The Neutral Interest Rate and the Stance of Monetary Policy in Brazil

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Abstract

The aim of this paper is to estimate the Brazilian neutral real interest rate and, based on the results, evaluate the stance of monetary policy in Brazil in the last decade. We present some different methodologies of estimations, such as: i) statistical filters; ii) a state space macroeconomic model. By making use of monthly data for the period 2002-2012, and taking into account the influence of the international interest rate in the Brazilian economy, we conclude that the country´s natural rate of interest is around 3.5%. As far as monetary policy stance is concerned a comparison between the interest rate and output gaps shows a countercyclical response to business cycle fluctuations. But a comparison between the interest rate and inflation gaps shows that recently a higher inflation rate has not been accompanied by a lower interest rate gap.

Key words: Natural Rate of Interest, Monetary Policy, State-Space Models

JEL Classification: E43, E52, C22

Resumo

O objetivo deste trabalho é estimar a taxa natural (ou neutra) de juros para a economia brasileira e, baseados no resultado encontrado, analisar a condução da política monetária do país na última década. Para tal, são apresentados diferentes métodos de estimação, tais como: i) filtros estatísticos, ii) modelo macroeconômico em espaço de estado. Os cálculos são feitos com dados mensais para o período 2002-2012. Também é levada em conta a influência da taxa de juros internacional na economia brasileira em algumas das estimações. Os resultados mostram a taxa de juros neutra (natural) do Brasil está em torno de 3.5%. Em termos de condução de política monetária, pode-se observar que a comparação entre o hiato dos juros e o hiato do produto mostra respostas contra cíclicas da autoridade monetária. No entanto, a comparação entre hiato dos juros e hiato da inflação mostra que, recentemente, uma maior taxa de inflação não tem sido acompanhada por respostas em termos de hiato dos juros.

Palavras-Chave: Taxa de Juros Natural, Política Monetária, Modelos em Espaço de Estado

Classificação JEL: E43, E52, C22

DISCLAIMER:

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1. Introduction

In the last few years, there has been an increasing range of articles analyzing the level of the natural rate of interest (or the neutral real interest) of several countries in the world. Such rate is the one that keeps economic activity growth on a sustainable path and, at the same time, keeps inflation stable.

Although the neutral interest rate is an unobservable variable, its calculation is of utmost importance in a context where central banks make use of short term interest rates as the main instrument to conduct monetary policy.

This is not different in Brazil. In fact, a lot has been said and written lately on issues regarding the Brazilian neutral real interest rate and how it has been declining over the last few years. This observed trend is a result of a credible conduct of monetary policy as well as a robust fiscal policy. As an example, the ex-ante real Selic interest rate went from over 30% in 1999 to around 2.00% in the end of 2012. If one looks at a longer-term interest rate, such as the 1-year Pre-DI Swap rate, the declining path is also observed. It lowered from over 20% in 1999 to less than 2% in the end of 2012. The most recent survey of analysts conducted by the Brazilian Central Bank[1] has shown that the country´s neutral interest rate is 5.5% (median estimate) and 5.43 (mean estimate). In November 2010, a similar investigation pointed to a median estimate of the neutral interest rate of 6.75%. Anyway, in any measure one looks at, it is clear that the country's natural rate of interest has been falling recently.

The aim of this paper is to estimate the Brazilian neutral real interest rate and, based on the results, evaluate the stance of monetary policy in Brazil in the last decade. We present some different methodologies of measuring the Brazilian neutral real interest rate: *i*) statistical filters; *ii*) a state space macroeconomic model. One of the main differences between this work and the ones written recently, regarding the Brazilian economy, is the inclusion of variables such as the real exchange rate, credit default swap and an international interest rate. This is important because these variables play an important role in the definition of our interest rate and, as a result, in the definition of our neutral interest rate. The period of analysis range from 2002 up to the end of 2012 and the results show that that the country´s natural rate of interest is around 3.5%. As far as monetary policy stance is concerned a comparison between the interest rate and output gaps shows a countercyclical response to business cycle fluctuations. But a comparison between the interest rate and inflation gaps shows that recently a higher inflation rate has not been accompanied by a lower interest rate gap.

Besides this introduction, this article is structured as follows. Section 2 presents the literature. Section 3 outlines the econometric methodology and the data. Section 4 reports the estimation results related to the Statistical Filters. Section 5 reports the results of the state space macroeconomic model, as well as an evaluation of the monetary policy stance in Brazil in the last decade. Section 6 concludes.

2. The Literature

2.1. The Concept of a Neutral Interest Rate

Wicksell (1907) was the first researcher to work with the concept of a natural interest rate. The author argued that the growth of average price levels was a result of increases in monetary base beyond output growth. With this definition of natural interest rate, Wicksell was able to analyze how the transmission mechanisms of monetary policy worked through the economic environment.

As time went along, central banks started to focus more on interest rates, instead of monetary aggregates, as their main instrument to conduct monetary policy. As a result, the concept of a neutral

[1] Banco Central do Brasil (2012). *Pesquisa sobre Política Monetária*. Disponível em: http://www.bancocentral.gov.br/Pre/ASIMP/bcimprensa/3416-Pesquisa%20Política%20Monetária.pdf

interest rate emerged again. Because of this, Woodford (2003) describes these new types of models as "Neo-Wicksellian".

It is important to make a distinction among three types of interest rates:

i) The actual real interest rate, the main monetary policy instrument, and set by Central Banks when they define their basic interest rate. As the New-Keynesian Macroeconomics advocates, monetary policy nowadays is usually specified as an interest-rule process, such as Taylor Rule (Taylor, 1993).

ii) The long-term equilibrium real interest rate, which is a result of macroeconomic fundamentals (e.g. productivity, population growth, savings).

iii) The neutral real interest rate, or the natural rate of interest, which is related to macroeconomic fundamentals but also to disturbances that influence supply and demand (i.e. the output gap) (Bernhardsen, 2007). It can deviate from long-term equilibrium real interest rate, as output can deviate from its long-term equilibrium, creating an interest rate gap. However, the neutral rate will always be around its long-term equilibrium counterpart.

According to Woodford (2003), a basic macroeconomic model consists of the following equations:

IS Curve: $$y_t^{gap} = E_t y_{t+1}^{gap} - \sigma(\hat{\imath}_t - E_t \pi_{t+1} - \hat{r}_t^n) \quad (1)$$

Phillips Curve: $$\pi_t = \delta y_t + \beta E_t \pi_{t+1} \quad (2)$$

where: 'y_t' is the country´s GDP; 'π_t' is the inflation rate; $\hat{r}_t^n$ = percentage deviation of the natural (Wicksellian) interest rate from its steady state value; $\hat{\imath}_t$ = nominal interest rate.

The system is closed with the specification of the monetary policy conduct in terms of an interest-rate rule, or a central bank's reaction function, which can be a useful way to determine the country's natural rate of interest. In his seminal work, Taylor (1993) set a monetary policy rule in which interest rates are adjusted according to the country's output gap and also to deviations of inflation from its target. In its original formulation, the Taylor Rule, set for the U.S. economy, was defined as follows:

$$r = p + 0.5y + 0.5(p - 2) + 2 \quad (3)$$

where: 'r' is the Federal Funds Rate, 'p' is the inflation rate and 'y' is the output gap.

The last component of Equation (3) is the natural rate of interest, which was set at 2.0% in the case of the U.S. economy. Such rate is observed when there is no gap between the actual output and its trend, and when inflation target is met.

A reaction function to close the small-scale macroeconomic model as proposed in Woodford (2003) can be depicted as follows:

Taylor rule: $$i_t^{nom} = \bar{\imath}_t + \alpha_\pi(\pi_t - \bar{\pi}) + \alpha_y \frac{(y_t^{gap} - \bar{y})}{4} \quad (4)$$

where: y_t^{gap} = output gap ; $\hat{\imath}_t$ = nominal interest rate; $\sigma > 0$ = measure of the intertemporal elasticity of substitution of the aggregate expenditure; π_t = inflation; $\bar{\pi}$ = inflation target; $\bar{\imath}_t$ = average nominal interest rate; $\bar{y}$ = steady state value of y_t^{gap} consistent with $\bar{\pi}$, so that in equilibrium $\bar{\pi} = \bar{\imath}_t$; $\hat{r}_t^n$ = percentage deviation of the natural (Wicksellian) interest rate from its steady state value (Woodford, 2003).

Therefore, the interest-rate gap can be written as:

$$r_t^{gap} = (\bar{\imath}_t + \hat{r}_t^n - \bar{\pi}) - E_t(\pi_{t+1} - \bar{\pi}) + \alpha_\pi(\pi_t - \bar{\pi}) + \frac{\alpha_y(y_t^{gap} - \bar{y})}{4} \quad (5)$$

Woodford (2003) goes on by saying that real interest rates may vary as a result of two different components: *i*) a neutral rate, which is unobserved, but its estimation (despite some uncertainties) is important once it can show whether a conduct of monetary policy is either contractionary or expansionary; *ii*) a real interest rate gap.

It is also important to say that the natural rate of interest, a crucial variable for the conduct of monetary policy, may vary along time and, it is closely linked to the trend growth rate of output (Laubach & Williams, 2003). Therefore, the Wicksellian real interest rate is the one consistent with the output equaling the flexible-price equilibrium level (Walsh, 2003).

Walsh (2003) shows a simpler way to make a close relationship between output gap, the difference between actual output and its potential, as a function of an interest rate gap. Therefore, the interest rate can be seen as the main monetary policy channel in which output is affected, and it can be represented by:

$$y_t^{gap} = E_t y_{t+1}^{gap} - \left(\frac{1}{\sigma}\right) r_t^{gap} \tag{6}$$

2.2. The Empirical Literature

Several articles have estimated the neutral real interest rate in different economies. Some of them make use of Dynamic Stochastic General Equilibrium (DSGE) models and some other estimate the natural rate of interest via a Taylor Rule. But the majority of the papers listed on Table 1 make use of a Kalman Filter to estimate a small-scale macroeconomic model comprised of a IS Curve (Aggregate Demand), a Phillips Curve (Aggregate Supply) and an interest rate rule.

Laubach & Williams (2003) apply the Kalman Filter approach on quarterly U.S. data over the period 1961:1 to 2000:4, and jointly estimate the U.S. natural rate of interest, its potential output and trend growth rate. The authors find a substantial variation in the U.S. neutral rate in the period analyzed. For instance, the rate found for the year 2000 was of 3%. The authors´ conclusion is that "*that policymakers' mismeasurement of the natural rate of interest can cause a significant deterioration in macroeconomic stabilization*".

The list of articles that make similar estimation for advanced economies show that the natural rate of interest for advanced economies is on average 2.5%, and around 5% for emerging market economies (Table 1).

Table 1
Neutral Real Interest Rate: Survey of Articles

Author	Country	Method	Period	Neutral Rate (%)
ADVANCED ECONOMIES				
Laubach & Williams (2003)	USA	Kalman Filter	2002	3.0
Crespo-Cuaresma et al. (2003)	Euro Area	Cycle-Trend Decomposition/ Kalman Filter	2002	1.5-2.0
Lam & Tkacz (2004)	Canada	DSGE	2002	1.25-2
Basdevant et al. (2004)	N. Zealand	Kalman Filter	2003	3.12
Garnier & Wilhelmsen (2005)	Euro Area	Kalman Filter	2004	2.0
Clark & Kozicki (2005)	USA	Kalman Filter	2005	2.5
Amato (2005)	USA	Time-Varying Parameter	2004	3.0
	UK		2004	4.0
	Germany		2004	2.75
Mésonnier & Renne (2007)	France	Kalman Filter	2002	1.5
Bernhardsen (2007)	Norway	Kalman Filter, Taylor Rule	2007	2.5

Table 1 (Cont.)
Neutral Real Interest Rate: Survey of Articles

EMERGING ECONOMIES				
Author	**Country**	**Method**	**Period**	**Neutral Rate (%)**
Brzoza-Brezezina (2004)	Poland	SVAR, Kalman filter	2003	4.0
Soto et al. (2007)	Colombia	Kalman Filter	2005	5.0
Muñoz & Tenorio (2007)	Costa Rica	Kalman Filter	2006	3.3
Fuentes & Gredig (2008)	Chile	Kalman filter	2008	2.8
BBVA (2008)	Mexico	Kalman Filter	2008	3.3
Humala & Rodríguez (2009)	Peru	Kalman Filter	2008	8.0
Öğünç & Batmaz (2011)	Turkey	Kalman Filter	2006	7.5

Several authors have also estimated the natural interest rate for the Brazilian economy. For instance, Muinhos & Nakane (2006) make use of a different set of methods to study important macroeconomic variables for a set of 20 countries using many different methods to compare measures of the real interest rate. For the Brazilian case, the author applied a Hodrick-Prescott (HP) Filter for the period 2000-2004, finding a 10% neutral interest rate.

Borges & Silva (2006) apply a structural VAR model to estimate the Brazilian natural rate of interest for the period ranging from September 2000 to December 2003. The authors find a natural rate of 9.97% for December 2003. They also conclude that the country's effective real interest rate was systematically higher and more volatile than the natural rate in the period analyzed.

Barcellos Neto & Portugal (2009) apply Laubach & Williams' (2003) approach, as well as a dynamic Taylor rule, to estimate the Brazilian neutral rate of interest for the period ranging from September 1999 to October 2005. For the Taylor rule estimation, they found a natural rate of 7.38%, whereas in the Kalman Filter approach, the result was 9.62%.

Duarte (2010) measures the natural interest rate in Brazil between 2000 and 2009 by means of statistical filters, as well as, a Taylor Rule approach. The author points out that Brazil experienced a considerable drop in its natural rate of interest, going from as high as 13.5% in 2001 and decreasing to 5.1% in late 2009. Umezu (2011) makes use of a Bayesian procedure to estimate for the Brazilian case and finds a natural rate of interest of around 5.0% at the end of 2009. Siqueira (2011) measures the Brazilian real equilibrium interest rate, for the period 1999-2010, using different methodologies. The author finds an annual natural rate between 6% and 7%.

More recent studies have been also carried out. By making use of several methods Magud & Tsounta (2012) find an average neutral interest rate of 5.1% for the month of May 2012. Nomura Securities (2012) estimates the level of neutral interest rate in Brazil, via a modified Taylor Rule where the constant term equals the nominal neutral rate. Once this is rate is calculated, the level of consumer prices (12-month expectations and past 12 months) is used to deflate the series. Real neutral rates are found to be between 4.2% and 5.3% (depending on the deflator).

Even the Central Bank of Brazil published a survey in February 2012[2] showing that market economists estimate median neutral rate of interest of 5.5 percent. This is much less than the previous survey, which estimated a rat e of 6.75 percent in November 2010 (Table 2).

[2] Banco Central do Brasil (2012). *Pesquisa sobre Política Monetária.*

Table 2
Neutral Real Interest Rate: Survey of Articles for the Brazilian Case

Author	Method	Period	Neutral Rate (%)
Muinhos & Nakane (2006)	HP Filter	2000-2004	10.0
Borges & Silva (2006)	Structural VAR	2003	9.97
Barcellos Neto & Portugal (2009)	Kalman Filter	2005	9.62
Duarte (2010)	Taylor Rule – Kalman Filter	2010	5.10
Umezu (2011)	Baysean Macro Model	2010	5.00
Siqueira (2011)	Kalman Filter	2011	7.00
Nomura Securities (2012)	Taylor Rule	2013	4.60
Banco Central do Brasil (2012)	Survey	2012	5.50
Magud & Tsounta (2012)	Average of several methods	2012	5.10

3. The Econometric Methodology and the Data

The econometric methodology applied in this work is basically the Kalman Filter[3], which is a recursive linear filter that uses the current observation of a variable to forecast the value of an unobservable variable the period ahead. As a matter of fact, the majority of models in Economics can have a state-space form. Once a model has been put in state-space form, the Kalman Filter allows their parameters to be estimated via maximum likelihood. In fact, modern control and system theory can help time series econometrics, once it makes possible to model the behavior of dynamic systems through states-space representations. Once the representation is set, the Kalman Filter is the statistical algorithm that will carry out the computation of the model (Asemota, 2010).

A state-space model consists of a signal (measurement) and a state (transition) equation. The signal equation is responsible for relating the observable data and the unobservable state variables and it can be represented by the following system:

$$y_t = Z_T \alpha_t + C_t x_t + u_t \tag{7}$$

$$\alpha_t = T_t \alpha_{t-1} + D_t x_t + R_t v_t \tag{8}$$

Equation (7) is the signal equation and Equation (8) is the state equation. The vector y_t represents the signal and x_t is a vector of exogenous (predetermined) observed variables. α_t is a vector of unobserved state variable, so that y_t and x_t are used to make inferences about α_t. 'T' is a matrix that describes the transition between the states. The dynamics of the models is a responsibility of the state equation (Asemota, 2010).

When one is working with state-space representations, the objective is to analyze how the observed values can be used to analyze the state part of the model. As for the Kalman Filter, it is a recursive minimum mean-square error estimator, meaning that, once the state from a previous step is estimated and the current signal is known, it is possible to estimate the current state. The gain in the estimation is the one that generates the minimum mean-square error.

The econometric estimations will be carried out by making use of monthly series for the period ranging from January 2002 to December 2012. The data used in the estimations are as follows:

- π_t = the 12-month CPI (IPCA) inflation
- π_t^{exp} = the 12-month ahead IPCA inflation expectation
- π_t^{target} is the inflation target

[3] See Harvey (1989), Hamilton (1994) and Durbin & Koopman (2001) for more on the topic.

- r_t = the Swap-pre DI rate (deflated by the 12-month ahead IPCA inflation)
- r_t^{gap} = the interest rate gap
- r_t^* = the natural rate of interest
- R^{USA} = the 3-Month (91 Day) U.S. Treasury Bill
- y_t= the log of monthly real GDP (seasonally adjusted) obtained via Chow & Lin´s method
- y_t^{pot} = the potential gap
- y_t^{gap} = the output gap
- $\Delta reer_t$ = the BIS real effective exchange rate (first difference of the log)
- g = the growth rate of the potential output
- z = random factor

It is worth mentioning that we made use of a dynamic version of the Chow and Lin´s method (Santos Silva & Cardoso´s method) in order to interpolate the missing data for the Brazilian quarterly GDP[4]. Santos Silva & Cardoso (2001) present an extension of the Chow and Lin's temporal disaggregation method based. The authors apply a method of disaggregation of the time series using dynamic models, adding considerable flexibility to the system, particularly when the series used are stationary or cointegrated. For the Brazilian case, the variables included in the estimations are: industrial production, ABCR index, Energy cost, Confidence Index FGV, Exports and Imports (quantum), SPC index, Cement production, M1, Central bank of Brazil Index (IBC-Br).

3.1. A State Space Macroeconomic Model for the Brazilian Economy

The model we estimate in this article has a lot to do with Öğünç (2006) and Öğünç & Batmaz (2011). Fuentes & Gredig (2007), Barcellos Neto & Portugal (2009) and Banco Central do Brasil (2010) also worked with something similar.

The first equation of the model is a IS Curve, representing the aggregate demand:

$$y_t^{gap} = \delta_1 y_{t-1}^{gap} + \delta_2 r_t^{gap} + \delta_3 \Delta reer_t + \varepsilon_t^{y^{gap}} \tag{9}$$

The second equation is a Hybrid New-Keynesian Phillips Curve, representing the supply side of the economy:

$$\pi_t = \beta_0 + \beta_1 \pi_{t-1} + \beta_2 \pi_t^{exp} + \beta_3 \Delta reer_t + \beta_4 y_{t-1}^{gap} + \varepsilon_t^{\pi} \tag{10}$$

The remaining set of equations are defined as follows:

$$r_t = r_t^* + r_t^{gap} \tag{11}$$

$$y_t^{pot} = y_t + y_t^{gap} \tag{12}$$

$$y_t^{pot} = y_{t-1}^{pot} + g_{t-1} + \varepsilon_t^{y^{pot}} \tag{13}$$

$$r_t^* = \theta_1 g_t + \theta_2 r_t^{USA} + z_t \tag{14}$$

$$z_t = \phi z_{t-1} + \varepsilon_t^{z} \tag{15}$$

[4] We are indebted to André Maranhão and André Minella for providing us with the data.

$$g_t = g_{t-1} + \varepsilon_t^g \tag{16}$$

$$r_t^{gap} = \rho r_{t-1}^{gap} + \varepsilon_t^{r^{gap}} \tag{17}$$

Equations (11) and (12) are identities. As mentioned by Öğünç & Batmaz (2011) for the Turkish case, countries like Brazil and Turkey suffer the heavy influence of the exchange rate on their economic activity. Therefore, movements on the country´s real effective exchange rate have to be taken into consideration. Equation (13) says that potential output follows a random walk with a drift and Equation (16) is the trend growth rate also following a random walk. Equation (17) says that the Brazilian interest rate gap follows an AR(p) process.

Equation (14) is related to our neutral real interest rate and, as in many other articles, is inspired in Laubach & Williams (2003), who model the rate as a function of "*g*", the growth rate for the potential output and a random factor, called "*z*", which is in the model to care for other determinants of the neutral rate[5].

Our approach differs from Laubach & Williams (2003), and also from Öğünç & Batmaz (2011), in some ways. Öğünç & Batmaz (2011) argue that *"the long-term course of the risk premium is considered to be one of the important determinants of neutral interest rate of Turkey"*. We argue that the international interest rates play an important role in the Brazilian financial system and, therefore, we model our neutral interest rate as in Equation (14). As in many other articles, the "*z*" component in Equation (15) follows an AR(p) process.

In order to perform the estimations, the equations above can be represented in the following state-space form:

Signal Equation

$$\begin{bmatrix} y_t \\ \pi_t \\ r_t \end{bmatrix} = \begin{bmatrix} 1 & -\delta_1 & 0 & 0 & 0 & \delta_2 \\ 0 & -\beta_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_t^{pot} \\ y_{t-1}^{pot} \\ r_t^* \\ z_t \\ g_t \\ r_t^{gap} \end{bmatrix} + \begin{bmatrix} 0 & \delta_1 & 0 & 0 & \delta_3 & 0 \\ \beta_0 & \beta_4 & \beta_1 & \beta_2 & \beta_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ y_{t-1} \\ \pi_{t-1} \\ \pi_t^{exp} \\ \Delta reer_t \\ r_t^{USA} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^{y^{gap}} \\ \varepsilon_t^{\pi} \\ 0 \end{bmatrix}$$

State Equation

$$\begin{bmatrix} y_t^{gap} \\ y_{t-1}^{gap} \\ r_t^* \\ z_t \\ g_t \\ r_t^{gap} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \phi & \theta_1 & 0 \\ 0 & 0 & 0 & \phi & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho \end{bmatrix} \begin{bmatrix} y_{t-1}^{pot} \\ y_{t-2}^{pot} \\ r_{t-1}^* \\ z_{t-1} \\ g_{t-1} \\ r_{t-1}^{gap} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \theta_2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ y_{t-1} \\ \pi_{t-1} \\ \pi_t^{exp} \\ \Delta reer_t \\ r_t^{EUA} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & \theta_1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_t^{y^{gap}} \\ 0 \\ 0 \\ \varepsilon_t^z \\ \varepsilon_t^g \\ \varepsilon_t^{r^{gap}} \end{bmatrix}$$

This first representation is the signal equation and the second one is the state equation of the macroeconomic model for the Brazilian Economy.

3.2. The Pile-Up Problem

Something else must be taken into account when performing these set of estimations. As mentioned in several articles that perform similar regressions as in this article, for instance, Laubach & Williams (2003) and Barcellos Neto & Portugal (2009), the estimations generated in these types

[5] These could be factors such as: population growth, technological progress, etc.

of regressions can be biased, as a result of the so-called pile-up problem, as discussed by Stock (1994) and Stock & Watson (1998). As mentioned by Laubach & Williams (2003), it is possible to estimate the model, as proposed here and in many other articles, via maximum likelihood by means of a Kalman Filter approach. However, if the variations some of the variables used in the estimation do not contribute considerably to overall variability in the data, the maximum likelihood estimates tend to be biased. This is related to the pile-up problem, as mentioned by Stock (1994).

Therefore, as in Laubach & Williams (2003) and Barcellos Neto & Portugal (2009), for the Brazilian case, we follow the three-step procedure proposed by Stock and Watson (1998). Firstly, a preliminary estimation of the potential output is obtained, via Kalman Filter, omitting the interest rate gap and the real exchange rate in the calculation, and assuming that the growth rate of the potential output is constant.

$$y_t = y_t^{pot} + \varepsilon_t^{y} \quad (18)$$

$$y_t^{pot} = y_{t-1}^{pot} + g_{t-1} + \varepsilon_t^{y^{pot}} \quad (19)$$

$$g_t = g_{t-1} \quad (20)$$

Secondly, we estimate previous equations again, but at this time an error term is added in equation (20) and the potential output calculated in the first step is included as an exogenous variable,

$$y_t = y_t^{pot} + \varepsilon_t^{y} \quad (21)$$

$$y_t^{pot} = y_{t-1}^{pot} + g_{t-1} + \varepsilon_t^{y^{pot}} \quad (22)$$

$$g_t = g_{t-1} + \varepsilon_t^{g} \quad (23)$$

Thirdly, we estimate the parameter λ_t, which is multiplied by the error term in the state equation related to the output:

$$\lambda_t = \frac{\varepsilon_t^{g}}{\varepsilon_t^{y^{pot}}} \quad (24)$$

4. The Natural Rate of Interest in Brazil: Results of Statistical Filters

The first set of estimations is the simplest way to approach our problem in order to have a first view of the country's neutral interest rate. This can be done by estimating neutral rates based on well-known statistical filters, such as the Hodrick-Prescott (HP) Filter and even the Kalman Filter approach. We decided to make use of the Swap Pre-DI interest rate, instead of the benchmark Selic rate, even though previous calculations have showed that there is no significant difference when one or another interest rate is used. The ex-ante rate was obtained by deflating both series by the 12-month ahead IPCA inflation expectation (the Brazilian Broad Consumer Price Index).

Figures 1 and 2 show the results for estimations related to the HP and Kalman Filters approaches. As it is discussed even by Wicksell in his seminal paper, the natural interest rate is not fixed, but it is subject to change over time. It is rather subject to changes along time. Both of the rates show a similar path for the Brazilian neutral interest rate, starting at around 17% in 2002 and showing end-of-period neutral rates of 2.10% and 2.12%, respectively. These results are much smaller than the previous calculations reported in Table 1 and they show a very clear downward trend of the country´s natural rate of interest.

Figure 1
The Brazilian Natural Rate of Interest: HP Filter Estimation

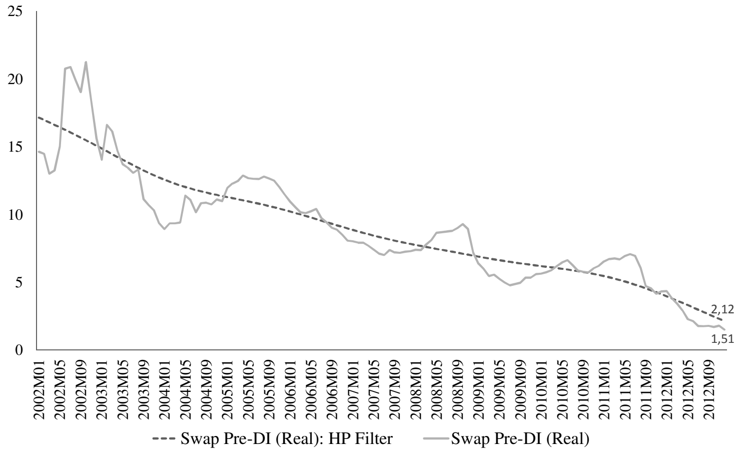


Figure 2
The Brazilian Natural Rate of Interest: Kalman Filter Estimation

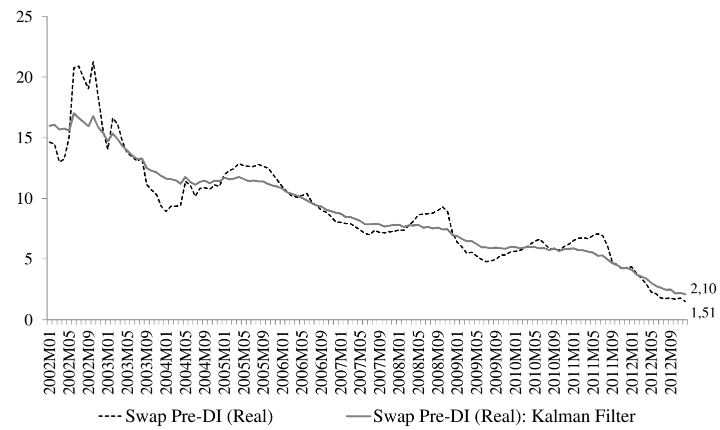
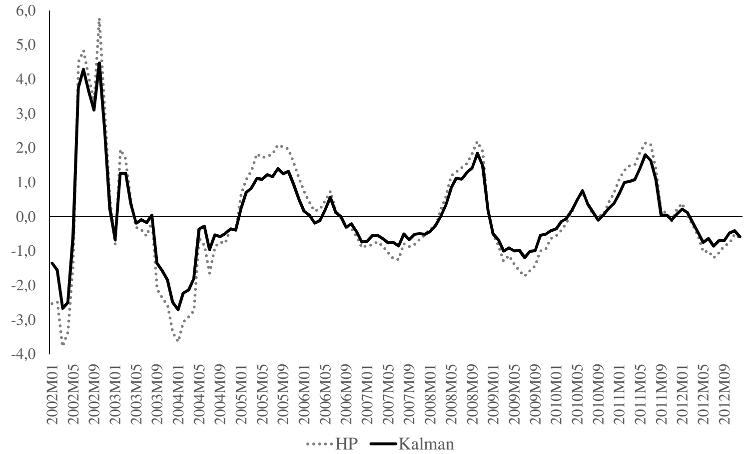


Figure 3 depicts the interest rate gap, that is, the difference between the actual rate and the natural rate, for both estimations (HP e Kalman Filters). This analysis is important once it shows how the natural rate is behaving in relation to the actual real interest rate. Both filters deliver a similar interest rate gap for the Brazilian economy, but it is clear to see that the HP Filter has higher maximum and lower minimum compared to the Kalman Filter. For the period analyzed, the natural interest rate was above the actual rate in approximately half of the period, no matter what type of filter we take into account.

Figure 3
Interest Rate Gap: HP and Kalman Filters



5. The Natural Rate of Interest: Results of the State Space Macroeconomic Model

Figure 4 depicts the results for the Brazilian neutral interest rate for the period ranging from 1999 to 2012 and taking into account the influence of the international interest rate in the Brazilian economy. As a result, the country´s real neutral interest rate dropped from above 20 percent in the beginning of 2002 to something around 3.30 percent in the end of 2012. This is quite close to the percentages reported in Table 1 for advanced and emerging economies.

The figure below shows a similar pattern as in Figures 1 and 2, but one can observe that the natural rate calculated, taking into account other important variables, is higher than the neutral rate calculated via univariate process as reported in section 4.1. It is also interesting to notice that our calculations showed a neutral rate of around 20 percent in 2002. In the end of 2005, it dropped to a single digit and it has been below 5 percent since the middle of 2010.

Figure 4
The Brazilian Neutral Interest Rate (% Year): State Space Estimations

5.1. Evaluating the Stance of Monetary Policy in Brazil

Figure 5 depicts the interest rate gap which results from the estimation of the macroeconomic model, that is, the difference between the actual Swap Pre-DI rate (in real terms) and the natural rate of interest. This analysis is important because we are able to measure the stance of monetary policy in Brazil. In order to do this, we can look at the inflation gap, which is the deviation of inflation from its target set by the Brazilian Monetary Policy Committee, and the output gap, which shows how the Brazilian Central Bank has been responding to business cycles fluctuations.

Figure 5
Brazil: Interest Rate Gap (p.p.)

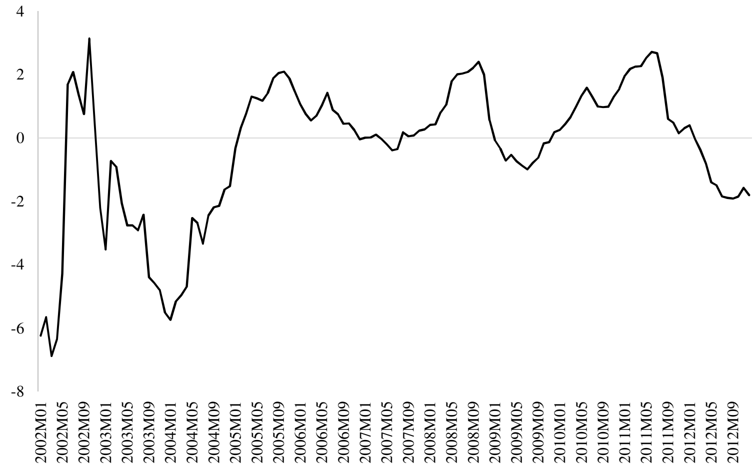
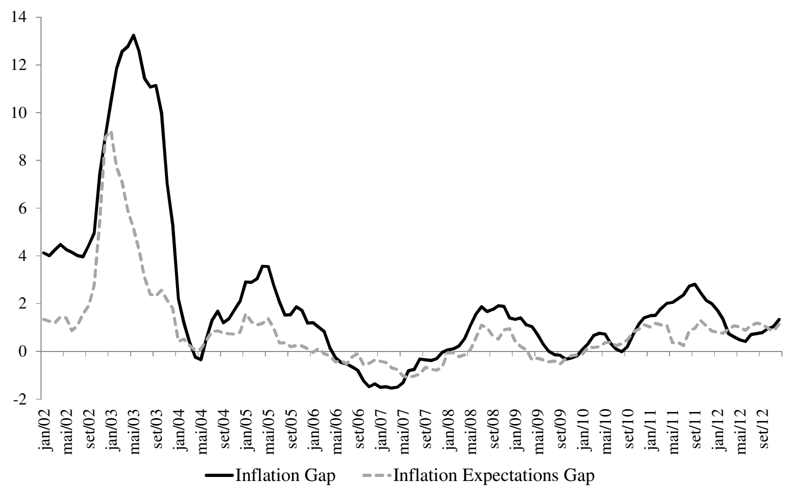


Figure 6 shows the Brazilian output gap. One can see that monetary authorities in Brazil have, in general, been countercyclical when responding to business cycles fluctuations. In general, there is a correlation between the interest rate gap (Figure 5) and the output gap (Figure 6). In other words, periods of low economic activity are usually marked by a negative interest rate gap, that is, an actual rate below the neutral rate. But what is interesting in this comparison between interest rate and output gaps is that, especially after the second half of 2011 up to the end of 2012, monetary policy in Brazil remained very stimulative, with actual interest rates below neutral. However, in spite of this, economic activity has not been able to pick up, and inflation is still resilient.

Figure 6
Brazil: Output Gap (p.p.)

Figure 7 reports the results for the inflation gap (we also made the same calculations using inflation expectations). As the Central Bank of Brazil has been targeting inflation since 1999 and uses interest rate rules, primarily, to reach its objectives, we can infer that there might be a correlation between the interest rate gap and inflation gap. In fact, if inflation is higher than its target, monetary authorities will usually increase interest rates and, as a result, such restrictive monetary policy will lead to a higher interest rate gap. After the problems faced by the Brazilian economy between 2002 and 2004, we noticed that there is a correlation between inflation and interest rate gaps. The only difference to be taken into consideration is the year 2012, which reported a neutral interest rate higher than the actual rate, but Figure 7 shows a higher inflation gap.

Figure 7
Brazil: Inflation Gap (p.p.)



It is important, at this point, to make an analysis of the reasons why the country have been able to lower its neutral interest rate recently, and why it took so long to do so. There is no doubt this is due to the efforts undertaken to strengthen the country´s macroeconomic fundamentals, as well as a successful fiscal consolidation process. Therefore, it is important to analyze why the country's interest rate was so high in the beginning of the 2000's and how it went down through the years. For instance, the year 2002 was hit by a mixture of election uncertainties, which led to high exchange rate depreciation and, consequently, more inflation and higher inflation expectations. For example, the 12-month ahead IPCA inflation expectation was 4.76% in January 2002, and reached 13.24% in December 2012.

After the presidential election, in October 2002, the new government released a primary surplus target and announced a reform in the country´s social security system. These actions sent a positive signal to economic agents and made real interest rates decline, as shown in Figure 4. But real rates started to climb in the first half of 2004 and kept climbing until the second half of 2005. This was due to the monetary policy conduct in 2004, as the benchmark Selic interest rate went from 16.20% in the middle of 2004 to 19.75% in May 2005. The nominal Swap Pre-DI interest rate behaved the same way, reaching 19% in March 2005. As inflation started to cool down in the middle of 2005, inflation expectations also started to decline, after reaching 5.87% in April 2005.

Due to lower interest rates, economic activity picked up in 2006 and Brazil´s GDP reached 4.0% in 2006 (with IPCA annual inflation of 3.1%) and 6.1% in 2007 (with IPCA inflation of 4.5%) (Figure 8). As a consequence, the Brazilian Monetary Policy Committee (Copom) stopped decreasing the benchmark Selic rate in the middle of 2007 and started to raise it sometime after (from 11.25% in April 2008 to 11.75% in April 2008). This also caused an impact in the Swap Pre-DI rates (Figure 4).

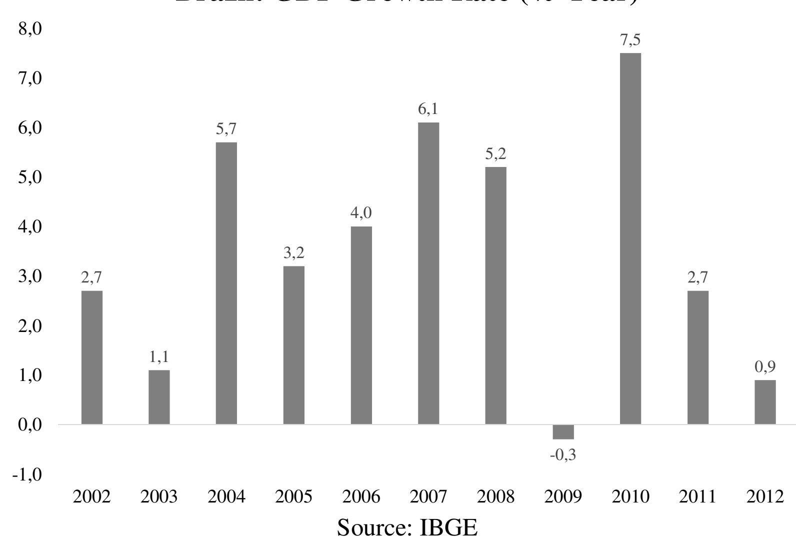
Such trend went on until the start of the global financial turmoil in September 2008, which affected the Brazilian economy in the second half of the year, this can be seen by the fourth quarter GDP growth (-3.6% compared to the third quarter 2008). In spite of this, the country´s annual GDP growth reached 5.2% in 2008, mainly because of the strong economic activity up to the third quarter of 2008. But the effects of the financial crisis impacted heavily the Brazilian economic activity in 2009, resulting in an annual negative GDP growth of -0.3 percent (Figure 9).

Figure 8
Brazil: IPCA Inflation Rate (% 12-month)

Source: IBGE

As far as interest rates are concerned, they started to decline in response to the financial crisis, but there was a considerable lag, once the Brazilian government did not have a clear sign about the real effects of the crisis, The Selic rate declined only in January 2009, from 13.75% to 12.75%. But the future rates (Swap Pré-DI) started a downward trend in November 2008, reaching 9.2% in August 2009.

Figure 9
Brazil: GDP Growth Rate (% Year)



Source: IBGE

The year 2010 was marked by economic recovery, which made GDP grow 7.5 percent. But inflation also picked up, reaching 5.9% at the end of the year. In order to cool down economic activity the Central Bank of Brazil started increasing the Selic rate in the middle of 2010, from 8.75% in March 2010 to 9.5% in the following month, reaching 12.50% in July 2011. This made future rates behave the same way and increased the interest rate gap, as shown in Figure 5. Macroprudential policies also played an important role in helping the Brazilian government to deal with heavy inflows of financial capital, especially to curb the effects of monetary expansion used by central banks in advanced economies. The economy recovered from the previous year´s downturn, with GDP growing 7.5% in 2010 (Figure 9).

In 2011, the global economy continued to feel the worsening effects of the world economic slowdown, heavily influenced by the unsolved sovereign debt problems in many European countries. The financial market also continued to be affected by the quantitative easing policy undertaken by the central banks of the major advanced countries, putting extra pressure on commodity prices and on the exchange rate market. Macroprudential policies and fiscal measures were used by the Brazilian economic authorities to stop the acceleration of prices. A fiscal consolidation program was launched in the beginning of 2011, in order to ensure the fulfillment of the primary surplus target. As well as that, a strong management of capital inflows helped decreasing the exchange rate volatility and, as a result, helped the conduct of the country´s monetary policy. But it did not mean that the interest rate was set aside as the main monetary policy instrument. In fact, the Selic interest rate kept on going up until August 2011, reaching 12.50% in the middle of the year. But rates started to decrease two months later, reaching 7.25% in December 2012.

In the beginning of 2012, the Brazilian economy was showing signal that it would recover from the 2.7% GDP growth rate in 2011. But economic activity picked up only in the second half of the year and, as a result, Brazil's GDP grew just 0.9 percent in 2012. Even so, the consumer price inflation did not moderate, reaching 5.8% in 2012. As for the nominal interest rate, the benchmark Selic rate, reached 7.25 percent at the end of the year.

Final Remarks

This article estimated the Brazilian neutral real interest rate and, based on the results, evaluated the stance of monetary policy in Brazil in the last decade. We made use of statistical filters and a state space macroeconomic model for the country´s monthly data for the period 2002-2012. We also took into account the influence of the international interest rate in the Brazilian economy.

We came to the conclusion that the country´s real neutral interest rate dropped from above 20 percent in the beginning of 2002 to something around 3.30 percent in the end of 2012. This is quite close to the percentages reported for advanced and emerging economies.

We also found a correlation between the country´s interest rate gap and output gap, showing that periods of low economic activity are usually marked by an actual interest rate below the neutral rate. However, especially after the second half of 2011 up to the end of 2012, monetary policy in Brazil remained highly stimulative and, in spite of this, economic activity did not pick up, and inflation did not cool down. In other words, a comparison between the interest rate and inflation gaps shows that, recently, a higher inflation rate has not been accompanied by a lower interest rate gap.

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