**Endogenous Labor Effort and Wage Differentials in a Dynamic Model of Capacity Utilization and Economic Growth**

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Abstract: Motivated by a considerable (experimental and empirical) evidence on endogenous labor effort and inter-industry wage differentials, this paper explores implications for capacity utilization, economic growth and macroeconomic stability of firms using different strategies to elicit effort from workers. The frequency distribution of the effort-eliciting strategies in the population of firms is governed by a replicator dynamics that generates wage differential as a long-run, evolutionary equilibrium outcome. Although firms willing to elicit more labor effort have to compensate workers with a higher wage rate, a larger proportion of firms adopting such a strategy will not necessarily produce higher rates of capacity utilization and economic growth. The intuition is that, depending on the resulting increase in average labor productivity, the share of wages in income (and hence aggregate effective demand) may vary negatively with the proportion of firms paying higher wages.

Keywords: economic growth, capacity utilization, labor effort, labor productivity, wage differentials.

Resumo: Motivado por uma considerável evidência (experimental e empírica) sobre esforço de trabalho endógeno e diferenciais de salários, o artigo explora implicações em termos de utilização da capacidade, crescimento econômico e estabilidade macroeconômica de um contexto em que as firmas usam diferentes estratégias para extrair esforço dos trabalhadores. A distribuição de frequência dessas estratégias na população de firmas é governada por uma dinâmica de replicação que gera diferencial de salários como um resultado de equilíbrio evolucionário de longo prazo. Embora as firmas que desejam extrair mais esforço tenham que pagar um salário mais elevado, uma maior proporção de firmas que adotam essa estratégia não necessariamente gera taxas mais elevadas de utilização da capacidade e crescimento econômico. A intuição é que, dependendo da elevação resultante na produtividade do trabalho média, a parcela dos salários na renda (e, portanto, a demanda efetiva agregada) pode variar negativamente com a proporção de firmas que pagam salários mais elevados.

Palavras-chave: crescimento econômico, utilização da capacidade, esforço de trabalho, produtividade do trabalho, diferenciais de salário.

**J.E.L. Classification Codes:** O41, O11, J31.

**Anpec Classification**: Área 6 – Crescimento, Desenvolvimento Econômico e Instituições.

## 1. Introduction

Clearly, a firm's labor productivity is endogenous to the level of effort with which its workers perform their tasks. Even when workers combine their labor with identical units of physical capital, and are homogeneous in all other relevant dimensions, the commitment and effectiveness with which they participate in the production process is rather heterogeneous. Indeed, there is compelling empirical and experimental evidence on labor effort being endogenous (especially to wage compensation).

Meanwhile, following Dickens and Katz (1987) and Krueger and Summers (1988), there has always been renewed interest among labor economists in analyzing inter-industry wage differentials. There is considerable empirical evidence on persistence of inter-industry wage differentials, even after controlling for observable characteristics (schooling or human capital, gender, years of experience, etc). The key results from estimations for a large number of countries, different time periods, and using different specifications of the empirical earnings function, are that inter-industry wage differentials do exist, are of a non-negligible magnitude, and are highly persistent over long periods of time.[1] In fact, large and persistent wage differentials have also been found to exist across establishments within industries, even after controlling for standard covariates and individual fixed effects (e.g., see Groshen, 1991).

Insightfully, in the contested-exchange approach elaborated by Bowles and Gintis (1990) it is shown that there needs to be a "labor extraction function" specifying how much actual labor firms obtains from a given labor input, since the labor contract is not costlessly enforced. As the labor contract alone cannot ensure the employer that work will be performed as desired and expected by him, the labor exchange is, as Bowles and Gintis (1990) phrase it, "contested". Meanwhile, the interesting Akerlof (1982) gift-exchange model portrays the offer of employment by a firm as an offer to "exchange gifts", with the worker's effort level therefore indicating the size of the reciprocal gift. In fact, the gift-exchange game has established that, in the laboratory, higher wages offered by an employer lead to considerably more costly effort provision. Fehr and Falk (1999), for instance, conducted experimental double auctions to conclude that workers' effort is positively related to the wage level, with workers choosing low effort levels in response to low wages and high (i.e., more costly) levels in response to high wages. Fehr et. al. (1998), meanwhile, conducted experiments in the context of an experimental labor market with firms as wage setters and workers as effort setters. In this market, labor contracts were incomplete because effort was not stipulated in the contracts and there was the possibility for workers to reciprocate high wages by nonminimal effort choices. The results of their experiments show that reciprocal behavior of workers is a persistent phenomenon, with workers' efforts varying positively with the wage payments. Charness (2004) also find a positive relationship between effort and wage, with distributional concerns and reciprocity playing a major role, while the laboratory experiments conducted by Charness and Kuhn (2007) show that workers' effort choices are highly positively sensitive to their own wages.

Motivated by all such (experimental and empirical) evidence on endogenous labor effort and inter-industry wage differentials, this paper sets forth a dynamic model to explore the implications for capacity utilization, economic growth and macroeconomic stability of firms adopting different strategies to elicit effort from workers. The frequency distribution of the existing effort-eliciting strategies in the population of firms is not parametric, though, being governed by a replicator dynamics that generates a wage differential as a long-run,

[1] Dickens and Katz (1987) offer a comprehensive review of early studies documenting the existence of inter-industry wage differentials, while Caju et al. (2005) provide a more updated literature review on the same issue.

evolutionary equilibrium outcome (a result which is in keeping with the empirical evidence reported above on the persistence of inter-industry wage differentials). Although firms willing to elicit more labor effort have to compensate workers with a higher wage rate, a larger proportion of firms adopting such a strategy will not necessarily produce higher rates of capacity utilization and economic growth. The intuition is that, depending on the resulting differential effort (and hence the ensuing rise in average labor productivity), the wage share in income (and hence aggregate effective demand) may vary negatively with the proportion of firms paying higher wages.

The remainder of the paper is organized in the following manner. Section 2 describes the structure of the model, while Section 3 analyses its behavior in the short and long runs. The paper closes with a summary of the main conclusions derived along the way.

## 2. Structure of the model

The economy is a closed one and with no government activities, producing only one good for both investment and consumption purposes. Output production is carried out by imperfectly-competitive firms that combine capital and labor through a fixed-coefficient technology. Firms have some leverage on their price but are small with respect to the overall market. They produce (and hire labor) according to effective demand, which is assumed to be insufficient for any of them to produce at full capacity at prevailing prices. The economy is populated by homogeneous workers whose effort in performing labor tasks is nonetheless endogenous. Firms are also homogeneous except with respect to the strategy for eliciting effort from workers they choose to follow, which determines the wage rate they are willing to pay. At a given short run each firm chooses between paying a lower wage rate $w_l \in \mathbb{R}_{++}$ or paying a higher wage rate $w_h > w_l$. A firm that decides to pay a higher wage rate is termed $h$-firm, while a firm that decides to pay a lower wage rate is referred to as $l$-firm. An $h$-firm is willing to pay a higher wage rate because it allows it to elicit more effort from workers. As a result, in a given short run there is a proportion $\lambda \in [0,1] \subset \mathbb{R}$ of $h$-firms, while the remaining proportion, $1-\lambda$, is composed by $l$-firms. Workers hired by a given type of firm, and hence receiving the same wage rate, behave alike as regard effort provision. However, while labor effort (and hence productivity) is homogeneous across firms of a certain type, labor effort (and productivity) is heterogeneous across the two types of firms. Meanwhile, the resulting labor productivity differential is (intertemporally) endogenous, varying over time with the frequency distribution of effort-eliciting strategies played by firms.

Having chosen a given wage compensation strategy, a firm makes a take-it-or-leave-it offer to available workers to hire as many workers it needs to produce its demand-determined level of output. These workers, who are always in excess supply, not only take the received offer, but also deliver the labor effort ensuring that their actual productivity is equal to the expected one by firms when they decide what wage compensation strategy to offer. As a result, workers actually have a higher productivity if the hiring firm pay they them a higher wage.

To keep focus on the dynamics of the distribution of wage compensation strategies and its implications for capacity utilization and growth, we simplify matters by assuming that the wage rate remains constant over time. The distribution of effort-eliciting strategies across firms, $(\lambda, 1-\lambda)$, which is given in the short run as a result from the previous dynamics of the economy, changes over time according to a replicator dynamics. All firms charge the same price for the (homogeneous) good they produce, with the general price level, $P$, remaining constant over time. In the short run, for given values of wage rate differential, labor effort differential (as proxied by the labor productivity differential) and frequency distribution of effort-eliciting strategies across firms, individual markups vary so as to ensure that individual prices are equalized. Hence, over time changes in the frequency distribution of effort-eliciting strategies across firms, by leading to changes in the average markup and the average labor productivity (and hence in the average wage rate and the share of wages in income), result in changes in aggregate effective demand and therefore in the short-run equilibrium values of capacity utilization and economic growth.

Let $w \equiv \lambda w_h + (1-\lambda) w_l$ be the average wage rate, so that the differential between the higher wage rate and the average wage rate is given by $w_h - w = (1-\lambda)(w_h - w_l) \geq 0$ for all $\lambda \in [0,1] \subset \mathbb{R}$. We assume that the extent to which labor productivity in $h$-firms is greater than labor productivity in $l$-firms varies positively with the relative wage rate differential given by $w_h - w$. Formally:

$$\alpha \equiv \frac{a_h}{a_l} = f\left((1-\lambda)(w_h - w_l)\right), \tag{1}$$

where $a_i = X_i / L_i$ denotes labor productivity in firms $i = h,l$, $X_i$ is total output of firms $i = h,l$, and $L_i$ is total employment of firms $i = h,l$. In the above expression, $f(0) = 1$ and $f'(\cdot) > 0$ for all $\lambda \in [0,1] \subset \mathbb{R}$. In fact, if all firms are of the $h$-type $(\lambda = 1)$, the relative wage rate differential given by $w_h - w = (1-\lambda)(w_h - w_l)$ vanishes. In this case, as labor productivity is uniform across firms, it follows that $\alpha = f(0) = 1$. Meanwhile, if all firms are of the $l$-type $(\lambda = 0)$, the relative wage rate differential $w_h - w = (1-\lambda)(w_h - w_l)$ takes its maximum value. In this case, an $l$-firm which decides to switch effort-eliciting strategy to become an $h$-firm is therefore able to elicit the largest additional effort, since $f(w_h - w_l) > f\left((1-\lambda)(w_h - w_l)\right)$ for all $\lambda \in (0,1] \subset \mathbb{R}$. Moreover, the greater the proportion of $h$-firms, the smaller the labor productivity differential between the two types of firms: in fact, as $w_h > w_l$ and $f'(\cdot) > 0$ for all $\lambda \in [0,1] \subset \mathbb{R}$, it follows that $(d\alpha / d\lambda) = -(w_h - w_l) f'\left((1-\lambda)(w_h - w_l)\right) < 0$.

The following rationales can be offered for the above specification of the productivity differential. First, the average wage rate, $w$, can be interpreted as workers' expected outside option or fallback position. Therefore, workers who are offered a higher wage rate perform their tasks with an additional effort (relatively to the level of effort they would provide if offered a lower wage) that varies positively with the excess of the higher wage rate over their expected outside option. Second, the average wage rate can be seen as determining the reservation wage of workers who are offered a higher wage rate (while workers who are offered a lower wage rate implicitly have a reservation wage which is equal to zero). Indeed, Brown and Taylor (2011), using individual-level data for the UK, find that the reservation wage and the expected wage are highly correlated at 0.89, which is statistically significant at the 1% level. They also find that the mean of the logarithm of the predicted market wage (4.47) is lower than that of either the reservation wage (4.65) or the expected wage (4.60). Meanwhile, Brown and Taylor (2013), also using individual-level data for the UK, find that expected wages are positively associated with reservation wages. In fact, they find that the mean log hourly reservation wage and expected wage are, respectively, 1.44 and 1.55. They also find that the elasticity of reservation wages with respect to expected wages is positive and elastic (1.479).

As all firms produce the same good, they sell it at the same price, $P$, which, for simplicity, is normalized to one. This price is set as a markup over the unit labor cost:

(2) $$1=(1+z_h)\frac{w_h}{a_h}=(1+z_l)\frac{w_l}{a_l},$$

where $z_i > 0$ stands for the markup set by firms $i = h,l$. For simplicity, and without loss of generality, we normalize $a_l$ to one. Hence, an $h$-firm, by having decided to follow a higher-wage effort-eliciting strategy, is able to use the corresponding productivity differential given by $\alpha$ to (try to) set a higher markup without eroding its price competitiveness (and therefore without compromising its ability to sell as much output as $l$-firms). But, as such productivity differential varies over time with the frequency distribution of effort-eliciting strategies across firms, so does therefore the average markup given by $z \equiv \lambda z_h + (1-\lambda) z_l$. Meanwhile, the short-run equilibrium values of the individual markups can be obtained by combining (1) and (2):

(3) $$z_h^* = \frac{f\left((1-\lambda)(w_h - w_l)\right)}{w_h} - 1$$

and

(4) $$z_l^* = \frac{1}{w_l} - 1.$$

Note that $z_h^* > z_l^*$ if $f\left((1-\lambda)(w_h - w_l)\right) > \frac{w_h}{w_l}$, a condition that will not be satisfied if $\lambda$ is sufficiently high. Indeed, if the proportion of $h$-firms is relatively high, the productivity differential $a_h > a_l = 1$ will not be enough to offset the wage differential $w_h - w_l > 0$.

The total real profits of $h$- and $l$-firms are given, respectively, by:

(5) $$R_h \equiv X_h - w_h L_h = \left(1 - \frac{w_h}{a_h}\right) X_h$$

and

(6) $$R_l \equiv X_l - w_l L_l = (1 - w_l) X_l .$$

Using (1), (5), and (6), the shares of real profit in total real income of $h$- and $l$-firms are given by:

(7) $$\pi_h \equiv \frac{R_h}{X_h} = 1 - \frac{w_h}{f\left((1-\lambda)(w_h - w_l)\right)}$$

and

(8) $$\pi_l \equiv \frac{R_l}{X_l} = 1 - w_l .$$

Therefore, although the share of profits in income of $l$-firms is constant, the share of profits in income of $h$-firms varies with the proportion of these firms:

(9) $$\frac{\partial \pi_h}{\partial \lambda} = -\left[\frac{w_h - w_l}{(1-\lambda) f(\cdot)}\right] \xi < 0,$$

for all $\lambda \in [0,1) \subset \mathbb{R}$, where $\xi \equiv \frac{f'(\cdot)(1-\lambda)w_h}{f(\cdot)}$ is the elasticity of the labor productivity differential with respect to the higher wage rate, with $\xi > 0$ for all $\lambda \in [0,1) \subset \mathbb{R}$. The intuition behind (9) is that the greater the proportion of $h$-firms, the smaller the differential between the higher wage rate and the average wage rate and hence the smaller the eliciting of additional labor effort.

As shown formally later, firms accumulate capital at the same rate, which implies that the aggregate capital stock, $K$, remains uniformly distributed across firms. It then follows that:[2]

(10) $$\frac{K_h}{\lambda} = \frac{K_l}{1-\lambda} = K,$$

where $K_i$ is the total capital stock of firms $i = h,l$. Given (10), it follows that the proportion of the aggregate capital stock that is available to the firms of each type is proportional to the share of each type in the population of firms, that is, $K_h / K = \lambda$ and $K_l / K = 1-\lambda$.

As prices are equalized across firms, the aggregate effective demand is uniformly distributed not only across firms, but also across wage compensation strategies. Therefore, individual nominal demand (or individual nominal revenue), which is the same for all firms playing a given wage compensation strategy, is also equalized across wage compensation strategies. As a result, capacity utilization is likewise equalized across wage compensation strategies:

(11) $$u_h = u_l = u = \frac{X}{K},$$

where $u_i \equiv X_i / K_i$ is (capital) capacity utilization of firms $i = h,l$, while $u$ denotes average capacity utilization and $X$ average output.

Using (5), (6), (7), (8), (10) and (11), the profit rates of $h$- and $l$-firms can then be expressed as follows:

(12) $$r_h \equiv \frac{R_h}{K_h} = \left(1 - \frac{w_h}{a_h}\right)\frac{X_h}{K_h} = \pi_h u,$$

and

(13) $$r_l \equiv \frac{R_l}{K_l} = (1-w_l)\frac{X_l}{K_l} = \pi_l u.$$

While the frequency distribution of wage compensation strategies is given in the short run, it varies over time according to an evolutionary dynamics based on expected payoffs. More precisely, an individual firm revises periodically its strategy for eliciting labor effort in a manner described by the following replicator dynamics:

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[2] The meaning of the implied assumption (10) can be explained as follows. Let $F$ be the total measure of firms in the economy and $F_h$ the measure of $h$-firms. As the aggregate capital stock is uniformly distributed across firms, it follows that $\frac{K_h}{F_h} = \frac{K_l}{F - F_h} = \frac{K}{F}$. By definition, $\lambda = \frac{F_h}{F}$, so we obtain (10) by multiplying both sides of these equalities by $F$.

(14) $$\dot{\lambda} = \lambda(r_h - r) = \lambda(1-\lambda)(r_h - r_l) = \lambda(1-\lambda)(\pi_h - \pi_l)u .$$

where $r \equiv \lambda r_h + (1-\lambda) r_l$ is the average profit rate, and the last equality are both obtained using (12) and (13). Under the replicator dynamics, therefore, the frequency of the higher-wage effort-eliciting strategy in the population of firms increases exactly when it has above-average payoff.

Using (7) and (8), the replicator dynamics (14) then becomes:

(15) $$\dot{\lambda} = \lambda(1-\lambda)\left[ w_l - \frac{w_h}{f\left((1-\lambda)(w_h - w_l)\right)} \right] u .$$

Note that there are two pure strategy equilibria, $\lambda = 0$ and $\lambda = 1$. There is also a mixed strategy equilibrium, $\lambda^* \in (0,1) \subset \mathbb{R}$, which is implicitly defined as follows:

(16) $$\pi_h - \pi_l = 0 \Leftrightarrow w_l = \frac{w_h}{f\left((1-\lambda)(w_h - w_l)\right)} \Leftrightarrow \phi(\lambda) \equiv f\left((1-\lambda)(w_h - w_l)\right) - \frac{w_h}{w_l} = 0 .$$

We can show that there is one, and only one, $\lambda^* \in (0,1) \subset \mathbb{R}$ such that (16) is satisfied, that is, such that $\phi(\lambda^*) = 0$. The function $\phi(\lambda)$ is continuous in the interval $[0,1] \subset \mathbb{R}$ because it is the sum of continuous functions. By (1), it follows that $\phi(0) \equiv f(w_h - w_l) - (w_h / w_l) > 0$ and $\phi(1) = 1 - (w_h / w_l) < 0$. Since $\phi(0) > 0$, $\phi(1) < 0$ and $\phi$ is continuous in the close interval $[0,1] \subset \mathbb{R}$, we can apply the intermediate value theorem and conclude that there is some $\lambda^* \in (0,1) \subset \mathbb{R}$ such that $\phi(\lambda^*) = 0$. Moreover, given that $f'(\cdot) > 0$ for all $\lambda \in [0,1] \subset \mathbb{R}$ and $w_h - w_l > 0$ for all $\lambda \in [0,1) \subset \mathbb{R}$ we have $\phi'(\lambda) \equiv -f'(\cdot)(w_h - w_l) < 0$ for all $\lambda \in [0,1) \subset \mathbb{R}$. Hence, since $\phi'(\lambda)$ is continuous in the interval $[0,1] \subset \mathbb{R}$, there is only one $\lambda^* \in (0,1) \subset \mathbb{R}$ such that $\phi(\lambda^*) = 0$.

Moreover, we can show that the equilibria given by $\lambda = 0$ and $\lambda = 1$ are both unstable, while the equilibrium given by $\lambda^* \in (0,1) \subset \mathbb{R}$ is asymptotically stable. Given that $\lambda(1-\lambda) > 0$ for all $\lambda \in (0,1) \subset \mathbb{R}$, the behavior of the state variable $\lambda$ depends on $\phi(\lambda)$. Given that $\phi'(\lambda) \equiv -f'(\cdot)(w_h - w_l) < 0$ for all $\lambda \in [0,1) \subset \mathbb{R}$ and $\phi(\lambda^*) = 0$, if $\lambda \in (0, \lambda^*) \subset \mathbb{R}$, then $\phi(\lambda) > 0$ and if $\lambda \in (\lambda^*, 1) \subset \mathbb{R}$, then $\phi(\lambda) < 0$. As a result, given (15) and (16), we can deduce that for any initial condition $\lambda \in (0,1) \subset \mathbb{R}$ the system converges to the mixed strategy equilibrium given by $\lambda^* \in (0,1) \subset \mathbb{R}$.

## 3. The behavior of the model in the short and long runs

The short run is defined as the time frame in which the population of firms, $F$, the capital stock, $K$, the labor supply, $N$, the wages rates, $w_h$ and $w_l$ (and hence the productivity levels, $a_h$ and $a_l$), the price level, $P = 1$, and the proportion of firms playing the higher-wage effort-eliciting strategy, $\lambda$, can all be taken as given. The short-run equilibrium values of the individual markups, $z_h$ and $z_l$, are given by (3) and (4), which we assume to be achieved fast enough for them to be taken as given as far as the income-generating process driven by aggregate effective demand is concerned. The existence of excess aggregate (and individual) capacity implies that aggregate (and individual) output adjusts in the short run to remove any excess aggregate (and individual) demand or supply in the economy (for any individual firm), so that in short-run equilibrium, aggregate savings, $S$, are equal to aggregate investment, $I$.

As regards saving and consumption behavior, we assume that workers spend all of their wage income on consumption, while firm-owner capitalists save a constant proportion $s \in (0,1) \subset \mathbb{R}$ of the corresponding real profits. Then, using (11), aggregate saving normalized by the aggregate stock of capital can be expressed as follows:

$$\frac{S}{K} = s\left(\frac{R_h}{K} + \frac{R_l}{K}\right) = s\left[\lambda \pi_h u_h + (1-\lambda)\pi_l u_l\right] = s\left[\lambda \pi_h + (1-\lambda)\pi_l\right]u . \tag{17}$$

Let us now turn to the derivation of the aggregate investment function. For simplicity, we have assumed above that workers consume all of their income no matter for what type of firm they work, while firm-owner capitalists have a homogeneous saving behavior no matter what wage compensation strategy they adopt. In the same spirit of simplicity, we assume that firms behave alike as far as desired investment is concerned:

$$\frac{I^d}{K} = \frac{I_h^d}{K} = \frac{I_l^d}{K} = \beta + \gamma r^e + \delta u^e , \tag{18}$$

where $r^e$ and $u^e$ denote, respectively, the (common) expected rates of profit and capacity utilization by $h$-firms and $l$-firms, while $\beta \in \mathbb{R}_{++}$, $\gamma \in \mathbb{R}_{++}$ and $\delta \in \mathbb{R}_{++}$ are parametric constants. We follow Kalecki (1935) and Robinson (1962) in assuming that the (average) rate of capital accumulation depends on the (average) expected profit rate, which we then proxy (as Kalecki and Robinson themselves often do) by the (average) current rate of profit. The rationale is that the current profit rate is not only an index of expected future earnings, but also provides internal funding for capital accumulation plans and makes it easier for firms to obtain external funding. Meanwhile, we follow Rowthorn (1981) and Dutt (1984), who in turn follow Steindl (1952), in making the (average) desired rate of capital accumulation to depend positively on the (average) rate of capacity utilization due to accelerator-type effects (recall that $u_h = u_l = u$). Therefore, as we assume that the expected rates of capacity utilization and profit are proxied by their current average values, it follows from (10)-(13) that:

$$u^e = \lambda u_h + (1-\lambda)u_l = u , \tag{19}$$

$$r^e = \frac{R_h + R_l}{K} = \lambda r_h + (1-\lambda) r_l = [\lambda \pi_h + (1-\lambda)\pi_l]u . \tag{20}$$

Substituting (19)-(20) in (18) we get the aggregate desired investment function:

$$\frac{I^d}{K} = \beta + \{\gamma[\lambda \pi_h + (1-\lambda)\pi_l] + \delta\}u . \tag{21}$$

Finally, by substituting (17) and (21) in the goods market equilibrium condition given by $S/K = I^d/K$, we can solve for the short-run equilibrium capacity utilization:

$$u^* = \frac{\beta}{(s-\gamma)\left[\lambda \pi_h + (1-\lambda)\pi_l\right] - \delta} . \tag{22}$$

Using (7) and (8), the preceding expression can be re-written as follows:[3]

(22-a) $$u^* = \frac{\beta}{(s-\gamma)\left\{\lambda\left[1-\frac{w_h}{f\left((1-\lambda)(w_h-w_l)\right)}\right]+(1-\lambda)(1-w_l)\right\}-\delta}.$$

Moreover, we can substitute (22) in (17) to obtain the short-run equilibrium growth rate:

(23) $$g^* = s\left[\lambda\pi_h+(1-\lambda)\pi_l\right]u^*.$$

In the long run, meanwhile, we assume that the short-run equilibrium values of the variables are always attained, while the frequency distribution of effort-eliciting strategies (and hence the average levels of labor productivity and markup) varies over time according to the replicator dynamics. Substituting (22) in (14) we obtain:

(23) $$\dot{\lambda} = \lambda(1-\lambda)(\pi_h-\pi_l)\left\{\frac{\beta}{(s-\gamma)\left[\lambda\pi_h+(1-\lambda)\pi_l\right]-\delta}\right\}.$$

This version of the replicator dynamics has the same three equilibria that we obtained for (15). As before, the two pure strategy equilibria of (23), which are given by $\lambda=0$ and $\lambda=1$, are unstable, and the mixed strategy equilibrium given by $0<\lambda^*<1$ is asymptotically stable. The reason is that it follows from (10) that:

(24) $$\left.\frac{\partial\dot{\lambda}}{\partial\lambda}\right|_{\lambda=\lambda^*} = \left[\frac{\beta}{(s-\gamma)(1-v_l)}\right]\left.\frac{\partial\pi_h}{\partial\lambda}\right|_{\lambda=\lambda^*}<0.$$

Let us now compute the rates of capacity utilization and economic growth associated with each of the three long-run equilibrium solutions. Corresponding to the mixed strategy long-run equilibrium given by $\lambda^*$, which is defined implicitly by the condition $\pi_h-\pi_l=0$, we obtain the following long-run equilibrium capacity utilization:

(25) $$u^*(\lambda^*)\equiv u^{**} = \frac{\beta}{(s-\gamma)\pi_l} = \frac{\beta}{(s-\gamma)(1-w_l)-\delta}.$$

Moreover, we can substitute both the condition $\pi_h=\pi_l=1-w_l$ and (25) in (17) to obtain the growth rate corresponding to the mixed strategy long-run equilibrium given by $0<\lambda^*<1$:

(26) $$g^*(\lambda^*)\equiv g^{**} = \frac{s(1-w_l)\beta}{(s-\gamma)(1-w_l)-\delta}.$$

Note that the rates of capacity utilization and economic growth corresponding to the pure strategy long-run equilibrium given by $\lambda=0$, in which the higher wage effort-eliciting strategy gets extinct, are also given by (25) and (26), respectively. However, in the other pure strategy long-run equilibrium ($\lambda=1$), in which only the higher wage effort-eliciting strategy survives, the corresponding values of capacity utilization and growth are given by:

[3] We are assuming that $(s-\gamma)\left[\lambda\pi_h+(1-\lambda)\pi_l\right]-\delta>0$ for all $\lambda\in[0,1]\subset\mathbb{R}$, which is the standard Keynesian stability condition in demand-driven models like the current one. This means that $u^*$ is positive and stable if average saving is more responsive than average desired capital accumulation to changes in average capacity utilization, which in turn requires that the denominator of the expression in (22-a) is positive.

(25-a) $$u^*(1) \equiv u^{***} = \frac{\beta}{(s-\gamma)(1-w_h)-\delta},$$

(26-a) $$g^*(1) \equiv g^{***} = \frac{s(1-w_h)\beta}{(s-\gamma)(1-w_h)-\delta}.$$

Several noteworthy implications follow from the preceding analysis. First, given that $w_h > w_l$, it follows that $u^{***} > u^{**}$, while the corresponding relationship between $g^{***}$ and $g^{**}$ is indeterminate. In words, while capacity utilization is unambiguously higher when all firms adopt the higher wage effort-eliciting strategy than when either no firm or the exact fraction $0 < \lambda^* < 1$ of the firms do so, economic growth can be higher in either case. Second, however, a short-run wage differential resulting from firms following different strategies to elicit effort from workers converges to a wage differential as a mixed strategy long-run, evolutionary equilibrium outcome (a result which is in accordance with the empirical evidence reported in the introduction on the persistence of inter-industry wage differentials). Therefore, unless all firms behave homogenously (as regards wage compensation) to begin with, heterogeneity in the choice of effort-eliciting strategy becomes a permanent feature. Third, the lower wage rate is the only distributive variable on which capacity utilization and growth depend when either no firm or the exact proportion $0 < \lambda^* < 1$ of the firms follow the higher wage effort-eliciting strategy, as shown by (25) and (26). Logically enough, however, the higher wage rate is the only distributive variable on which capacity utilization and growth depend when all firms follow the higher wage effort-eliciting strategy, as shown by (25-a) and (26-b). Fourth, in each of the three equilibrium configurations, capacity utilization and growth vary positively with the corresponding wage rate (due to the increase in the wage share in income) and the parameters of the investment function, and negatively with the saving rate. The intuition is that both capacity utilization and growth vary positively with aggregate effective demand.

## 4. Conclusions

Motivated by some compelling (experimental and empirical) evidence on endogenous labor effort and inter-industry wage differentials, this paper formulates a dynamic model to explore implications for capacity utilization, economic growth and macroeconomic stability of firms adopting different strategies to elicit effort from workers. The frequency distribution of the existing effort-eliciting strategies in the population of firms is governed by a replicator dynamics that generates a wage differential as a long-run, evolutionary equilibrium outcome.

The economy is populated by homogeneous workers whose effort in performing labor tasks is nonetheless endogenous. Firms are also homogeneous except with respect to the strategy for eliciting effort from workers they choose to follow, which determines the wage rate they are willing to pay. At a given short run each firm chooses between paying a lower wage rate or a higher wage rate. A firm may be willing to pay a higher wage rate because it allows it to elicit more effort from workers. Hence, average labor productivity is endogenous, varying over time with the frequency distribution of effort-eliciting strategies played by firms.

In line with the empirical evidence, the extent to which labor productivity in firms paying a higher wage rate is greater than labor productivity in firms paying a lower wage rate is assumed to vary positively with the relative wage rate differential given by the excess of the higher wage rate over the average wage rate. One rationale for this specification is that the average wage rate can be interpreted as workers' expected outside option or fallback position. Another rationale is that the average wage rate can be seen as determining the reservation wage of workers who are offered a higher wage rate.

The replicator dynamics driving the frequency distribution of effort-eliciting strategies has three equilibria: two pure strategy equilibria (only one strategy survives in each of them) and a mixed strategy equilibrium (co-existence of both strategies). While the two pure strategy equilibria are unstable, the mixed strategy equilibrium is asymptotically stable. Therefore, unless all firms behave homogenously (as regards wage compensation) to begin with, heterogeneity in the choice of effort-eliciting strategy becomes a permanent feature. Moreover, while capacity utilization is unambiguously higher when all firms adopt the higher wage effort-eliciting strategy than when either no firm or the exact proportion of the firms given by the mixed strategy solution do so, economic growth can be higher in either case. Meanwhile, in each of the three equilibrium configurations, capacity utilization and economic growth vary positively with the corresponding wage rate and the parameters of the investment function, and negatively with the saving rate. The intuition is that both capacity utilization and growth vary positively with aggregate effective demand.

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