Analysis of the strategic use of forward contracting in electricity markets

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ABSTRACT

When oligopolistic producers anticipate the effects of forward sales on spot prices, they may react in the forward market to compensate for the spot price reduction. In this situation, forward prices are not the (risk-affected) expectation of future spot prices, which is contrary to no-arbitrage pricing theory. However, we will show that it does not represent arbitrage opportunities, and that players' profits are equivalent to the ones obtained in the no-forward-trade case. The paper also considers that, when past spot prices are signals of future spot prices, the equilibrium often results in the spot prices of the no-trade case.

JEL classification: C72, C73, G13, L13, L14, L50, Q40
Key words: Forward markets, oligopoly, private information

1. Introduction

Absence of arbitrage can be thought of as the fundamental tool to describe financial markets. The no-arbitrage price of any financial contract represents players' valuation of the uncertain future income stream that will result from the contract. As a part of the calculation of these valuations, a risk-neutral probability might be defined that modifies the real probability to incorporate the effects of players' preferences (risk aversion, etc.), see for instance Duffie (2001). But the previous reasoning is based on considering future income streams as exogenously defined variables. When spot markets do not behave under the assumption of perfect competition, future income streams might depend on players' strategic interaction, and thus risk-neutral probabilities might be affected by such interaction. For instance, the risk-neutral price of a forward contract is the expected spot price at the expiration of the contract, with respect to the risk-neutral measure. This condition represents a situation where players cannot obtain riskless profits (arbitrage) just by benefiting from forward and spot price differences, as markets are supposed to rule out these opportunities. But the above argument rests on the assumption that, once players have entered into the contract, incentives in the spot market remain the same. To put it another way, the above reasoning implies that the effects of strategic behavior are restricted to the spot market or, equivalently, that all possible strategies concern only production decisions, whereas financial markets do not introduce any gaming opportunity. When assuming so, the usual same-price condition is still valid, so that the modeling of strategic behavior can be restricted to the analysis of spot behavior, regardless financial market decisions.

The relationship between forward and spot market has been widely discussed in the electricity industry with different aims. One of the most discussed issues is whether or not the use of forward market decreases players' opportunities to exercise market power. We cope with this problem and we show that, the strategic interaction that characterizes most power markets casts doubt on the possibility to separate spot and financial decisions, and hence on the direct application of no-arbitrage pricing to power markets.

After this introduction, we review the literature in section 2. Section 3 will describe the game for the interaction between the forward and spot markets by means of a two-period model. It is shown that, when producers set the forward price, there is a strategic premium associated with the anticipation of spot market responses to forward contracting, which makes forward prices depart from expected spot prices. Section 4 shows that the condition "forward price is equal to (risk affected) expected spot price" might not hold when players behave strategically in the spot market. The logic for this is that, although price differences might result in non-convex market sets, this does not imply the existence of arbitrage opportunities, as it happens under perfect competition[1], because market players cannot take advantage of such non-convexities. Section 5 then continues by showing that, when players anticipate that forward sales reduce spot price, they can react in the forward market to compensate for the spot price decrease, so that players profits are, considering both forward and spot markets, equivalent to the ones obtained in the case where no forward trading is allowed. In addition, section 6 develops a refinement of the game based on the consideration of private information. The logic for this model is that, when considering private information, past spot price reveals information regarding competitors' parameters, and thus they are signals of the probability of future spot prices. Consequently, a decrease in the spot price will make the forward price lower. Therefore, there is an additional incentive when playing in the spot market associated with the sensitivity of forward prices to past spot decisions. Section 7 collects some final remarks and analyzes the policy implications of the results obtained in the paper.

2. Interaction between forward and spot markets

Most of the early works on the interaction between forward and spot markets tackled the problem using a two-period game where players decided in both markets simultaneously --Anderson (1991) is a survey of these works. But Allaz (1992) and Allaz and Vila (1993) can be considered as the starting point of a long-lasting discussion on whether the existence of forward contracts limits the ability of players to exercise market power. Allaz and Vila (1993) explicitly modeled the forward-spot interaction as a sequence of markets, and concluded that the possibility to trade forward contracts, even in markets without uncertainty, forces market players to behave more competitively. Moreover, an infinite number of consecutive forward markets before the spot market clearing would result in perfectly competitive spot prices. The problem can be related to the more general analysis of Bulow, et al. (1985) and Fudenberg and Tirole (1984). Both works discussed on the problem of whether two consecutive markets result in more competitive outcomes than two simultaneous markets. According to these models, and assuming that forward sales lower rival's profits, the pro-competitive effects of sequential trading depend on whether spot market competition is modeled as a price or quantity game. From this standpoint, if firms are assumed to compete in quantities in the spot market, the results obtained in Allaz and Vila (1993)remain valid. Mahenc and Salanié (2004), on the contrary, considered a model where firms compete in prices. They obtain that, in this case, firms buy electricity in the forward market and the outcome is less efficient than in absence of forward trading. Along these lines, Holmberg (2008) investigates the consequences of considering supply function competition[2] in a two-period problem, and he

[1] See for instance Magill and Quinzii (2002)

[2] Originally, Klemperer and Meyer (1989) developed the concept of supply function equilibrium as a compromise between price and quantity competition, suggesting that in an uncertain environment firms

concludes, in agreement with Bulow, et al. (1985) and Fudenberg and Tirole (1984), that the strategic effects depend on the way in which players compete in the spot market. Newbery (1998) also developed a supply function model to describe forward and spot markets interaction. His work, nonetheless, aimed at analyzing the role of forward contracts as entry barriers. He concludes that the only incentive of producers to enter into forward contracts was to make the market less contestable. Green (1999) shows that considering supply functions competition in the spot market and Cournot conjectures in the forward market, result in no forward contracting for risk-neutral producers. Kamat and Oren (2004) and Yao, et al. (2007) develop models with the same strategic setup as the Allaz-Vila's model and obtain the same results concerning strategic effects, but they consider a very detailed representation of the power system characteristics.

All previous proposals build on the representation of a market with two periods. In addition, there are several refinements of the two-period model that are aimed at analyzing the effects of considering many periods. Ferreira (2003) considers a model where producers have the opportunity to trade forward in infinitely many periods before the spot market takes place. This model, although formally similar to the one analyzed in Allaz and Vila (1993), considers an infinite horizon directly instead of as the limit of the two-period case. In this setting, Ferreira (2003) obtains that many outcomes can be sustained in equilibrium. Moreover, when firms are not allowed to buy in the forward markets, the Cournot outcome is a renegotiation-proof[3] equilibrium. The infinite horizon model suggests considering the effects of repetition in the forward-spot interaction. This is the main idea behind Le Coq (2004) and Liski and Montero (2006). On the one hand, Le Coq (2004) considers that forward markets open once at the initial period, and then producers compete in prices in a repeated spot market. On the other, Liski and Montero (2006) analyzes the infinite repetition of the two-period game defined by the sequence of a forward and a spot market. Although these works differ in the setup of the game, their basic message is quite similar i. e. the repetition of the game facilitates to coordinate strategies. The logic for these models is that forward sales reduce the incentive of firms to deviate from collusion, as contracts limit the players' profits during the deviation period. A refinement of the previous argument is suggested in Green and Le Coq (2009), where the effects associated with the length of the forward contract is analyzed. When firms are allowed to enter into long-term forward contracts, there is an additional effect on the collusive behavior. In their analysis, long-term contracts are a protection during the punishment period, so that collusion is harder to sustain. Under this model, thus, it is difficult to anticipate the result of the trade-off between the pro- and the anti-collusive incentives.

On the other hand, private information has not played an important role in the analysis of forward-spot interactions. Hughes and Kao (1997)studies the role of observability in the analysis of the forward-spot interaction. In particular, they argue that if market players do not observe rivals' forward positions, the equilibrium of the game is the Cournot outcome (when considering Cournot competition in the spot market). Ferreira (2003) and Ferreira (2006), challenge the model by arguing that, even when forward positions are not observable, the results obtained by Allaz-Vila's model still hold. Zhang and Zwart (2006) study a model where a monopolist is forced to sell part of her production forward, and where her costs are private information. They show that, in this case, the monopolist has the incentive to raise the price, to build the reputation of having high costs, along the lines of Kreps and Wilson (1982) and Milgrom and Roberts (1982).

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would not want to commit with either of these strategies, but instead firms would specify supply functions, i. e. functions specifying the bid price corresponding to each possible output.

[3]Loosely, a renegotiation-proof equilibrium, which is defined in Bernheim and Ray (1989) and Farrell and Maskin (1989) for repeated games and extended in Ferreira (2003) for extensive form games, is the one that results in the highest profits with respect to the rest of sub-game perfect equilibria.

3. Two-period game setup and equilibrium conditions

3.1 Firms' behavior

This section will discuss the analysis of the financial problem in a two-period setting, considering that quantity decisions in the spot market affect spot prices, and hence forward prices. The time-uncertainty setting is defined by two periods and S possible alternatives at date 1 (spot market). The possible states at date 1 will be denoted by $s = 1, \dots, S$. Date 0 (forward market) will be considered to be state 0, so that we will have $S + 1$ states, $s' = 0,1, \dots, S$. This formulation aims to represent a situation where players decide on their contracts at date 0 subject to the uncertainty of date 1.

In order to motivate the need for considering spot market reactions to forward positions, let us first analyze the classical financial model where such reactions are not taken into account. The revenue stream at date zero will be defined by the income associated with forward sales, denoted by $R_0^i = p^F q^i$, where p^F is the forward price and q^i are firm i's forward sales[4]. At date 1, the spot market takes place, where the revenues in each future state $s = 1, \dots, S$ are denoted by $Spot_s^i$ and the price by π_s . In addition, player i has a preference ordering on the revenues set $R^i = R_0^i, \dots, R_S^i$, and we will assume that the ordering can be expressed by a utility function:

$$U^i : \mathcal{R}^{S+1} \longrightarrow \mathcal{R}$$

This utility function, thus, defines the producers' preferences for each state of nature, and it is assumed transitive, convex and complete. The profit-maximization problem of market players can be represented by the program:

$$\begin{array}{lll} max_{R_S^i, q^i} & U^i\left(R_{S'}^i\right) & \\ s.t. & R_0^i = p^F q^i & : \lambda_0^i \\ & R_S^i = Spot_S^i - \pi_S \, q^i & : \lambda_S^i \end{array} \tag{1}$$

The optimality of problem (1) with respect to revenues R_S^i gives the definition of the Lagrange multipliers:

$$\frac{\partial U^i(R_{S'}^i)}{\partial R_{S'}^i} = \lambda_{S'}^i, \;\; s' = 0,1, \dots, S \tag{2}$$

As Lagrange multipliers are marginal utilities at date 0 of an additional unit of spot profits, they can be thought of as discount factors for each state of nature. No-arbitrage conditions impose that each λ_S^i is a positive value, which is equivalent to the condition that problem (1) has a solution, see for instance Magill and Quinzii (2002). Besides, to keep the notation simple, we will assume hereinafter that $\lambda_0^i = 1$ (this simply says that, at date 0, market players' utility function is the identity function).

The optimality conditions of problem (1) with respect to forward sales give the definition of the forward price. When it is assumed that $\frac{\partial Spot_S^i}{\partial q^i} = 0$, the forward price is given by[5]

$$p^F = \sum_{s=1}^{S} \lambda_S^i \, \pi_S \tag{3}$$

[4] Note that producers are allowed to buy and sell forward; there are no limitations on the forward positions that producers may take.

[5] Note that the relationship between optimality condition (3) and risk-neutral probabilities needs just a normalization of the Lagrange multipliers, so that each of them represents artificially constructed probabilities, see for instance Duffie (2001).

The objective of this section is to identify how this result changes when an oligopolistic equilibrium is considered. In order to take into account the effects of market power in the spot market, we will consider a more detailed description of spot market revenues. Thus, we will define

$$Spot_s^i = \left(\pi_s \ - c_s^i\right) Q_s^i \tag{4}$$

where $Q_s^i$is the total production of firm $i$in state s, $c_s^i$is the corresponding variable cost. Hence, the detailed expression for revenues at each state of date 1 is given by:

$$R_s^i = \ \left(\pi_s \ - c_s^i\right) Q_s^i - \pi_s \ q^i \tag{5}$$

For notational simplicity, we will include the constraint $R_0^i = p^F q^i$in the primal problem by assuming the corresponding multiplier equal to one, so that the firms' profit-maximization problem can be described by the following model:

$$\begin{aligned} & max_{R_s^i, q^i} && p^F q^i + U^i\left(R_s^i\right) \\ & s.t. && R_s^i = \ \left(\pi_s \ - c_s^i\right) Q_s^i - \pi_s \ q^i \quad : \lambda_s^i \end{aligned} \tag{6}$$

Following the above reasoning, the Lagrange multipliers represent the players' marginal utility. The main difference with respect to the perfect competition problem is that, in general $\frac{\partial Spot_s^i}{\partial q^i} \neq 0$. In particular, spot market reactions are defined by the derivatives $\frac{\partial Q_s^i}{\partial q^i}and\frac{\partial \pi_s}{\partial q^i}$. In Vazquez (2011), these derivatives are obtained under both Cournot and conjectural variation competition, and the consequences of different assumptions on the forward-spot interaction. For the sake of exposition, this paper will assume that players compete à la Cournot, although no result developed hereinafter makes use of such assumption.

The firm's optimality of problem (6) with respect to forward sales is given by

$$p^F = \sum_{s=1}^{S} \lambda_s^i \pi_s \ - \sum_{s=1}^{S} \lambda_s^i \left\{ \frac{\partial Q_s^i}{\partial q^i} \left(\pi_s \ - c_s^i\right) + \frac{\partial \pi_s}{\partial q^i} \left(Q_s^i - q^i \ \right) \right\} \tag{7}$$

In contrast to the perfect-competition case, which states that forward prices are the present value of future payoffs –in this case, future spot prices–, optimality condition (7) states that present values are modified by the present values of spot market reactions. They are made up of two terms:

- $\frac{\partial Q_s^i}{\partial q^i}\left(\pi_s \ - c_s^i\right)$ represents the profits variation associated with the change of equilibrium production. Contracting an additional megawatt changes the production of the firm, which is affected by the difference between the spot price and the production cost
- $\frac{\partial \pi_s}{\partial q^i}\left(Q_s^i - q^i \ \right)$ is the change of profits associated with the change of spot price. That is, the change of spot prices affects the infra-marginal quantity $\left(Q_s^i - q^i \ \right)$

Equation (7) says that optimal forward prices, in general, take account of the spot market response, so they are determined not only by the present value of future prices, but in addition by the present value of market responses. Thus, the premium with respect to the (risk-affected) expected spot price depends both on the profits increase due to the increased production, and on the decrease of prices due to the loss of incentives to exercise market power. Hence, market players have two opposite incentives to deviate from spot price present value: on the one hand, forward contracting increase the firms' total production, and thus spot sales, so they can reduce forward prices to compensate for these extra profits; on the other, forward contracting reduces spot prices, so market players have the incentive to raise forward prices.

The spot market will be represented by a Cournot model. We will model a linear demand curve at each state of nature D_s , with a slope given at each state of nature by α_s :

$$D_s = D_s^0 - \frac{1}{\alpha_s}\pi_s$$

Under such conditions, it is possible to obtain, as in any other Cournot model, the spot optimality for the firms:

$$\pi_s = c_s^i + \alpha_s \left(Q_s^i - q^i\right) \tag{8}$$

where it can be observed that forward positions do affect spot output decisions. The consequences of assuming different behavior in spot markets are analyzed, among others, by Holmberg (2008), who used a supply function equilibrium, and by Vazquez (2011), who used a conjectural variations equilibrium.

The forward market is defined by an inelastic demand D^F .The main idea behind that representation is that power consumers are often significantly risk averse, so that they prefer to hedge certain parts of their portfolios in advance. This is the typical situation of a power retailer who needs to hedge the price risk. Thus, all price elasticity is represented in the spot market, whereas the forward position is given as a fixed parameter.

3.2 Equilibrium conditions

The set of optimality conditions defined by the equations corresponding to the optimality conditions of all players defined in (7) and (8), added to the two definitions of the quantity demanded both in the forward and the spot market, allows solving the game. The solution of this game provides forward and spot prices, in addition to forward and spot quantities. Qualitatively, the idea behind this equilibrium is players' bids in the forward market are defined by the marginal cost of the contract, as defined by (7), which is made up both of the price expectations and of the expectations on spot market responses. Thus, in this initial model, market players are assumed to bid competitively in the forward market, even when the spot market is oligopolistic. Section 5will relax this assumption, but the main objective of the following section is to show that just by considering oligopolistic spot markets, no-arbitrage conditions do not imply that the forward price is equal to the expected spot price.

4. Absence of arbitrage and market power

From the financial theory under perfectly competitive spot markets, if forward prices are different from the expected spot price, there will be arbitrage opportunities. We will investigate in this section whether that conclusion can be extended to the case of oligopolistic spot markets. To do so, we will consider the geometric interpretation of absence of arbitrage.

Under the assumption of perfect competition, no-arbitrage conditions can be obtained by representing all possible trades available in the market by means of the market set $\langle \mathcal{M} \rangle^{PC} = \{d \in \mathcal{R}^{S+1} / d = \mathcal{M}q\}$, where d is the income transfer associated with the forward contract, $\mathcal{M}$ is the payoff matrix[6], and $q$is the quantity contracted. To simplify the exposition, let me consider that there is just one future state, so that $s = \{0,1\}$. Hence, $d$will be a vector with two dimensions. That is, it will be made up of the income transfer associated with date 0, $d_0 = p^F q$, and

[6] In this case, it would be a vector containing the forward price and the spot prices at each state of nature with the negative sign.

the transfer associated with the unique state at date 1, $d_1 = -\pi q$. This market set is a linear subspace, and it is represented in Figure 1by the dotted line.

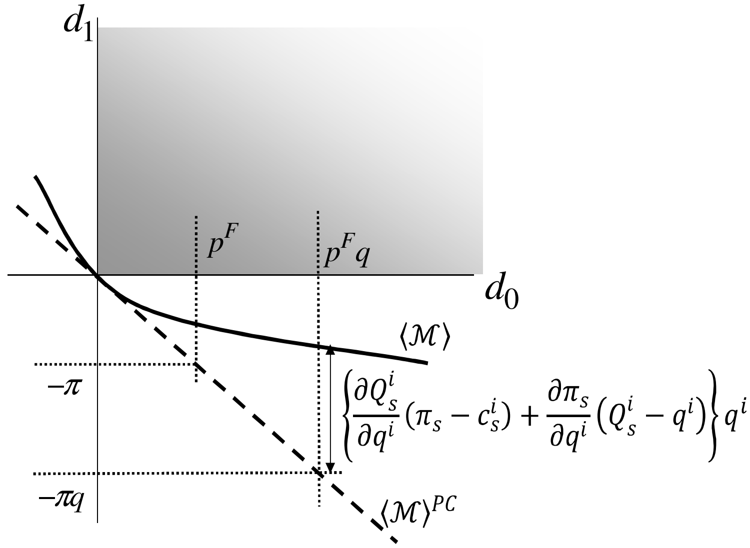

Figure 1. Market set with and without market power opportunities.

It is possible to state absence of arbitrage as a geometric property of the market set, namely that the market set does not intersect the non-negative orthant $\mathcal{R}^{S+1}$. That is, if the set representing all possible trades –the market set– intersects the non-negative orthant, trades in intersection are arbitrage opportunities, in the sense that they are opportunities to obtain riskless profits (e. g. the opportunity to obtain revenues at date 1 without a cost at date 0). Markets are assumed to rule out these opportunities for free profits. Formally, the definition of absence of arbitrage can be written as $\langle \mathcal{M} \rangle^{PC} \cap \mathcal{R}^{S+1} = \{0\}$. Hence, no-arbitrage implies that the market set must be separable from the positive orthant. From the Separating Hyperplane Theorem, see for instance Luenberger (1984), nonconvex sets cannot be separated in general. That is, when the trades available to market players are represented by a nonconvex market set, a possible strategy is to build a portfolio made up of linear combinations of the contracts lying in the market set, which in general would intersect the non-negative orthant (and thus would yield arbitrage opportunities). Therefore, under perfect competition, no-arbitrage implies that the trades available in the market must be represented by a convex set.

It is possible to use the same reasoning to analyze forward prices when the assumption of perfect competition in the spot market is relaxed. For the sake of explanation, let me consider that the total effect of spot price responses is a positive value:

$$\lambda^i \left\{\frac{\partial Q^i}{\partial q^i}(\pi - c^i) + \frac{\partial \pi}{\partial q^i}(Q^i - q^i)\right\} > 0 \tag{9}$$

In this case, equation (7) implies that $p^F < \lambda^i \pi$, and thus the income transfer at date 1 would be

$$d_1 = -\pi q + \left\{\frac{\partial Q_s^i}{\partial q^i}(\pi_s - c_s^i) + \frac{\partial \pi_s}{\partial q^i}(Q_s^i - q^i)\right\} q^i \tag{10}$$

as represented by the solid line inFigure 1.However, the market set represented by the model is not a convex one, which in the perfect-competition case implied arbitrage opportunities, because it was possible to build a portfolio made up of linear combinations that intersected the non-negative orthant.

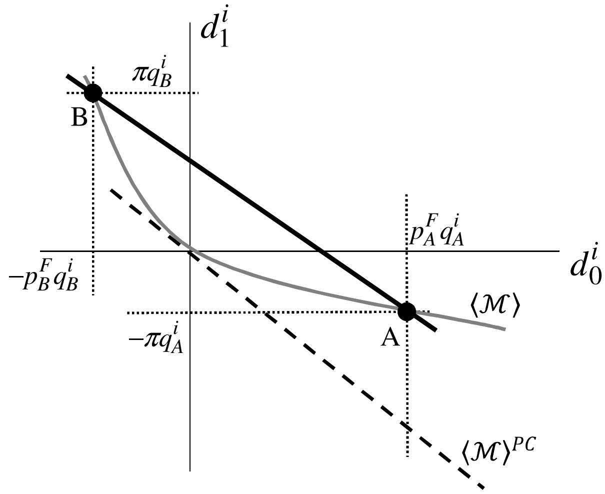

Figure 2. Convexity of the market set in oligopolistic spot markets.

Figure 2 represents the kind of portfolio required to benefit from the non-convexity of the market set. Such portfolio would be the combination of positions A and B(note that these positions correspond to the typical strategy used to motivate absence of arbitrage, i. e. buying and selling simultaneously the same commodity). However, both points require that the player behaves in the spot market according to each single contracting decision, i. e. that she decided her spot market productionQ^i as if she had bought quantity q_B^iin the forward market, and at the same time, as if she had sold quantity q_A^iin the forward market. But, as the spot market takes place once all forward decisions are made, the producer will play taking into account her net position, which is contrary to the assumptions regarding her contract position made to determine the forward prices (the ones represented by the market set). Therefore, assuming that players are sequentially rational means that is not possible to build a portfolio from the combination of two (or more) points of the market set. Or, conversely, that the market set changes when several positions are taken at the same time.

We can conclude that, when forward decisions have an influence on spot prices, nonconvex market sets arise that do not represent arbitrage opportunities, so having forward prices different from the expected spot prices can be a plausible market equilibrium.

This kind of analysis can be related to similar approaches in the context of General Equilibrium Theory. Building on the model described in Shapley and Shubik (1977), Koutsougeras (2003b) and Koutsougeras (2003a) have proposed that market prices might not be equalized when firms behave strategically. These works study the problem of price behavior when there is more than one market for each commodity (which they called trading posts). In the case of electricity markets, this corresponds to multiple forward markets to trade electricity. The conclusion of these works is that, when firms' strategies change future spot prices, absence of arbitrage does not necessarily imply that the forward price is the expected spot price. In addition, price differences do not represent arbitrage opportunities, because when players try to take advantage of them, prices react dissipating such arbitrage opportunities. Note that this is essentially the same conclusion obtained above. The problem above can be identified with a market with two different trading posts (the forward and spot markets). Although both prices might be different, if players try to take advantage of the arbitrage opportunities the prices react to compensate for the change of strategy. In contrast, Gobillard (2006) argued that the result in Koutsougeras (2003b) is motivated by players trading in both the supply- and demand-side of each market, which he considered implausible in real commodity markets. In the problem of this section, it is natural to consider players supplying electricity in the forward market and purchasing electricity in the spot market.

To support the price equality condition, it is often claimed that considering a significant number of arbitrageurs results in forward prices equal to the expectation of spot prices. The rationale behind this is that arbitrageurs can be modeled as players having almost no incentive to manipulate spot prices, so that their optimality conditions imply that forward are equal to spot prices. In addition, it is claimed that, even if the spot prices increase because of the strategies of oligopolists, the forward price is set by arbitrageurs' optimality. It is worth to analyze the conditions on the forward price formation required to satisfy such result. From the spot market point of view, arbitrageurs are assumed to trade a small quantity to have no effect on the oligopolistic prices. From the forward market point of view, arbitrageurs are assumed to trade a large enough quantity to set the forward price. Both assumptions are compatible only in the case that power producers trade a small enough quantity in the forward market. In such a case, all the strategic effects related to forward contracting discussed in this paper are irrelevant. In a more general case, with relevant forward trade by power producers, arbitrageurs will not be capable of eliminating all differences between forward and expected spot prices. Note that a possible active risk-neutral, non-strategic demand, choosing the cheapest market to buy electricity would play the same role as the arbitrageurs, and hence the above reasoning could be used also in that case.

5. Optimal forward price response to forward positions

The model described in section 3 has assumed that power producers have no incentive to raise forward prices. Relaxing this assumption requires a slight generalization of the price equation defined above. To do so, we will use the same profit-maximization problem (6), but considering the ability of producers to manipulate the forward price. In this case, the first-order optimality conditions are

$$p^F + \frac{\partial p^F}{\partial q^i} q^i = \sum_{s=1}^{S} \lambda_s^i \pi_s - \sum_{s=1}^{S} \lambda_s^i \left\{ \frac{\partial Q_s^i}{\partial q^i} \left(\pi_s - c_s^i \right) + \frac{\partial \pi_s}{\partial q^i} \left(Q_s^i - q^i \right) \right\} \tag{11}$$

where the new term representing forward market power $\frac{\partial p^F}{\partial q^i} q^i$ has been included. Note that the rest of the game, including the forward market clearing, has not changed. Equation (11) requires the definition of the forward price sensitivity $\frac{\partial p^F}{\partial q^i}$. The aim of this section is to show that the incentive to raise forward prices is associated with compensating for the spot market response to forward sales. Loosely, this effect follows from the fact that the optimal forward price is equal to the contract payoff, which is the spot price expectation plus the expectation of spot market responses to forward contracting. When those responses result in lower profits, there is an incentive to compensate for the loss of profits.

Specifically, we will consider that supply-side players in the forward market face an inelastic demand, compete in quantities and assume that rivals do not react to their sales decisions. The central point of this section is to show that, even in this case where there are no responses to forward market decisions, players face a reaction curve defined by their own behavior in the spot market. Relaxing this assumption will have only effects typically found in oligopoly models, and they will add to the ones described below.

Consider a player solving the problem (6) in order to decide on her bids in the forward market. In addition, consider first that the player, when doing so, assumes that forward prices have no sensitivity to forward sales decisions. In this case, she obtains the price equation (7). This price equation can be re-interpreted as expressing that forward price is equal to the marginal cost of the contract[7]. That is, the right-hand side of equation (11) represents the price that the producer would pay to undo her position. Let us denote this cost by $c^{F,i}$, so that

[7]Note that marginal cost refers to the contract payoff, i. e. it is not only the production cost but also includes the strategic behavior in the spot market.

$$c^{F,i} = \sum_{s=1}^{S} \lambda_s^i \pi_s - \sum_{s=1}^{S} \lambda_s^i \left\{ \frac{\partial Q_s^i}{\partial q^i} (\pi_s - c_s^i) + \frac{\partial \pi_s}{\partial q^i} (Q_s^i - q^i) \right\} \tag{12}$$

and equation (7) is $p^F = c^{F,i}$.

Using the spot optimality conditions (8), the previous equation can be expressed as

$$c^{F,i} = \sum_{s=1}^{S} \lambda_s^i \left\{ c_s^i + \alpha_s (Q_s^i - q^i) \right\} - \sum_{s=1}^{S} \lambda_s^i \left\{ \frac{\partial Q_s^i}{\partial q^i} \alpha_s + \frac{\partial \pi_s}{\partial q^i} \right\} (Q_s^i - q^i) \tag{13}$$

The marginal cost contains both the spot price and the spot price reactions to forward decisions. Hence, it is possible to represent the cost of the contract as in Figure 3, where the slope of the curve is $-\sum_{s=1}^{S} \lambda_s^i \left(\alpha_s - \frac{\partial Q_s^i}{\partial q^i} \alpha_s - \frac{\partial \pi_s}{\partial q^i} \right)$.

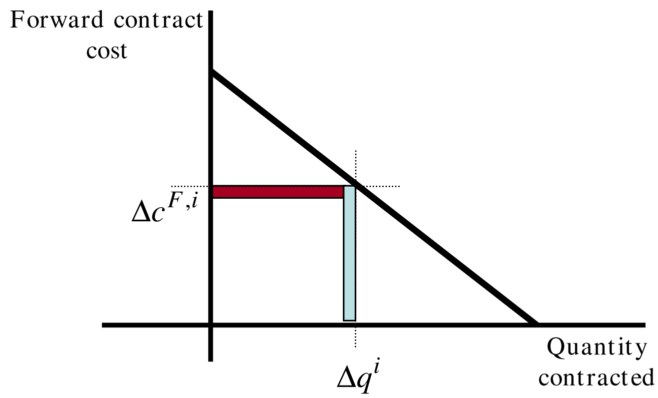

Figure 3. Marginal cost of forward contract as a function of forward sales.

If the player observes Figure 3, she knows that an extra megawatt sold in the forward market will lower the marginal cost of the contract. That is, as forward sales lower the incentive to raise the spot price, the cost of undoing her position will be lower. Therefore, they do reduce the contract cost and hence the forward price. In turn, as the forward price is reduced, the revenues associated with forward sales will be also reduced. Hence, the player modifies her conjecture to take account of the slope of the marginal cost curve, by observing equation (13), so that

$$\frac{\partial p^F}{\partial q^i} = -\sum_{s=1}^{S} \lambda_s^i \left(\alpha_s - \frac{\partial Q_s^i}{\partial q^i} \alpha_s - \frac{\partial \pi_s}{\partial q^i} \right) \tag{14}$$

Using this conjecture, the optimality conditions of problem (6) are

$$p^F = \sum_{s=1}^{S} \lambda_s^i \left(c_s^i + \alpha_s Q_s^i \right) - \sum_{s=1}^{S} \lambda_s^i \left(\frac{\partial Q_s^i}{\partial q^i} \alpha_s + \frac{\partial \pi_s}{\partial q^i} \right) Q_s^i \tag{15}$$

which differs from (7) in the fact that the relevant quantity is the total output Q_s^i instead of $Q_s^i - q^i$. In that case, the conjecture is consistent with the final optimality condition.

By considering (14), the agent translates to the forward market the loss of market power opportunities derived from forward contracting. To put it another way, if the quantity q^i were sold in the spot market, the equilibrium would imply greater prices and lower productions. When selling q^i in the forward market, the player sets a price that compensates for the loss of profits, so that her profits selling forward are equal to the ones that she obtained when she did not enter into any forward contract. Loosely, when market players sell forward and do not consider forward price sensitivity, they are implicitly loosing market power opportunities. Selling forward implies that this amount of energy will not be affected by the manipulation of spot prices, so that, under the assumption of no-forward-price-sensitivity, producers move production from a market (spot) where they are oligopolists to a market (forward) where prices are not manipulated. Expression (15) shows that forward prices are the expected spot price considering that all production is sold in the spot market, but discounting the spot market response: the extra profits from the increased production and the cost of price decrease. In addition, it can be alternatively thought of as the no-trade price modified by the change of the spot market equilibrium. Therefore, although spot prices are actually reduced, forward prices compensates for the corresponding loss of profits. Informally, this is a revenue-equivalence-like result: agents' profits will be the same regardless the market where they sell their output. Hence, although the efficiency –in terms of system costs– can be improved from the viewpoint of an isolated market, the whole set of markets, in absence of uncertainty, will be equivalent to the case without forward trading.

6. The role of private information

The previous section has analyzed the forward-spot interaction assuming perfect information. In particular, it showed that when players are allowed to set their optimal forward price, market players internalize the spot market response to forward sales, resulting in an outcome equivalent to the no-trade case. Nonetheless, each equilibrium results in the same profits as the corresponding no-trade solution, but forward prices depend on the particular forward position. This section is aimed at introducing an additional refinement of the game. In particular, the objective is to show the effects of considering private information.

The spot market outcome can be thought of as the aggregated signal about rivals' costs and strategic parameters. Describing the uncertainty in a multi-period market by an event-tree[8], the primary idea behind the model of this section is that the probability associated with each node of the event-tree is a personal belief rather than input data, and such beliefs change when players observe spot prices. From this viewpoint, a similar problem can be found in auction theory models. Auctions are often thought of as games with private information: bidders observe some private signal, which is then used to infer the value of the auctioned object. This is usually modeled in the context of Bayesian games, using an objective probability distribution giving the signal probability conditional to the object value. The same idea can be used to model the problem of this section. That is, the probabilities that each producer assigns to the states of nature are conditioned to the last spot price observed in the market:

$$\psi = \psi^i(n/\pi(n^-)) \tag{16}$$

where $\psi^i$is the probability distribution of player i, n is the node of the event-tree and $\pi(n^-)$is the spot price at the antecessor of n. These probabilities are objective probabilities, and hence the utility defined over the states are objective. In general, the probability distribution will be different for each market player. In addition, we assume that players update their beliefs according to the Bayes' Law.

6.1 Spot market game under private information

The representation is based on considering that, at each node of the event-tree, a spot market takes place and, after the spot market clearing, market players have the opportunity to trade in a forward market. Such forward market is assumed to have neither liquidity constraints nor transaction costs. Thus, the portfolio evolution can be represented by means of its capital value. In addition, the forward price is zero at terminal nodes. In this situation, the problem can be represented by the multi-period version of the problem (6):

[8] The event-tree notation is described in detail in the Appendix.

$$
\begin{aligned}
max \quad & \left(\pi(n_0) - c^i\ (n_0)\right) Q^i\ (n_0) + p^F\ (n_0) q^i(n_0) + U^i\left(R^i\ (n)\right) \\
& R^i\ (n) = \left(\pi(n) - c^i\ (n)\right) Q^i\ (n) + p^F\ (n) q^i(n) - \left(\pi(n) + p^F\ (n)\right) q^i(n^-) \quad : \lambda^i(n), n \notin N^T \\
& R^i\ (n) = \left(\pi(n) - c^i\ (n)\right) Q^i\ (n) - \pi(n) q^i(n^-) \quad : \lambda^i(n), n \in N^T
\end{aligned}
$$

Let us analyze spot market decisions at the intermediate nodes $n \notin N^T$. The first-order optimality conditions with respect to production decisions $Q^i\ (n)$ are the following:

$$
\pi(n) = c^i\ (n) - \frac{\partial \pi(n)}{\partial Q^i\ (n)}\left(Q^i\ (n) - q^i(n^-)\right) - \frac{\partial p^F\ (n)}{\partial Q^i\ (n)}\left(q^i(n) - q^i(n^-)\right) - \frac{1}{\lambda^i(n)} \frac{\partial U^i\left(R^i\ (n^+)\right)}{\partial Q^i\ (n)}
$$

First, it is important to highlight that the statement of probabilities dependent on past spot prices introduces a signaling game between consecutive spot markets. In the model developed in this section, these effects might be expressed by means of the utility derivative in the next period. That is, it would be necessary to take into account that $\frac{\partial U^i\left(R^i\ (n^+)\right)}{\partial Q^i\ (n)} \neq 0$. The effects of that term are not directly related to the analysis of the spot-forward strategic interaction, as it would exist even if there were no possibility to trade forward. That is, if this effect were considered, it would represent an additional strategic effect in the spot market, i. e. the incentive to raise the spot price to increase future spot profits, which would appear even without the opportunity to trade forward. As the main objective of this thesis is to analyze the forward-spot interaction, we will disregard the effect. Therefore, the optimality can be simplified to yield

$$
\pi(n) = c^i\ (n) - \frac{\partial \pi(n)}{\partial Q^i\ (n)}\left(Q^i\ (n) - q^i(n^-)\right) - \frac{\partial p^F\ (n)}{\partial Q^i\ (n)}\left(q^i(n) - q^i(n^-)\right) \tag{17}
$$

The last term of the previous equation represents the effects of spot market decisions on the subsequent forward market. It is the analogue to the term representing the spot market power: the ability to manipulate the forward price by withholding spot production times the quantity sold forward in that node of the event-tree.

We will assume that the incentive to manipulate the forward price at a certain node of the event-tree is defined just by the quantity sold in that node, $q^i(n)$. To see the logic for this assumption, consider that no renegotiation is possible: then, the capital value is intended to represent that there is no change in the portfolio over time, so both quantities $q^i(n) = q^i(n^-)$ are approximately the same and the last term would be small. This is equivalent to a two-period model, and this case was analyzed in the previous section. The other extreme case is when all contracts in the forward market expire in the next period (modeled as all successors). In this case, the capital value is zero. We will concentrate herein in the latter situation in order to analyze the effects related to the renegotiation of the contracts. When liquidity constraints are considered, which is the typical situation in power markets (where the forward contracts available in the market are often written on the average spot price of several periods), the effects studied below would involve only the part of the portfolio that can be renegotiated. In general, the results will be something intermediate between the two-period case and the results of this section. Therefore, when the quantity representing the capital value of the portfolio is not included, the spot market optimality can be written as

$$
\pi(n) = c^i\ (n) - \frac{\partial \pi(n)}{\partial Q^i\ (n)}\left(Q^i\ (n) - q^i(n^-)\right) - \frac{\partial p^F\ (n)}{\partial Q^i\ (n)} q^i(n) \tag{18}
$$

As in the two-period case, We will represent the spot market competition by a Cournot model, so that equation (18) can be recast as

$$\pi(n) = c^i\ (n) + \alpha(n)\left(Q^i\ (n) - q^i(n^-) + \frac{\partial p^F\,(n)}{\partial \pi(n)} q^i(n)\right) \tag{19}$$

The right-hand side of equation (19) can be interpreted as the marginal cost at node n plus the effect of exercising market power in the spot market. That is, $\alpha(n)$is the ability of player i to manipulate the spot price at node n, and the rest of the terms are the incentives to manipulate it. These terms have changes with respect to the two-period case. The first two terms are the same, namely the production sold at the spot price. The last term is a new effect related to the consideration of private information. It represents the fact that players' beliefs about the probabilities of future states of nature change when they observe the spot price.

Hence, it is necessary to define the forward price derivative with respect to output decision. To do so, let us consider the optimality condition with respect to forward quantities:

$$p^F\,(n) = \frac{1}{\lambda^i(n)} \sum_{n' \in N^+(n)} \lambda^i(n') \begin{Bmatrix} \pi(n') + \frac{\partial Q^i\,(n)}{\partial q^i\,(n'^-)}\left(\pi(n') - c^i\,(n')\right) + \\ \frac{\partial \pi(n\prime)}{\partial q^i\,(n\prime^-)}\left(Q^i\,(n) - q^i\,(n'^-)\right) \end{Bmatrix} \tag{20}$$

6.2 Players maximizing expected utilities

The main effect that this section analyzes is that, in presence of private information, $\frac{\partial p^F\,(n)}{\partial \pi(n)} \neq 0$. To put it another way, after producers observe the spot market results at node n, they update their beliefs about future probabilities, which in turn change the utility associated with each state of nature. Therefore, at all successors $\frac{\partial \lambda^i(n^+)}{\partial \pi(n)} \neq 0$ and the forward price will be affected.

Thus, compared to the case without forward trading, players have two opposite incentives. On the one hand, their actual forward position $q^i(n^-)$lowers the quantity that benefits from a higher spot price. On the other, a high spot price makes more likely a high price in the future. Next, We will analyze in detail the latter incentive. We will analyze the case where the utility function of all market players is the expected utility, and they define their probabilities to be conditioned to the observation of past spot price. In addition, players are risk-neutral. The possible consequences of the generalization of this model will be explored in section

In this case, the producers' marginal utility function is

$$\lambda^i(n) = \frac{\partial U^i\left(R^i\,(n)\right)}{\partial R^i\,(n)} = \psi^i(n/\pi(n^-)) \tag{21}$$

Hence, the forward price derivative can be expressed as

$$\frac{\partial p^F\,(n)}{\partial \pi(n)} = \frac{1}{\lambda^i(n)} \sum_{n^+} \frac{\partial \lambda^i(n^+)}{\partial \pi(n)} \begin{Bmatrix} \pi(n^+) + \frac{\partial Q^i\,(n^+)}{\partial q^i\,(n)}\left(\pi(n^+) - c^i\,(n^+)\right) + \\ \frac{\partial \pi(n^+)}{\partial q^i\,(n)}\left(Q^i\,(n^+) - q^i\,(n)\right) \end{Bmatrix} \tag{22}$$

In addition, using equation (21), the forward price sensitivity is

$$\frac{\partial p^F\,(n)}{\partial \pi(n)} = \frac{1}{\psi^i} \sum_{n^+} \frac{\partial \psi^i(n^+)}{\partial \pi(n)} \begin{Bmatrix} \pi(n^+) + \frac{\partial Q^i\,(n^+)}{\partial q^i\,(n)}\left(\pi(n^+) - c^i\,(n^+)\right) + \\ \frac{\partial \pi(n^+)}{\partial q^i\,(n)}\left(Q^i\,(n^+) - q^i\,(n)\right) \end{Bmatrix} \tag{23}$$

The previous expression is made up of three terms:

- The first one is the change in the risk-affected expectation of future spot prices
- The second one is the perceived sensitivity of forward prices to production decisions
- The third one is the perceived sensitivity of forward prices to spot price reactions

The last two terms represent the change in the valuation of spot market reactions to forward contracting, which follows from changes in output decisions in the present spot market. When power producers are deciding on the production of the present spot market, they take into account that higher spot prices today will likely imply higher spot prices tomorrow. But this would also imply, in general, different needs for internalization in the next forward market. Nonetheless, it is more difficult for power producers to take into account the change in spot reactions than the change in the expected spot price. Furthermore, it is important to highlight that this would have no effect in players' revenue, as any change in the future spot market situation can be internalized in the next forward market.

We will assume hereinafter that players take account of the change in the expected spot price, but they disregard the change in its derivatives at future nodes of the event-tree. Formally,

$$\sum_{n^+} \frac{\partial \psi^i(n^+)}{\partial \pi(n)} \left\{ \frac{\partial Q^i\ (n^+)}{\partial q^i\ (n)} \left(\pi(n^+) - c^i\ (n^+) \right) + \frac{\partial \pi(n^+)}{\partial q^i\ (n)} \left(Q^i\ (n^+) - q^i\ (n) \right) \right\} = 0$$

Consequently, it is possible to assume that power producers do not update their perceptions of future reactions, because they are able to internalize in the next forward price any possible loss of market power related to forward sales. To put it another way, it is possible to consider bounded rationality, and to assume that players take account of the change in the expected spot price, but they disregard the change in its derivatives. In addition, it will be shown that, under some further assumptions, spot prices do not depend on forward contracting decisions, and thus there is no spot market sensitivity to forward sales. Hence, assuming no update of spot reactions would be sequentially rational. The above expression can be understood as a situation where the market players assume that the net effect of their spot signals is a change in the (risk-affected) expected spot price. Therefore, the forward price sensitivity is

$$\frac{\partial p^F\ (n)}{\partial \pi(n)} = \frac{1}{\psi^i} \sum_{n^+} \frac{\partial \psi^i(n^+)}{\partial \pi(n)} \pi(n^+) \tag{24}$$

Denoting the term defined by equation (24) by $\delta(n)$, equation (19) can be rewritten as

$$\pi(n) = c^i\ (n) + \alpha(n) \left(Q^i\ (n) - q^i(n^-) + \delta(n) q^i(n) \right) \tag{25}$$

Thus, the effect of the next markets is to reduce the relevant forward position and, hence, increase the incentives to raise the spot price. In fact, let us consider that

- The expected update of future probabilities after the observation of present spot prices, time the spot price at each node of the event-tree, is equal to one, $\delta(n) = 1$
- The forward position remains stable from one period to the next, $q^i(n^-) = q^i(n)$

In this situation, when producers are risk neutral when deciding on spot market, the equilibrium is the same as the one obtained in the no-trade case.

On the one hand, $\delta(n) = 1$ represents that an increase of 1€ in the spot price today will change the price expectation for tomorrow in 1€. This implies that the market conditions that caused a high spot price will remain the same in the next period and raise the next spot price. For instance, a change in the strategy of any certain player that caused a 1€ increase in the price of today, will cause the same increase in the price of tomorrow, assuming that the rest of the conditions remain stable. Therefore,

$$\pi(n) = c^i\ (n) + \alpha(n)\left(Q^i\ (n) - q^i(n^-) + q^i(n)\right) \quad (26)$$

On the other hand, from (25), it is possible to observe that the relevant quantity in terms of market power is the portfolio change with respect to the previous period. In periods of stable forward strategies, it yields the Cournot solution. When the forward portfolio is being reduced, the spot market is more competitive than Cournot, but in periods of an increase of the forward position the spot prices are higher than Cournot. In practice, forward positions do not change dramatically over time, except for certain few specific periods (often linked to a regulatory change in the liberalization process). For the general analysis, these specific periods may be disregarded. Most trading periods, on the other hand, would show stable portfolios.

Assuming both stable portfolios and equation (25), the spot optimality yields

$$\pi(n) = c^i\ (n) + \alpha(n)Q^i\ (n) \quad (27)$$

which is the same optimality result that is obtained when there is no forward trading. As, in addition, spot prices do not depend on forward positions, forward prices are given by the traditional no-arbitrage condition.

6.3 General utility functions

Previous results have been obtained by assuming that players maximize their expected utility, and that they are risk-neutral. Note that this does not imply that market players are risk-neutral, but that they consider risk neutrality when deciding in the spot market. Let us discuss on the consequences of considering more general utility functions. In order to determine the forward price sensitivity to spot prices, it is necessary to define $\frac{\partial\lambda^i(n^+)}{\partial\pi(n)}$, which is quite a difficult task in the general case, especially because the Lagrange multipliers $\lambda^i(n)$, representing the marginal cost of profits at nodes $n$with respect to the initial nodes, are not just the probability of the node. They take account of the risk attitude of producers. It is worth to use as an example the case of quadratic utility, so that

$$U^i\left(R^i\ (n^+)\right) = \sum_{n^+}\psi^i(n^+/\pi(n))R^i\ (n^+) - \frac{1}{2}\beta\sum_{n^+}\psi^i(n^+/\pi(n))R^i\ (n^+)^2 \quad (28)$$

In this case, it is possible to define

$$\lambda^i(n^+) = \frac{\partial U^i\left(R^i\ (n^+)\right)}{\partial R^i\ (n^+)} = \psi^i(n^+)\left[1 - \beta R^i\ (n^+)\right] \quad (29)$$

so that

$$\frac{\partial p^F(n)}{\partial\pi(n)} = \frac{1}{\lambda^i(n)}\sum_{n^+}\frac{\partial\psi^i(n^+)}{\partial\pi(n)}\pi(n^+) - \frac{\beta}{\lambda^i(n)}\sum_{n^+}\frac{\partial\psi^i(n^+)}{\partial\pi(n)}\pi(n^+)R^i\ (n^+) \quad (30)$$

The first term in the right-hand side represents the present value of the sensitivity of the expected profits to actual spot prices. In the case of risk-neutral producers, this term was approximately equal to one. In this case, however, the present value contains the attitude towards risk of market players, so that the valuation of this sensitivity will depend on their preferences. To put it another way, when valuating the expected profits sensitivity, producers use a discount factor that represents their preferences. In this case, the discount factor is $1 - \beta R^i\ (n^+)$. Therefore, this term is in general different from one, and thus there is a net effect of forward sales on spot strategies. The second term represents the present value of the sensitivity of the profits covariance with respect to spot prices. As shown in Bessembinder and Lemon (2002), such covariance can be related, by means of a Taylor expansion, to spot price variance and skewness. That is, risk-averse producers will take account of the fact that manipulating the moments of future spot prices changes their valuation of future profits. Moreover, general utility functions might imply the consideration of other moments (kurtosis...).

7. Final remarks and policy implications

One of the primary objectives of all previous developments is to analyze the incentives for forward trading in oligopolistic markets. In particular, they aim at answering the question of whether forward trading is motivated only by hedging purposes or even risk-neutral players have strategic incentives to trade forward. Most of the works analyzing the forward-spot interaction assume that the no-arbitrage condition under perfect competition directly generalizes to the oligopolistic case. Thus, the model imposes that forward and spot prices are the same. This serves as the condition to clear the forward market regardless the particular model for the forward demand. However, this simplified version of no-arbitrage implicitly defines forward incentives, and thus determines the strategic behavior in forward markets. In this paper, we have relaxed this assumption to show that it is not enough that forward trading results in lower spot prices to obtain outcomes that are more efficient. It is also necessary that players do not raise the forward price to compensate for the lower spot prices, and that there is no incentive to give signals to future forward negotiation through present spot prices.

The paper has shown that the assumption that the incentive to manipulate the forward price at a certain node of the event-tree is defined just by the quantity sold in that node, requires liquidity of the power market. To obtain the result that firms' profits are the same as in the no-trade case we assumed that the portfolio remains stable. When these assumptions do not hold, there is a dependence of forward prices and spot strategies, and then the effect described in the two-period model is activated. What we have done is two separate two effects: when there is renegotiation, then the private info model is activated. If there is no renegotiation, just the two-period model. Typically, the real situation would be a combination of the two. In both cases, the pro-competitive effect is not present.

We will use these ideas to investigate the consequences on policy making. Actually, the introduction of forward markets has been proposed as a means to mitigate market power, and usually these proposals are based on the theoretical model proposed in Allaz and Vila (1993). This paper has shown that such pro-competitive effect rests on very particular assumptions. Thus, regulated forward contracting can be analyzed from the viewpoint of the following criteria:

- Whether the regulatory measure avoids the internalization in the forward price of the loss of spot market power opportunities
- Whether the measure destroy the signaling incentive of private information

There is and have been a wide range of regulatory measures that imply some kind of regulated forward contracting. From the viewpoint of the analysis developed in the paper, there are, on the one hand, measures imposing the obligation of writing forward contracts, but at a price determined by some kind of market-based mechanism -e. g. an auction-, and on the other, measures imposing the obligation of forward contracting at a regulated price. The difference between them can be studied under their effects on the internalization incentive. In the former kind of measure, the situation can be thought of as a two-period market, so that players can internalize the loss of market power by raising the forward price, so that the exercise of market power is equally high (in terms of system costs) as in the case with contracting obligation. In the latter kind of measure, the ability to internalize the loss of market power disappears. Therefore, with a regulated price for forward contracts, the opportunities of exercising market power are reduced.

Another relevant feature of these measures is the contract duration, because it affects the signaling incentive. Actually, short duration contracts imply that there is an incentive to raise spot prices caused by the signaling game. On the other hand, large durations eliminate the incentive, as players cannot manipulate the forward price by driving up the spot price. In the case of forward contracting at a regulated price, if such price is constant, players have no ability to respond to the loss of market power associated with the contract, and thus the duration of the contract seems to be irrelevant.

Nonetheless, these prices are often actualized every time the contract expires. This price actualization can be thought of as a renegotiation of the contract, which might be manipulated by players by manipulating the corresponding spot prices. Therefore, in this case, short duration contracts will not destroy the signaling incentive, and the market will not be more competitive. Good examples of long-term contracts allocated through an auction are reliability forward contracts, or Virtual Power Plant contracts. In principle, they will not be able to eliminate the internalization effect, so that the profits of power producers will be the same as in the case without the contracting obligation. However, in presence of uncertainty, the internalization process will be more difficult for large durations. In particular, if players are risk averse, they will internalize the value of the loss of market power. If they are risk averse and the uncertainty is large, the value of this loss will be negligible. Vesting contracts are typical examples of a forward contract with regulated price. In this case, as both the price and the quantity are fixed, the two incentives disappear regardless the duration of the contract. If, as in the previous case, the renegotiation process can be manipulated, then short durations will imply absence of the pro-competitive effect.

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Appendix: Detailed event-tree notation

As in the two-period model, the uncertainty associated with each period is described by means of a finite set of states $\mathrm{S} = \{1,\ldots,S\}$, but in this case, the two-period model needs to be extended in two ways:

- To consider more time periods, namely $\mathrm{T} = \{1,\ldots,T\}$
- To define the concept of partial information

The latter objective can be achieved by considering partitions of the set of states. Thus, the unfolding of information is described by means of a sequence of partitions $\{\mathrm{F}_0, \mathrm{F}_1, \ldots, \mathrm{F}_T\}$ where $\mathrm{F}_0 = \{1,\ldots,S\}$, $\mathrm{F}_T = \{\{1\},\ldots,\{S\}\}$. An additional condition on the way the information unfolds is that it increases over time. Thus, F_t is finer than F_{t-1}. Therefore, the information available at time t is the subset of the partition F_t in which the state s lies. **Erro! Fonte de referência não encontrada.** represents an example of this idea.

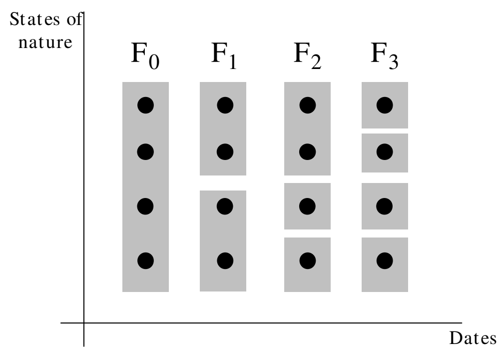

Figure 4.A sequence of partitions for 4 periods and 4 states.

We will use an event-tree approach to represent this information process, which is based on considering each subset of the sequence $\{\mathrm{F}_0, \mathrm{F}_1, \ldots, \mathrm{F}_T\}$ as a single node of an event-tree. **Erro! Fonte de referência não encontrada.** shows the event-tree associated with the previous example.

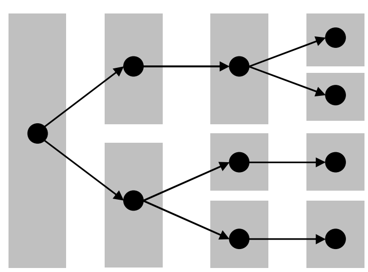

Figure 5.Event-tree corresponding to the sequence of partitions in **Erro! Fonte de referência não encontrada.**.

Next, we will define some concepts and notation that will be used below. These definitions are represented in **Erro! Fonte de referência não encontrada.**.

- Event-tree: let $\mathrm{F} = \{\mathrm{F}_0, \mathrm{F}_1, \ldots, \mathrm{F}_T\}$ be a sequence of partitions of S, with $\mathrm{F}_0 = \{1,\ldots,S\}$, and F_t finer than F_{t-1}. For each $t \in \mathrm{T}$ and for each $\sigma \in \mathrm{F}$, the pair $n = (t, \sigma)$ is a node. The set N consisting of all nodes is called an event-tree

$$\mathrm{N} = \bigcup_{\substack{t \in \mathrm{T} \\ \sigma \in \mathrm{F}}} (t, \sigma)$$

- Predecessor: the unique node $n = (0, \mathrm{S})$ is called the initial node. The set of non-initial nodes is denoted by N^+. For each $n = (t, \sigma) \in \mathrm{N}^+$ there is a unique subset $\sigma' \in \mathrm{F}_{t-1}$ such that $\sigma' \supset \sigma$. Thus, the node $n^- = (t-1, \sigma')$ is called the predecessor of n
- Successor: a node $n = (T, \sigma)$ with $\sigma \in \mathrm{F}_T$ is called a terminal node, and the set of all terminal nodes is $\mathrm{N}^T = \bigcup_{\sigma \in \mathrm{F}_T} (T, \sigma)$; the set of all non-terminal nodes is denoted by N^-. For each $n = (t, \sigma) \in \mathrm{N}^-$, the immediate successors are defined by the set

$$n^+ = \left\{ n' \in \mathrm{N} \middle/ n' = (t+1, \sigma'), \quad \sigma' \subset \sigma \right\}$$

- Subtree: for any node $n \in \mathrm{N}$, the set of all nodes that succeed n is called the subtree $\mathrm{N}(n)$ starting at n

$$\mathrm{N}(n) = \left\{ n' \in \mathrm{N} \middle/ n' \geq n \right\}$$

- The set of all strict successors of n is denoted by

$$\mathrm{N}^+(n) = \left\{ n' \in \mathrm{N}(n) \middle/ n' > n \right\}$$

- And the set of all non-terminal successors is

$$\mathrm{N}^-(n) = \left\{ n' \in \mathrm{N}(n) \middle/ n \in \mathrm{N}^- \right\}$$

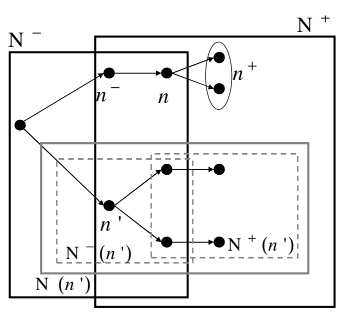

Figure 6. Event-tree definitions.