometric literature that the use of covariates with measurement error causes attenuation bias toward zero.

The rest of the paper is organized as follows. Section 2 presents stylized facts. Section 3 presents the empirical semi-structural model. Section 4 presents the estimation results, the estimated NAIRU and NAICU paths, and the three estimated gaps. Section 5 concludes.

## 2 Stylized facts

The unemployment rate, $U_t$, as measured by the Brazilian Institute of Geography and Statistics (IBGE), has been decreasing since mid-2003, as shown in panel (A) of Figure 1. The reduction in frictions, the Phillips curve includes the unemployment gap in addition to the aggregate output gap (e.g. Alves (2012), Blanchard and Gali (2010), Christiano et al. (2011), Gali (2010), Gertler et al. (2008), Gertler and Trigari (2009), Ravenna and Walsh (2008, 2012), Thomas (2008, 2011) and Walsh (2005)).

\textsuperscript{3}Important references are found in Cogley and Sbordone (2008), Coibion et al. (2012), Coibion and Gorodnichenko (2011), Linde (2005), Rabanal and Rubio-Ramirez (2005), Rudd and Whelan (2005) and Smets and Wouters (2003, 2005, 2007), among others.

\textsuperscript{4}In Brazil, estimates of Phillips curves with measures of marginal cost are rare (e.g. Alves and Areosa (2005), and Areosa and Medeiros (2007)).

\textsuperscript{5}Good references are found in Mendonca et al. (2012) and Minella et al. (2003).
unemployment also seems to have been almost acyclical and immune to many shocks that hit the Brazilian economy in the period. On the other hand, the industrial capacity utilization rate, $CU_t$, as measured by the National Confederation of Industry (CNI), has been quite volatile around an almost constant average.

![Figure 1](image)

**Figure 1:** Unemployment, capacity utilization, GDP, industrial production and inflation rates

**Nota:** Panel (A): Industrial capacity utilization rate (red circles), Unemployment rate and its linear trend (blue).
Panel (B): Industrial production growth rate (YoY) (red circles), GDP growth rate (YoY) (blue), first difference of unemployment rate (black stars).
Panel (C): Traded goods 12-month inflation rate (red circles), Non-traded goods 12-month inflation rate (blue).

Panel (B) shows that year-over-year changes in the unemployment rate remain negative in most part of the period and almost acyclical, while GDP$^6$ and industrial production (general industry)$^7$ growth rates vary widely. In particular, the large decline in the Brazilian economic growth during the 2007-2009 financial crisis was accompanied only by a slight increase in unemployment. There was also no increase in unemployment in 2011-2012, when GDP and industrial production growth have slowed down. The industrial capacity utilization rate, on the other hand, has strongly decreased during the period.

This evidence suggests a dichotomy in the Brazilian economy in recent years: low and declining unemployment rates coexist with low GDP and industrial production growth rates. In fact, after growing 7.5% in 2010, Brazil’s GDP slowed to 2.7% in 2011 and 0.9% in 2012, while industrial production grew only 0.3% in 2011 and decreased 2.7% in 2012. On the other hand, the unemployment rate, which was around 6.7% in the 2010 average, declined to 6.0% and 5.5% in 2011 and 2012, respectively, and reached 4.6% in December 2012, the lowest level of the series so far.

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$^6$Measured and released by IBGE.
$^7$Measured and released by IBGE.
Panel (C) shows the different paths of traded and non-traded goods 12-month inflation rates. While prices of traded goods increased 4.4% in 2011 and 4.5% in 2012, prices of non-tradables rose 8.6% and 8.5%. The production of the first class of goods is associated with the industrial sector, which is intensive in capital, while the latter is associated with the non-industrial sector, which is intensive in labor. Therefore, pressures coming from the labor market are particularly important for the nontraded goods sector, especially the service sector, where payroll represents a significant portion of total production costs.\(^8\) The sector of traded goods is more exposed to competition from imported products, which limits their ability to adjust prices. The distinct inflation dynamics of both sectors are a direct consequence of the sectoral dichotomy and suggests that it is not possible to characterize the economy as a whole using a model with a single aggregate Phillips curve.

3 The model\(^9\)

For simplicity, we assume that the inflation rates for traded and non-traded goods are good proxies for the inflation rates of the industrial and non-industrial sectors. To avoid collinearity problems between covariates, we only consider the most intensive production factor in each sector. Thus, the relevant demand variable for the Phillips curve of non-traded goods is the unemployment gap with respect to NAIRU, while the relevant demand variable for the Phillips curve of traded goods is the capacity utilization gap with respect to NAICU. Both Phillips curves are jointly estimated by full-information maximum likelihood, while the estimates of NAIRU and NAICU are obtained by Kalman filtering. We acknowledge the fact that no filtering method is free of problems (Canova (1998) and Canova and Ferroni (2011)). However, we assume that filters containing greater economic structure have more chances to extract the correct information.

In one approach we consider in this paper we use auxiliary measures obtained by the HP filter as initial paths for the estimation of NAIRU and NAICU. The Kalman filter is used to fine-tune those initial paths. This strategy helps the convergence of the Kalman filter.

We consider a dynamic linear model (DLM), in which the trajectories of the latent variables NAIRU and NAICU are described as random walks with stochastic drifts.\(^10\) In the best specification, the model has the following form:

\[
\begin{align*}
\pi^nt_t &= \lambda_1 \pi^{tr}_{t-1} + \lambda_2 E_t \pi^{nt}_{t+1} + \lambda_3 \pi^*_t + \lambda_4 u_t + \beta' X_{t-j} + \xi^nt_t \\
\lambda_1 + \lambda_2 + \lambda_3 &= 1; \xi^nt_t \sim N(0, \sigma^2_{nt}) \\
\pi^{tr}_t &= \gamma_1\pi^{tr}_{t-1} + \gamma_2 E_t \pi^{tr}_{t+1} + \gamma_3 \pi^*_t + \gamma_4 \hat{c}_{t-2} + \theta' Z_{t-1} + \xi^{tr}_t \\
\gamma_1 + \gamma_2 + \gamma_3 &= 1; \xi^{tr}_t \sim N(0, \sigma^2_{tr}) \\
\pi^{nt}_t &= \omega^{nt}_t (\pi^nt_t - \xi^{nt}_t) + (1 - \omega^{nt}_t) (\pi^nt_{t-1} - \xi^{nt}_{t-1}) + \xi^{nt}_t \\
\xi^{nt}_t &\sim N(0, \sigma^2_{nt})
\end{align*}
\]

The Phillips curve for the inflation rate of non-traded goods, \(\pi^nt_t\), is described in equation (1), where \(E_t (\cdot)\) is the expectation operator conditional on the information set available at period \(t\), \(u_t\) is the unemployment gap (defined below) and \(X_t\) collects zero-meaned proxies for supply shocks.

\(^8\) Of course, wage pressures generated by a booming labor market end up affecting the costs in the tradable sector as well.

\(^9\) The model we present and estimate in this paper is meant for academic purposes only. It is not meant for forecasting and monetary policy implementation by the Banco Central do Brasil.

\(^10\) Great references on dynamic linear models (DLM) and inference using the Kalman filter are Hamilton (1994), Prado and West (2010) and West and Harrison (1997). For DLMs applications in macroeconomic models, see Basdevant (2003).
Equation (2) is a Phillips curve for the inflation rate of traded goods, \( \pi_t^{fr} \); \( \hat{c}_t \) is the capacity utilization gap (defined below) and \( Z_t \) collects zero-meaned proxies for supply shocks (\( X_t \) and \( Z_t \) may have non-empty intersection). Equation (3) imposes consistency of sectorial inflation rates with the aggregate free-market inflation rate, \( \pi_t^{fr} \), where \( \omega_t^{fr} \) is the time-varying weight of the inflation rate of traded goods and \( \xi_t^{fr} \) is a modelling error term. Moreover, \( \pi_t^{\ast} = (\Delta e_t + \pi_t^I) \) is the inflation rate of imported goods, in domestic currency prices, measured by the (log) variation of the nominal exchange rate \( \Delta e_t \), added to the external inflation rate \( \pi_t^I \); \( \xi_t^{ut} \) and \( \xi_t^{fr} \) are error terms, and \([\lambda_1, \lambda_2, \lambda_3, \lambda_4, \gamma_1, \gamma_2, \gamma_3, \gamma_4, \beta, \theta, \sigma_{\text{com}}^2, \sigma_{\text{comp}}^2, \sigma_{\text{icov}}^2, \sigma_{\text{d}}^2\) is the parameter vector to be estimated. We impose verticality restrictions on the coefficients of the Phillips curves. Finally, \( u_t \equiv -\log (1 - U_t) \) and \( c_t \equiv \log (UCI_t) \) are logarithmic transformations of the unemployment rate, \( U_t \), and the capacity utilization rate, \( UCI_t \).

Equation (3) deserves some comments. Note that the terms in parentheses are the explained component of the other two equations. The sum of those values, weighted by their respective time-varying weights, equates the explained component of the aggregate free-market inflation rate. If the weights \( \omega_t^{com} \) were constant over time, this equation would not add any information to the system. Moreover, the variance-covariance matrix of the error terms would be singular and the joint estimation of the three equations would be impossible. It is the fact that the weights are time-varying that allows us to jointly estimate the system.

Now we describe the way we compute the unemployment gap, \( \hat{u}_t \), and the capacity utilization gap, \( \hat{c}_t \). In this work we use two different strategies and compare their results. The first is to adopt the standard approach for estimating latent variables. We will call this strategy as model \( M_S \) (standard approach). In this approach, we define the unemployment gap as the difference between the unemployment rate and the NAIRU, and directly model the NAIRU as a state variable. The same procedure is used for the capacity utilization gap and the NAICU. Thus, in model \( M_S \) we jointly estimate the system (1)-(3) with the following state variables:

\[
(M_S) \quad u^n_t = u^n_{t-1} + u^{dr}_{t-1} \quad ; \quad c^n_t = c^n_{t-1} + c^{dr}_{t-1} \\
\quad u^{dr}_t = u^{dr}_{t-1} + \xi^{ud}_t \quad ; \quad c^{dr}_t = c^{dr}_{t-1} + \xi^{cd}_t \\
\quad \xi^{ud}_t \sim N(0, \sigma_d^2) \quad ; \quad \xi^{cd}_t \sim N(0, \sigma_d^2)
\]

(4)

where \( u^n_t \equiv -\log (1 - \text{NAIRU}_t) \) and \( c^n_t \equiv \log (\text{NAICU}_t) \) are logarithmic transformations of NAIRU and NAICU, and the gap variables are defined as \( \hat{u}_t \equiv u_t - u^n_t \) and \( \hat{c}_t \equiv c_t - c^n_t \).

In the second approach, we use series obtained by purely statistical filters as initial paths for the NAIRU and NAICU. The Kalman filter fine tunes over those initial guesses. For simplicity, we chose the HP filter as the auxiliary filter. We will call this procedure as model \( M_A \) (auxiliary variables). To see how the procedure works, we write the NAIRU and NAICU as:

\[
(M_A) \quad u^{hp}_t = u^{hp}_t + \text{cor}^u_t \quad ; \quad c^{hp}_t = c^{hp}_t + \text{cor}^c_t
\]

(5)

where \( u^{hp}_t \equiv -\log (1 - \text{NAIRU}^{hp}_t) \) and \( c^{hp}_t \equiv \log (\text{NAICU}^{hp}_t) \) are estimates obtained by HP filtering in a first step, and \( \text{cor}^u_t \) and \( \text{cor}^c_t \) represent corrections obtained by the Kalman filter. Thus, in model \( M_A \), we jointly estimate system (1)-(3) with the following state variables:

\[
(M_A) \quad \text{cor}^u_t = \text{cor}^u_{t-1} + \text{cor}^{ud}_{t-1} \quad ; \quad \text{cor}^c_t = \text{cor}^c_{t-1} + \text{cor}^{cd}_{t-1} \\
\quad \text{cor}^{ud}_{t} = \text{cor}^{ud}_{t-1} + \eta^u_{t-1} \quad ; \quad \text{cor}^{cd}_{t} = \text{cor}^{cd}_{t-1} + \eta^c_{t-1} \\
\quad \eta^u_{t} \sim N(0, \sigma_d^2) \quad ; \quad \eta^c_{t} \sim N(0, \sigma_d^2)
\]

(6)

where \( \text{cor}^u_t = u^n_t - u^{hp}_t \) and \( \text{cor}^c_t = c^n_t - c^{hp}_t \). In order to simplify the estimation process of model \( M_A \), we impose \( \sigma_d^2 = \sigma_{d}^2 \), where the latter was estimated in model \( M_T \).

There are at least one apparent advantages in using model \( M_A \) over \( M_S \). The corrections are stationary. Thus, it will be easier for the Kalman filter to infer them and fine tune the auxiliary
paths of NAIRU and NAICU.

Note that random walks with stochastic drift specifications are able to capture the vast majority of stationary and non-stationary processes in finite samples. Thus, the model allows the latent variables to have stationary patterns in some parts of the sample and non-stationary patterns in others.

Both methods use a recursive procedure, consisting of two steps, which can improve identification. In the first step, we use the central values of the smoothed NAIRU and NAICU latent variables, obtained in the previous step, to estimate the model parameters. In the second step, we fix the parameters of the unemployment and capacity utilization gap obtained in the previous step, and infer the dynamic distributions of the NAIRU and NAICU. To start the recursive process, we use the auxiliary measurements obtained by the HP filter. The two steps are repeated until the system parameters satisfy a convergence criterion.

4 Estimation

We estimate the system using full-information maximum likelihood, while the estimates of NAIRU and NAICU are obtained by Kalman filtering. The estimation is carried out using seasonally adjusted quarterly data in the period 1999Q2–2012Q4. The measures of inflation for free prices, traded and non-traded goods are from the Broad National Consumer Price Index (IPCA), released by IBGE. Time-varying weights come from the correspondents IPCA basket. We have used slightly different measures of inflation rates for traded and non-traded goods, as described in Banco Central do Brasil (2011). The method incorporates the new structure of consumption patterns, according to the IBGE’s Household Budget Survey (POF) 2008-2009. The measure of foreign inflation is given by the variation of the Commodity Research Bureau (CRB) Index.

The capacity utilization variable is released by CNI. The measure of unemployment adopted for most of the period is the rate of open unemployment, with a reference period of 30 days from the IBGE’s Monthly Employment Survey (PME). IBGE conducted important methodological changes in the calculation of unemployment in 2002 to conform its measurement to international standards, which means that this information is available only for the period from March 2002 on. To obtain a longer data series, the unemployment data of IBGE were combined with the series of aggregate unemployment measured by the Survey of Employment and Unemployment (PED), released by DIEESE/Fundação SEADE-SP (from April 1999 to February 2002). This series measures the unemployment rates in the metropolitan regions of Belo Horizonte, Fortaleza, Porto Alegre, Recife, Salvador, São Paulo and Distrito Federal. Due to this change in the unemployment series, we also performed an estimation using data from IBGE only, i.e., constraining the sample to 2002Q2–2012Q4.

There are also additional variables proxying supply shocks in the Phillips curves, represented by the vectors $X_t$ and $Z_t$. Several variables were tried as controls for these shocks, such as changes in relative prices, in commodity prices, in oil prices, changes in the minimum wage etc. Of those variables, only two were significant in the equation for inflation of traded goods: the variable $de_t$, that captures the misalignment of prices at wholesale and retail (measured by the difference between the (log) Wholesale Price Index – IPA-FGV and the (log) Consumer Price Index – CPI-FGV); and $shtr_t$, capturing shocks to commodity prices, as measured by the CRB index (measured in Brazilian currency) gap from its HP trend.

The presence of inflation expectations terms, $E_t\pi_{t+1}^{nr}$ and $E_t\pi_{t+1}^{tr}$, in the Phillips curves cause an endogeneity problem that needs to be addressed. Thus, we use a two-stage procedure to estimate

\[\tilde{u}_t = E\left(u_t\mid \{Y_t\}_T^{T=1}\right)\] and

\[\tilde{c}_t = E\left(c_t\mid \{Y_t\}_T^{T=1}\right),\] where $Y_t$ is the vector of observable variables (endogenous and exogenous) in period $t \in \{1, T\}$.

11 Smoother values consider the whole information set, i.e., central values are obtained by $\tilde{u}_t = E\left(u_t\mid \{Y_t\}_T^{T=1}\right)$ and $\tilde{c}_t = E\left(c_t\mid \{Y_t\}_T^{T=1}\right)$, where $Y_t$ is the vector of observable variables (endogenous and exogenous) in period $t \in \{1, T\}$.

12 We also performed estimations using data from Fundação Getulio Vargas – FGV. The results did not change qualitatively.
the model. The first stage involved regressing the actual values of inflation of tradable and non-traded goods in the period \( t + 1 \) on instrumental variables. Then, using the fitted components as the expectations variables in the second stage. In this second stage, all the equations of the model are jointly estimated. The instruments used for \( E_t \pi_{tr}^{nr1} \) were \( (1/2) \sum_{j=1}^{2} \pi_{t-j}^{nr1}, \pi_{t-1}^{tr} \) and \( d_{t-1} \). The instruments for \( E_t \pi_{hp}^{tr1} \) were \( \pi_{t-1}^{nr1}, \pi_{t-1}^{tr}, \pi_{t-1}^{ipca} \) and \( d_{t-1} \).

The two specifications of the model were estimated. In model \( M_S \), NAIRU and NAICU are estimated directly as the latent variables in the Kalman filter. Model \( M_A \) uses the recursive procedure described above, with initial auxiliary variables for the NAIRU and NAICU obtained by HP filtering. The state variables in this model are the deviations of the initial trajectories. The estimation results of the two models are reported in Table 1. We also present the results of the first step of the recursive estimation of model \( M_A \), i.e., the step in which the gaps are simply those obtained by HP filtering: \( \hat{u}_{hp}^t \equiv u_t - \hat{u}_{hp}^t \) and \( \hat{c}_{hp}^t \equiv c_t - \hat{c}_{hp}^t \). We call this specification as model \( M_0 \).

The first result that worth mentioning concerns the unemployment gap in the Phillips curve of non-traded goods. Not only the coefficients are very significant in models \( M_A \) and \( M_S \), as their magnitudes are large (\(-0.31 \) in both models). This result suggests that the labor market plays an important role in inflation dynamics: reductions in unemployment below the NAIRU directly affect the inflation rate of non-traded goods and, consequently, the aggregate inflation rate.

Some economists have reported difficulties in obtaining significant parameters for the unemployment gap when estimating aggregate Phillips curves, which could suggest a weak (or nonexistent) relationship between inflation and unemployment in the short run (e.g. Mendonça et al. (2012), Delfim Netto (2013) and Minella et al. (2003)). Our results suggest that these difficulties may arise from the approach used by the authors to capture this relationship. This evidence is reinforced by model \( M_0 \), which suggests that simply using the HP-filtered unemployment gap, without considering any economic structure such as the ones we impose in the Kalman filter in this paper, seems not to be a good strategy. Indeed, the coefficient estimated in model \( M_0 \) is much smaller (\(-0.17 \)) than those obtained in models \( M_A \) and \( M_S \) and is not statistically significant (P-value equal to 0.21). This fact is a strong evidence that the HP-filtered unemployment gap has measurement errors. It is a well-known result in the econometric literature that an attenuation bias towards zero arises when a regressor has measurement errors (e.g. Wooldridge (2010, cap. 4)).

The estimated coefficients for the capacity utilization gap in models \( M_A \) and \( M_S \) (both equal to 0.15) are also very significant, indicating that this variable plays as well an important role in inflation dynamics. Note, however, that if we have considered only the HP-filtered gap (model \( M_0 \)) the estimated coefficient would be significant only at the 10\% level. The central estimate would be about the same as the ones obtained in models \( M_A \) and \( M_S \). This suggests that the dynamics obtained by Kalman filtering is more consistent with inflation dynamics, even though both the Kalman filter and HP-filtered capacity utilization gaps have similar average volatilities.

Regarding for the other estimated coefficients in models \( M_A \) and \( M_S \), most are statistically significant and have theoretically expected signs. Commodity shocks and changes in relative prices seem to be important for inflation dynamics. Also worth noting are the magnitudes of the pass-through coefficients, from international to domestic prices, in the Phillips curves of traded and non-traded goods. While the short-run coefficient is estimated at around 7\% for traded goods, it is only about 2\% for non-traded goods.

Finally, the estimated coefficient on inflation inertia of non-traded goods (\( \lambda_1 = 0.36 \)) is smaller than that of inflation of traded goods (\( \gamma_1 = 0.45 \)). This is not an expected result for Brazil due

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13 The estimation for 2002Q2-2012Q4 is reported in Table 2 in the appendix. In general, the results are pretty similar.

14 This result complements those found in Banco Central do Brasil (2013), although in that study the impact of the labor market on inflation is captured directly through wages.

15 Mendonça et al. (2012) use both the unemployment gap, obtained by HP filtering, and the actual unemployment rate in various specifications of the Phillips curve. Their results suggest that, although very small, the relationship between inflation and unemployment in Brazil exists in the short run.
to the existence of indexation rules for readjusting part of wages in the sector of non-traded goods. However, a Wald test is not able to reject the null of $\lambda_1 \geq \gamma_1$.

### Table 1: Estimated Parameters

<table>
<thead>
<tr>
<th></th>
<th>$M_S$</th>
<th>$M_0$</th>
<th>$M_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_{t-1}^m$</td>
<td>0.367***</td>
<td>0.358***</td>
<td>0.357***</td>
</tr>
<tr>
<td></td>
<td>(0.065)</td>
<td>(0.079)</td>
<td>(0.066)</td>
</tr>
<tr>
<td>$E_t \pi_{t+1}^m$</td>
<td>0.609***</td>
<td>0.618***</td>
<td>0.619***</td>
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<tr>
<td></td>
<td>(0.063)</td>
<td>(0.081)</td>
<td>(0.064)</td>
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<tr>
<td>$\pi_{t-1}^v$</td>
<td>0.024**</td>
<td>0.024***</td>
<td>0.024**</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.014)</td>
<td>(0.012)</td>
</tr>
<tr>
<td>$\bar{u}_t^p$ or $\bar{u}_t$</td>
<td>-0.306***</td>
<td>-0.166***</td>
<td>-0.314***</td>
</tr>
<tr>
<td></td>
<td>(0.067)</td>
<td>(0.131)</td>
<td>(0.066)</td>
</tr>
<tr>
<td>$d e_t$</td>
<td>0.380***</td>
<td>0.373***</td>
<td>0.371***</td>
</tr>
<tr>
<td></td>
<td>(0.124)</td>
<td>(0.140)</td>
<td>(0.127)</td>
</tr>
</tbody>
</table>

### Phillips Curve: Traded Goods

<table>
<thead>
<tr>
<th></th>
<th>$M_S$</th>
<th>$M_0$</th>
<th>$M_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_{t-1}^m$</td>
<td>0.445***</td>
<td>0.436***</td>
<td>0.446***</td>
</tr>
<tr>
<td></td>
<td>(0.121)</td>
<td>(0.126)</td>
<td>(0.121)</td>
</tr>
<tr>
<td>$E_t \pi_{t+1}^m$</td>
<td>0.485***</td>
<td>0.493***</td>
<td>0.484***</td>
</tr>
<tr>
<td></td>
<td>(0.118)</td>
<td>(0.123)</td>
<td>(0.117)</td>
</tr>
<tr>
<td>$\pi_{t-1}^v$</td>
<td>0.070***</td>
<td>0.072***</td>
<td>0.070***</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td>(0.017)</td>
<td>(0.016)</td>
</tr>
<tr>
<td>$c_{t-2}^p$ or $c_{t-2}$</td>
<td>0.145***</td>
<td>0.148***</td>
<td>0.145***</td>
</tr>
<tr>
<td></td>
<td>(0.049)</td>
<td>(0.087)</td>
<td>(0.049)</td>
</tr>
<tr>
<td>$d e_t$</td>
<td>1.154***</td>
<td>1.155***</td>
<td>1.156***</td>
</tr>
<tr>
<td></td>
<td>(0.240)</td>
<td>(0.250)</td>
<td>(0.242)</td>
</tr>
<tr>
<td>$s h t_{t-1}$</td>
<td>0.069***</td>
<td>0.069***</td>
<td>0.068***</td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.022)</td>
<td>(0.017)</td>
</tr>
</tbody>
</table>

### Log-likelihood Step 1

- Log-likelihood: 597.850
- Sample: 1999Q3 – 2012Q4

### Log-likelihood Step 2

- Log-likelihood: 535.666
- Parenthesis: st. dev.; Signif.: * (10%), ** (5%), *** (1%)

Note: Sample: 1999Q3 – 2012Q4

Parenthesis: st. dev.; Signif.: * (10%), ** (5%), *** (1%)

$M_0$: step 1, iteration 1, HP\textsubscript{1600} filtering with initial values for NAIRU and NAICU, no corrections

$M_A$: corrections for NAIRU and NAICU inferred by Kalman filtering using auxiliary series (HP\textsubscript{1600})

$M_S$: NAIRU and NAICU inferred by Kalman filtering

### 4.1 Comparing the models

The evidence shown in the previous section suggests that HP-filtered gaps may have measurement errors, which generate difficulties in estimating the relationship between unemployment, capacity utilization and inflation. Therefore, we expect model $M_0$ to be inferior to the other two. In fact, its log-likelihood is much smaller than the log-likelihood of models $M_A$ and $M_S$.\(^{16}\) Indeed, differences

---

\(^{16}\)A more precise analysis requires the use of marginal log-likelihoods. However, the fact that the confidence intervals of the coefficient on the unemployment gap are much narrower in models $M_A$ and $M_S$ than in model $M_0$ suggests that this criterion would provide the same conclusion.
greater than \( \log(100) = 4.60 \) are very large and can be interpreted as decisive evidence against model \( M_0 \) as opposed to \( M_A \) or \( M_S \).\(^{17}\)

The difference between \( M_A \) and \( M_S \) is more tenuous, since their log-likelihood values are quite close. Using Kass and Raftery (1995) criterion, model \( M_A \) has a slightly advantage over \( M_S \). Moreover, convergence of the iterative process is faster when using model \( M_A \). Therefore, we choose this specification as our benchmark model.

### 4.2 Non-accelerating inflation rates

Now we present our estimates of NAIRU and NAICU using the benchmark model \( M_A \).\(^{18}\) Panels (A) and (B) of Figure 2 show the NAIRU and NAICU (smoothed series) for 2001Q1 to 2012Q4,\(^{19}\) the unemployment and capacity utilization rates, and their HP-filtered series. Panels (C) and (D) show 95% confidence intervals (smoothed series).

The results suggest the NAIRU has substantially decreased in Brazil over the last years. The estimates indicate that while it was close to 11–12% at the beginning of the sample, it has decreased to something close to 6.3% in late 2012.\(^{20}\) Our central estimates suggest that the unemployment rate has been below the NAIRU since mid-2010. As in any inference of latent variables, however, our central estimates have a high degree of uncertainty (as suggested by the confidence interval shown in Panel (C)) and should be interpreted with care.

Our estimates also suggest that the actual capacity utilization rate has been well below the NAICU in recent years, despite the fact that both the level of capacity utilization and the NAICU have overall increasing trends in the sample. A similar remark about the uncertainty in the estimating the NAICU (as shown in panel (D)) holds here.

da Silva Filho (2008), using data from 1996Q2–2006Q4, finds evidence of a constant NAIRU (ranging at 7.4–8.5%), but highlights the fact that his estimates are quite sensitive to including or not proxies to supply shocks in the Phillips curve. Note that our estimates are consistent with da Silva Filho (2008) in the 2001Q1–2006Q4 period, for our central estimates show relatively little variation given the confidence intervals: it starts at about 11% by 2001, slightly increases to 12% in mid-2003, and returns to about 10% in late 2006. Indeed, we can not reject the null that the NAIRU has effectively remained constant until the end of 2006. However, when considering the whole sample, the evidence suggests that the NAIRU has significantly decreased in Brazil. In a recent extension (covering March 2002 to March 2011), da Silva Filho (2012) estimates a larger, but still constant, level for the NAIRU (9.6%). This result, however, is difficult to be reconciled with our estimates.

There are important differences between our estimated paths for the NAIRU and NAICU and those obtained using HP filtering. Our NAIRU estimates and the HP trend are quite similar over 2003 to 2008. Nevertheless, our beginning-of-sample estimates are smaller than HP ones (our NAIRU estimates oscillate around 11.3%, whereas the HP trend oscillates around 12% before 2002Q4), and the opposite happens at the sample end (our NAIRU estimates oscillate around 6.7%, whereas the HP trend oscillates around 5.8% from 2011Q1 to 2012Q4). In the NAICU case, we also observe important end-of-sample differences. From 2011Q1 to 2012Q4, our NAICU estimates average 84.0%, whereas the HP trend oscillates around 82.5%.

\(^{17}\)This decision rule is described in Kass and Raftery (1995) and is based on Jeffreys (1961) suggestions.

\(^{18}\)The estimates of NAIRU and NAICU using model \( M_S \), as well as an alternative estimation of model \( M_A \) using the sub-sample 2002Q2-2012Q4, are depicted in Figures 4 e 5 in the appendix. The results, however, seem no to change much.

\(^{19}\)The model was estimated using data from the 1999Q3 to 2012Q4. However, we discard the initial values because inferring the states in the initial periods are subject to errors of the initial distribution used by the Kalman filter.

\(^{20}\)There are few studies with updated data on NAIRU in Brazil. The two most recent estimates are found in da Silva Filho (2008, 2012). Older analyses are found in e.g. Portugal and Madalozzo (2000) and Lima (2003).
Figure 2: NAIRU and NAICU, benchmark model

Note: Panel (A): Unemployment rate (blue), NAIRU (black circles), HP trend (red stars).
Panel (B): Capacity utilization (blue), NAICU (black circles), HP trend (red stars).
Panel (C): Unemployment rate (blue), NAIRU (black circles), 95% confidence interval (black).
Panel (D): Capacity utilization (blue), NAICU (black circles), 95% confidence interval (black).

There is an important message from those results. In line with recently observed larger inflation rates for non-traded goods, our estimated NAIRU indicates that the labor market is tighter in recent years than what is suggested by HP filtering. On the other hand, also consistent with smaller inflation rates for traded goods, our estimated NAICU suggests that the industrial sector has been operating in a more sluggish way than what is suggested by HP filtering.21

4.3 The three gaps

In this section, we describes how to retrieve an aggregate measure of output gap using our estimates for the NAIRU and NAICU. We assume that GDP $Y_t$ is produced according to a Cobb-Douglas technology, $Y_t = A_t \left(K_t C_t\right)^{1-\alpha} \left[L_t (1 - U_t)\right]^\alpha$, where $A_t$ is the exogenous technology shock, $K_t$ is the capital stock, $C_t$ stands for capacity utilization, $L_t$ is the labor force, $U_t$ is the unemployment rate, and $\alpha = 0.67$, as estimated in Gomes et al. (2005). The potential output is given by $Y_t^n = A_t \left(K_t C_t^n\right)^{1-\alpha} \left[L_t (1 - U_t^n)\right]^\alpha$, where $C_t^n$ is the NAICU and $U_t^n$ is the NAIRU.

Our assumptions give us a simple way to compute the (gross) output gap $Y_t / Y_t^n$ without relying

21 The differences stem from the fact that our approach takes into account the interaction between the inflation rates of traded and non-traded goods when estimating the NAIRU and NAICU, while HP filtering is just a low-pass filter with no economic reasoning.
on inferring the labor force, capital stock or the technology shock: \( \frac{Y_t}{Y_n} = \left( \frac{C_t}{C_n} \right)^{1-\alpha} \left( \frac{1-U_t}{1-U_n} \right)^{\alpha} \).

In log-notation, we have:

\[
\hat{y}_t = (1 - \alpha) \hat{c}_t + \alpha \hat{e}_t = (1 - \alpha) \hat{c}_t - \alpha \hat{u}_t,
\]

where \( \hat{y}_t \) is the output gap, \( \hat{c}_t \) is the capacity utilization gap, \( \hat{u}_t \) is the unemployment gap and \( \hat{e}_t \simeq -\hat{u}_t \) is the employment gap. Therefore, the output gap is a combination of the capacity utilization and employment gaps.

\begin{align*}
\text{(A)} & \quad \text{(B)} \\
\text{(C)} & \quad \text{(D)}
\end{align*}

Figure 3: Gaps

Note: Panel (A): Employment gap (red stars), Capacity utilization gap (blue circles), Output gap (black).
Panel (B): Output gap (black), Quasi GDP gap obtained by HP\(_{1600}\) filtering (black stars).
Panel (C): Capacity utilization gap (blue circles), Quasi Capacity utilization gap obtained by HP\(_{1600}\) filtering (black).
Panel (D): Employment gap (red stars), Quasi Employment gap obtained by HP\(_{1600}\) filtering (black).

Panel (A) of Figure 3 shows the three gaps: the employment gap, the capacity utilization gap and output gap. Panel (B) compares our estimates of the output gap with the one obtained by using the HP filter for extracting the GDP trend. Panels (C) and (D) compare, respectively, our estimates of the capacity utilization and employment gaps with HP filter gaps. Figure 3 summarizes our main findings: (i) even though suggesting an economic slowdown by the end of 2012, our estimates for the output gap clearly suggests that the Brazilian economy was better than what is implied by simply HP-filtering the GDP time series (Panel (B)); (ii) by the same period, our estimates suggest that the industrial sector was more sluggish than what HP-filtering implies (Panel (C)); and (iii), on the other hand, we find evidence that the labor market was more heated than what predicted by HP-filtering (Panel (D)).
Both measures of output gap display similar dynamics throughout the sample, as shown in Panel (B). However, the HP-filter gap has more pronounced movements, especially after 2007. From the world economic crisis of 2007/2008 on, the HP gap measure became extremely volatile, whereas our measure remained with the same volatility level as before the crisis.

5 Conclusions

Brazil has been experiencing an odd economic instance in recent years. In the aggregate, economists have been facing a hard time to find a positive correlation between employment and inflation, which should be a trivial task in most countries. Hence, a puzzle arises in understanding the lack of such correlation.

In order to solve this puzzle, we have looked into disaggregate measures. In particular, we have explored the dichotomy in the Brazilian economy: Brazil has been characterized by low and decreasing unemployment rates along the last decade, on the one hand, and low capacity utilization and GDP growth rates, on the other hand. Simultaneously, the inflation rate in the sector of non-traded goods has been persistently high in recent years, up to 2012, while the inflation rate in the sector of traded goods has been much lower.

In this context, we have tested a semi-structural empirical model to study the relationship between unemployment, capacity utilization and inflation in Brazil. The key feature of the model is that it has two separate Phillips curves, for the inflation rate of traded and non-traded goods, which are jointly estimated by full maximum likelihood method. In the sector of traded goods, we assume that production is much more intensive in capital, whereas it is intensive in labor in the sector of non-traded goods.

Therefore, we assume that the Phillips curves for the sectors of traded and non-traded goods have the capacity utilization gap and the unemployment gap as appropriate demand variables, respectively. We use the Kalman filter to infer the non-accelerating inflation rate of unemployment (NAIRU) and non-accelerating inflation rate of capacity utilization (NAICU), by means of adding economic structure in inference. Using a simple production function, we combine those results to obtain three gaps: unemployment gap, capacity utilization gap and output gap.

Our results suggest that the labor market does have a significant impact on the dynamics of inflation, especially through the non-traded goods sector. The impact of capacity utilization, through inflation of traded goods, is also relevant. We argue that the evidence of a weak relationship between unemployment and inflation in Brazil, reported by many economists, may be due to: (i) standard analyses using aggregate quantities, which disregard the idiosyncrasies of different sectors; and (ii) measurement errors in the variable used as the unemployment gap, obtained by a naive HP-filtering, which cause attenuation biases during inference.

We find that the industry is more sluggish than what HP-filtered capacity utilization gaps suggest, whereas the labor market is tighter than what HP-filtered unemployment gaps predict.

Finally, our results emphasize the dichotomy experienced by Brazilian economy. They suggest that the NAIRU has substantially reduced in Brazil in recent years, but not as much as the actual unemployment rate has. Our results also indicate that since mid-2010 the labor market has been operating at full-employment and pushing up the inflation rate of non-traded goods. The estimated NAICU path confirms that the industrial sector has been slowing down, and that has been keeping the inflation rate of traded goods at bay.
References


A Additional results

Figure 4: NAIRU and NAICU, standard model

Note: Panel (A): Unemployment rate (blue), NAIRU (black circles), HP trend (red stars).
Panel (B): Capacity utilization (blue), NAICU (black circles), HP trend (red stars).
Panel (C): Unemployment rate (blue), NAIRU (black circles), 95% confidence interval (black).
Panel (D): Capacity utilization (blue), NAICU (black circles), 95% confidence interval (black).
Table 2:Estimated Parameters, sample starting at 2002Q2

<table>
<thead>
<tr>
<th>Phillips Curve: Non-Traded Goods</th>
<th>$M_0$</th>
<th>$M_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_{t-1}$</td>
<td>0.346*** (0.101)</td>
<td>0.342*** (0.081)</td>
</tr>
<tr>
<td>$E_t \pi_{t+1}$</td>
<td>0.629*** (0.089)</td>
<td>0.634*** (0.079)</td>
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<td>$\pi_{t-1}$</td>
<td>0.025 (0.017)</td>
<td>0.024 (0.014)</td>
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<tr>
<td>$\hat{\pi}_{t-2}$ or $\hat{\pi}_t$</td>
<td>$-0.234^*$ (0.137)</td>
<td>$-0.288^{***}$ (0.091)</td>
</tr>
<tr>
<td>$d_t$</td>
<td>0.311* (0.164)</td>
<td>0.314** (0.131)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Phillips Curve: Traded Goods</th>
<th>$M_0$</th>
<th>$M_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_{t-1}$</td>
<td>0.492*** (0.130)</td>
<td>0.481*** (0.125)</td>
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<tr>
<td>$E_t \pi_{t+1}$</td>
<td>0.420*** (0.128)</td>
<td>0.435*** (0.116)</td>
</tr>
<tr>
<td>$\pi_{t-1}$</td>
<td>0.088*** (0.017)</td>
<td>0.084*** (0.017)</td>
</tr>
<tr>
<td>$\hat{\pi}_{t-2}$ or $\hat{\pi}_t$</td>
<td>0.151 (0.095)</td>
<td>0.202*** (0.058)</td>
</tr>
<tr>
<td>$d_t$</td>
<td>0.966*** (0.242)</td>
<td>0.935*** (0.227)</td>
</tr>
<tr>
<td>$shtr_{t-1}$</td>
<td>0.046** (0.025)</td>
<td>0.041*** (0.014)</td>
</tr>
</tbody>
</table>

| log ($\sigma_{ul}^2$) | $-11.054^{***}$ (0.397) | $-11.222^{***}$ (0.346) |
| log ($\sigma_{ur}^2$) | $-9.639^{***}$ (0.546) | $-9.743^{***}$ (0.576) |
| log ($\sigma_{fr}^2$) | $-10.609^{***}$ (0.624) | $-10.677^{***}$ (0.653) |
| log ($\sigma_d^2$) | - | $-10.657^{***}$ (0.550) |

Log-likelihood Step 1: 489.964 497.286
Log-likelihood Step 2: 415.871

Nota: Sample: 2002Q2 – 2012Q4
Parenthesis: st. dev.; Signif.: *(10%), **(5%), *** (1%)
$M_0$: step 1, iteration 1, HP$_{1600}$ filtering with initial values for NAIRU and NAICU, no corrections
$M_A$: corrections for NAIRU and NAICU inferred by Kalman filtering using auxiliary series (HP$_{1600}$)
Figure 5: NAIRU and NAICU, benchmark model, sample starting at 2002Q2

Note: Panel (A): Unemployment rate (blue), NAIRU (black circles), HP trend (red stars).
Panel (B): Capacity utilization (blue), NAICU (black circles), HP trend (red stars).
Panel (C): Unemployment rate (blue), NAIRU (black circles), 95% confidence interval (black).
Panel (D): Capacity utilization (blue), NAICU (black circles), 95% confidence interval (black).