Assessing day-to-day volatility: Does the trading time matter?*

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Abstract

The aim of this study is to examine whether investors who trade daily but at different times have distinct perceptions about the risk of an asset. In order to capture the uncertainty faced by these investors, we define the volatility perceived by investors as the distribution of standard deviations of daily returns calculated from intraday prices collected randomly. We find that this distribution has a high degree of dispersion. This means that different investors may not share the same opinion regarding the variability of returns of the same asset. Moreover, the close-to-close volatility is often less than the median of the volatility distribution perceived by investors while the open-to-open volatility is greater than that statistic. From a practical point of view, our results indicate that volatilities estimated using traditional samples of daily returns (i.e., close-to-close and open-to-open returns) may not do a good job when used as input in financial models since they may not properly capture the risk investors are exposed.

Keywords: Volatility, risk.
JEL Code: G1.

Resumo

O objetivo deste estudo é examinar se investidores que negociam diariamente, mas em momentos diferentes, têm percepções distintas acerca do risco de um ativo. A fim de capturar as incertezas enfrentadas por esses investidores, definimos a volatilidade percebida como a distribuição de desvios-padrão de retornos diários calculados a partir de preços intradiários coletados aleatoriamente. Nós concluímos que essa distribuição tem um alto grau de dispersão. Isso quer dizer que diferentes investidores podem não ter a mesma opinião sobre a variabilidade dos retornos do mesmo ativo. Além disso, a volatilidade close-to-close é muitas vezes menor que a mediana da distribuição de volatilidade percebida pelos investidores, enquanto a open-to-open é maior que essa estatística. De um ponto de vista prático, nossos resultados indicam que as volatilidades estimadas com o uso de amostras tradicionais de retornos diários (ou seja, retornos close-to-close e open-to-open) podem não fazer um bom trabalho quando empregadas em modelos financeiros, já que podem não captar os riscos aos quais os investidores estão expostos.

Palavras chave: Volatilidade e risco
Código JEL: G1.

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1 Introduction

Since the seminal work of Markowitz (1952) the volatility of asset returns plays an important role in the modern finance theory, particularly in pricing models, portfolio selection, and risk management. The volatility is an unobservable variable that reflects the degree of variation in prices of a given asset at a certain time period. Undoubtedly, it is the simplest way to quantify the uncertainty of an asset payoff.[1]

Several studies propose models to estimate volatility.[2] In general, when these volatility models are used in practical applications they are estimated with daily data, commonly using the closing price, as noted by Goodhart and O'Hara (1997). However, this procedure has some drawbacks. For example, Parkinson (1980) shows that the volatility estimated using the highest and lowest prices of the days is superior to the close-to-close volatility. Wood, McInish and Ord (1985) and Lockwood and Linn (1990) show that higher standard deviations are observed at the beginning and at the end of the trading session, i.e., the volatility has a U-shaped pattern throughout the day. Brown (1990) argues against the use of the closing prices since it can be influenced by the lack of trading at the close or by "marked on the close" orders. Guillaume, Dacorogna and Pictet (1994) and Andersen and Bollerslev (1998) observe that the series of intraday returns have different characteristics for different periods of the day. They warn that this intraday seasonality should be corrected to avoid distortions in the volatility estimation.

In this paper we revisit these criticisms by a different point of view. As in other studies, we calculate volatility from a series of daily prices. Therefore, we have the same amount of information used by models which estimate the volatility from closing or opening prices. However, instead of prices being collected at a fixed time, we randomly select the time that it is observed in each day. With these prices, we calculate the realized volatility of the asset return in a given period (set as one month in the empirical exercise of Section 3).[3] Next we repeat this experiment, i.e., we draw another sequence of random daily prices and a new volatility is calculated for the same period. From a sequence of draws, we build a probability distribution of volatility. We denominated this distribution as the volatility distribution perceived by investors (VDPI). This distribution is the focus of our paper.

Note that the selection procedure described in the previous paragraph better approximates the daily volatility perceived by investors. An investor does not trade (or look at the market) only at the beginning or at the end of the trading session. In fact, there is no reason for the buying and selling decisions to occur at a specific time. In this sense we understand the volatility calculated from a random sampling of daily prices as the volatility perceived by investors.

A tale may shed some light on the reason why the volatility calculated with prices collected in random moments represents the volatility perceived by an investor. Suppose that an investor negotiates a single share over a month once a day. The time of the trade is determined, for example, when the price reaches thresholds (above an upper limit she sells and below a lower limit she buys). If at the end of this month we ask the investor

[1]There is no consensus of what is risk and uncertainty. Several definitions of these concepts have been proposed. For instance, Knight (1921) distinguishes risk and uncertainty. Knightian uncertainty is risk that is immeasurable. Bekaert et al. (2009) define uncertainty as changes in the fundamentals. In this work we follow a more casual approach. The terms risk and uncertainty are used interchangeably. Risk and uncertainty just mean the dispersion of an asset payoff.

[2]Among others, we can cite the works of Engle (1982) and Bollerslev (1986), which led to the development of the GARCH models and its variants.

[3]The realized volatility of the asset return in a given period is the standard deviation of the series of returns observed in this period.

what the asset volatility perceived by her is, the answer will be the standard deviation of daily returns calculated based on the prices actually negotiated. In other words, her perception of uncertainty is one draw from VDPI. If this volatility is much higher or much lower than the close-to-close volatility, the investor's perception of the uncertainty of this share is undoubtedly different from that calculated by a market analyst who uses closing prices.

Let's look at a more realistic example. Suppose that a hedge fund evaluates its risk by the Value-at-Risk (VaR) metric. Suppose further that VaR is estimated from a parametric model in which the volatility of the portfolio is obtained from the closing prices. However, the hedge fund does not necessarily negotiate assets at the closing of the market. The trades are conducted according to the manager's strategy and can happen at any time during the session. Therefore, the VaR model may not do a good job, i.e., it can not adequately capture the uncertainty faced by the hedge fund.

Our goal in this paper is to study the dispersion of the distribution of the daily volatility perceived by investors. Additionally, we compare the volatility calculated from daily prices randomly selected with the volatility calculated from opening and closing prices. More specifically, we investigate the location of the open-to-open and the close-to-close volatilities on the VDPI. It allows us to evaluate whether this volatilities can be a good representation of the uncertainty perceived by an arbitrary investor.

Harris (1986), Amihud and Mendelson (1987), Lockwood and Linn (1990), Hong and Wang (2000) among others present comparisons between the open-to-open and close-to-close volatilities. They show that the open-to-open returns are more volatile than close-to-close returns. Besides replicating this result our study extends these works since the open-to-open and close-to-close volatilities are just samples of VDPI. The approach proposed here allows us to investigate several kinds of patterns of daily returns volatilities since VDPI encompasses all daily sampling of prices.

The database of this study consists of all intraday prices of 84 shares traded on the Brazilian stock exchange (BM&FBovespa) between July 2006 and April 2009. BM&FBovespa centers the negotiations of Brazilian stocks. The Brazilian stock exchange had a daily average turnover of $ 3.9 billion in June 2008, which places it as one of the largest stock markets in the world and the largest in Latin America according to World Federation Exchanges.[4]

In a nutshell, our results show that the volatility perceived by investors has a high degree of dispersion. The difference between the lowest and highest volatility of the VDPI of a stock in the same month can be greater than 100%. Thus investors have different perceptions about the uncertainty of the same share. Regarding the open-to-open and close-to-close volatilities, both these volatilities can be far away from the median of VDPI. Furthermore, the open-to-open is often located on the right side of the VDPI while the close-to-close is on the left side. For example, we find that the frequency in which the close-to-close volatility is on the VDPI 5% percentile is 11% and the frequency in which the open-to-open is on the VDPI 95% percentile is 84.5%.

Our findings imply that the usual practice of estimating volatility from opening and closing prices can distort the measures of risk since the day-to-day uncertainty perceived by an investor can be quite different from these volatilities. In this way, our results are important not only for investors and risk managers but also for financial regulators. For instance, as capital requirement for market risk is based on the close-to-close volatility, it may not be enough to cover daily losses.[5] In order to account for the dispersion of

[4]See the website http://www.world-exchanges.org/.

[5]Although beyond the scope of this work, we can state that our results present arguments in favor of the incorporation of intraday prices information into the volatility estimation. Many studies advocate

the VDPI, we propose to adjust capital requirements by the ratio between a high VDPI percentile (e.g. 95%) and the close-to-close volatility.

The rest of the paper is organized as follows. Section 2 presents the data description. Section 3 describes the procedure proposed to assess the volatility perceived by investors. Section 4 discusses the empirical results. Section 5 provides some concluding remarks.

2 Database

The database was provided by BM&FBovespa. It consists of a series of all intraday trade prices of 84 stocks of Brazilian firms. These firms were chosen due to the liquidity of their shares. Together they represent more than 85% of BM&FBovespa traded volume. Only the most liquid share for each firm is included in the sample.[6,7] The data cover the period from July 3, 2006 to April 30, 2009, corresponding to 739 trading days. Not all stocks are traded in every month and the average number of stocks is 77.03 per month.[8] It is worth noting that we do not collect data according to fixed time intervals. Instead of, we include all trades in the sample. Therefore, the size of the price series varies with the share liquidity. For example, the price series of Petrobras and Vale, the two most liquid stocks in the sample, contain 5,875,374 and 4,793,357 observations, respectively. Besides the price, the database includes the date, the time (recorded with precision of seconds) and the volume of each trade. To compute the volatilities, we adjust the price series of each stock to splits, inplits and dividends.

Another issue to consider about our database is that some records refer to the same order. Consider, for instance, that there are two limit orders for selling, one of 200 shares at \$40.00 and another of 100 shares at \$40.30. Assume also that both orders have the lowest sell prices in the limit order book. An order to buy 300 shares at \$40.30 generates two trades in the BM&FBovespa database. We modify the database so that this order only generates one trade of 300 shares at \$40.10 (average price per share of the trade).

3 Methodology

In this section we describe the selection mechanism of the daily prices as well as the volatility evaluation methodology. For each share, we generate several paths of daily prices. The price of each day is chosen according to the following procedure. First, we randomly generate a number from a discrete uniform distribution between one and the number of trades occurred in the day. Next, we select the price of the trade that corresponds to the number randomly drawn. The number of observations for each day is set as the number of trades held between 10:00am and 5:00pm (or 11:00am and 6:00pm during the daylight

using of high frequency data. Currently, the realized volatility theory is the workhorse of this strand of the literature (see Andersen, 2000).

[6]For example, Petrobras has two types of shares, PETR3 (with voting rights) and PETR4 (without voting rights). Only PETR4 is in our sample because its liquidity is higher than that of PETR3.

[7]Liquidity is measured by the liquidity ratio defined by $100\frac{p}{P}\sqrt{\frac{n}{N}\frac{v}{V}}$, where p is the number of days that there was at least a trade with the stock within the chosen period; P is the total number of days in the selected period, n is the number of trades of the stock within the chosen period, N is the number of trades of all stocks within the chosen period, v is the traded volume (in monetary units) of the stock within the chosen period, and V is the traded volume of all stocks within the chosen period. This definition of liquidity is used by BM&FBovespa to select shares that compose the Bovespa index (Ibovespa).

[8]The initial public offering of some firms, as for example Ecodiesel and JBS Friboi, occurred after the initial date of our database.

savings time). By repeating this procedure we obtain several daily prices paths in a way that each price represents a trade actually occurred.

The next step is the construction of a series of daily logarithmic returns calculated using the prices drawn. From these series we calculate the realized daily volatility (the standard deviation of the daily returns) of each stock in each month of the sample. This is the volatility of one path and possibly the volatility observed by an investor who trades in each day during the trading hours. If $P^i_{t,1}, \ldots, P^i_{t,N}$ is a series of stock i daily prices randomly selected in month t, the realized volatility of this path is

$$vol(i,t) = \sqrt{\frac{\sum_{n=1}^{N}\left[\ln\left(\frac{P^i_{t,n+1}}{P^i_{t,n}}\right)\right]^2 - \left[\sum_{n=1}^{N}\ln\left(\frac{P^i_{t,n+1}}{P^i_{t,n}}\right)\right]^2}{N}}.$$

For each stock in each month we generate 400,000 paths and calculate 400,000 daily realized volatilities according to the procedure described above. These volatilities represent the ones perceived by investors. The intuition is that investors trade at random times and not at fixed times such as the opening and closing of the market. From the sequence of draws, we build a histogram of volatility. As a result, we have a probability distribution of the realized volatility of each stock for each month of the sample. We denominated this distribution as the volatility distribution perceived by investors (VDPI).

Our goal is to investigate the dispersion of the VDPI. It allows us to answer the following question: Can the uncertainty perceived by different investors (who trade daily but possibly at distinct times) be quite different? If so, it places an important issue: Do a regulator or a risk manager who require capital to cover market risk using volatilities computed from prices observed at specific times (such as at the closing or opening of the market) indeed capture the risk faced by investors? Moreover we want to compare the close-to-close and open-to-open volatilities with the median of the VDPI. For this reason, besides the VDPI, we also compute the daily returns volatilities from opening prices (open-to-open volatility) and closing prices (close-to-close volatility).

For example, Figure 1 shows the VDPI of Vale (VALE5) in January 2009.[9] The numbers 1 and 2 on the histogram represent, respectively, the open-to-open (4.47%) and close-to-close (3.56%) volatilities. The fact that the open-to-open volatility is greater than the close-to-close is in agreement with many papers. For instance, Harris (1986), Amihud and Mendelson (1987), Lockwood and Linn (1990) and Hong and Wang (2000) report that opening returns exhibit greater dispersion than closing returns. Note also that in this month both the close-to-close and open-to-open volatilities are far from the median of the VDPI (4.05%).

The volatility of daily returns of Vale in January 2009 ranges from 2.41% to 5.97%, depending on the prices path selected. The 5% and 95% percentiles of Vale VDPI in January 2009 are equal to 3.36% and 4.85%, respectively. This variation can yield a significant difference between the volatility perceived and the one used in models for quantifying risk such as Value-at-Risk (VaR). An investor could be trading their shares with the volatility of 5.97%, but considering, for purposes of risk measuring, the volatility of the closing price. The volatility of 5.97% is 68% higher than the close-to-close volatility.

Now, let's see what can happen over the months of the sample (July 2006 until April 2009). Figure 2 illustrates the evolution of the open-to-open and close-to-close volatilities of Vale (VALE5). It also shows the 5% percentile, median and 95% percentile of Vale VDPI. We can observe that the open-to-open volatility is sometimes less than the 5% percentile (September 2006) and sometimes greater than the 95% percentile (March 2008,

[9] Although this VDPI is bell-shaped, resembling a normal, the Jarque-Bera test discard this hypothesis.

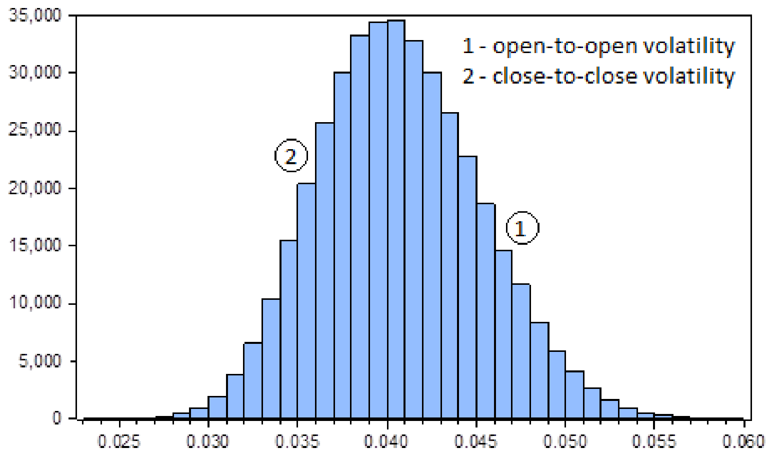

Figure 1: Histogram of VALE5 VDPI

This figure contains the histogram of Vale (VALE5) VDPI in January 2009. It also presents the open-to-open (labeled by the number 1) and the close-to-close (labeled by the number 2) volatilities. The histogram represents the frequency of daily volatilities calculated from 400,000 paths of prices of Vale in January 2009. Each path is obtained by a random process which draw an observed price in each day of the month. The bin width is 0.001.

for example), i.e., occasionally this volatility is far from the median of the VDPI. Similarly, in other months the close-to-close volatility is very distant from the median of the VDPI. In January 2008 the close-to-close volatility is greater than the 95% percentile of the VDPI. On the other hand, in November 2006 it is less than the 5% percentile. In addition, note that the highest distance between the 5% percentile and 95% percentile of the VDPI occurs in October 2008 (the epicenter of the subprime crisis). If we had computed the volatility by the traditional way, i.e., using the closing price (Goodhart and O'Hara, 1997), we would have probably subestimated the risk of this share in this month.

4 Results

In this section we present a detailed description of the results of the empirical exercise described in Section 3. Our results consist of a series of unbalanced panel data containing observations of VDPI percentiles for each stock in each month of the sample period. The panel has the form $vol\alpha(i,t)$, where $\alpha \in \{1\%, 5\%, 10\%, 25\%, 50\%, 75\%, 90\%, 95\%, 99\%\}$ is the percentile of the VDPI, i is the spatial dimension (stock) and t is the time dimension (month). For example, $vol95$(VALE5,August 2008) is the VDPI 95% percentile of the firm Vale in August 2008.

4.1 Spatial dimension analysis

Table A in the Appendix presents the median of the VDPI percentiles of each stock calculated over the sample period. Each entry in Table A is given by

$$vol\alpha(i) = \underset{t}{\texttt{Med}} \; vol\alpha(i,t),$$

where `Med` stands for the median operator.

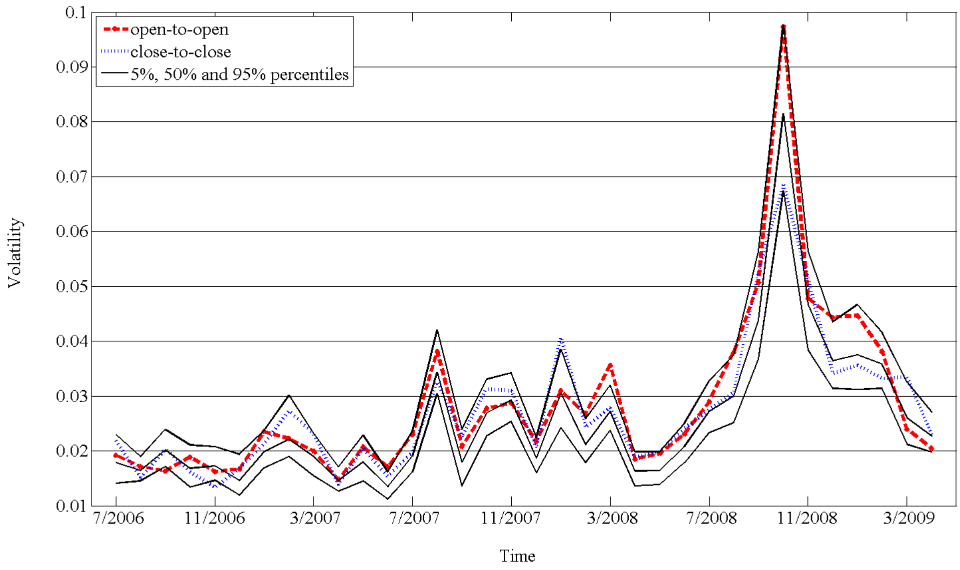

Figure 2: VDPI percentiles of Vale

This figure contains the time evolution of the open-to-open and close-to-close volatilities, the 5% percentile, the median, and the 95% percentile of VALE VDPI from July 2006 to April 2009. The VDPI for a month is obtained simulating 400,000 paths of daily random prices. From each path we calculate the realized volatility of the returns.

We can observe that the median of percentiles has a high dispersion across the stocks. For example, the median of the 1% percentile ranges from 1.17% (CGAS5) to 3.25% (ECOD3), while the median of the 99% percentile ranges from 2.19% (CGAS5) to 5.34% (MRVE3). As we can see, the highest median of the 1% percentile is greater than the lowest median of the 99% percentile. We can also note a high difference between the medians of the extreme percentiles (1% and 99%) for all stocks which suggests that the volatility perceived by two investors can be quite different.

In order to investigate further the dispersion of the VDPI, we compute the ratio between some VDPI percentiles for each stock in each month. Then, we calculate the median of this ratio for each stock, i.e.

$$\frac{vol\alpha_1}{vol\alpha_2}(i) = \underset{t}{\mathtt{Med}} \ \frac{vol\alpha_1(i,t)}{vol\alpha_2(i,t)}.$$

Table B in the Appendix presents the results. For example, the median of the ratio of the 99% percentile to 1% percentile of Natura (NATU3) VDPIs $\left(\frac{vol99}{vol1}(\text{NATU3})\right)$ is 1.73. This indicates that the volatility perceived by an investor can be 73% higher than another investor in the same month for this share. Furthermore, the median of the difference between the lowest and highest volatility of the VDPI for all stocks is greater than 100%. On the other hand, note that the percentiles ratios do not vary significantly across the stocks. This indicates that the VDPI dispersion is almost the same for all stocks.[10]

[10]The Coefficient of Variation, i.e., the ratio between the standard deviation and the mean, is much smaller for the ratios of the VDPI percentiles than for the VDPI percentiles. For example, for the 95% and 5% percentiles the coefficient is around 0.20 whereas for the ratio of the 95% percentile to 5% percentile is 0.03.

Table 1 presents some descriptive statistics of Table B. It shows that the highest value of the ratio $\dfrac{vol99}{vol1}(i)$ is 1.86 (this value comes from AES Tietê (GETI4), see Table B in Appendix) and the lowest is 1.55 (i =Bradespar (BRAP4)). The highest median of the ratio of the 75% percentile to the 25% percentile $\left(\dfrac{vol75}{vol25}(i)\right)$ is 1.20 (i =Ecodiesel (ECOD3)), which means that the differences of the volatilities of the first to the last quartile of Ecodiesel are greater than 20%.

	Max/Min	$vol99/vol1$	$vol95/vol5$	$vol90/vol10$	$vol75/vol25$
Minimum	2.27	1.55	1.37	1.28	1.14
1th Quartile	2.54	1.62	1.42	1.31	1.15
Mean	2.68	1.68	1.45	1.34	1.16
Median	2.66	1.68	1.45	1.34	1.16
3th Quartile	2.80	1.73	1.48	1.36	1.18
Maximum	3.34	1.86	1.54	1.40	1.20

Table 1: Descriptive statistics of VDPI percentiles ratios.
This table presents descripitive statistics of the median of the percentiles ratios. $\frac{vol\alpha_1}{vol\alpha_2}$ is a series of the median (calculated over time) of the ratios between the VDPI α_1 and α_2 percentiles.

4.2 Time dimension analysis

After we discuss the spatial dimension properties of the VDPI, we proceed to investigate the time behavior of the percentiles $vol\alpha(i,t)$. Table C in the Appendix shows the medians of the VDPI percentiles calculated on the stock dimension, that is,

$$vol\alpha(t) = \underset{i}{\texttt{Med}} \; vol\alpha(i,t),$$

for $\alpha = 1\%, 5\%, 10\%, 25\%, 50\%, 75\%, 90\%, 95\%, 99\%$.

We can note that the perceived volatility can fluctuate through the months. For example, for the 50% percentile, it varies from 1.68% in December 2006 to 8.29% in October 2008 (the epicenter of the subprime crisis).

For each stock and for each month, we also compute the ratio between some percentiles. Then we calculate the median of these ratios over the stock dimension, i.e.,

$$\frac{vol\alpha_1}{vol\alpha_2}(t) = \underset{i}{\texttt{Med}} \; \frac{vol\alpha_1(i,t)}{vol\alpha_2(i,t)}.$$

Table 2 presents the results. The minimum of the median of the ratio $\frac{vol99}{vol1}(t)$ is 1.49, confirming by another point of view that the volatility perceived by investors can vary significantly. Interestingly, this minimum ratio occurs in October 2008 (the epicenter of the subprime crisis). However, since in time of crisis the volatility is high, the maximum difference between $vol99(t)$ and $vol1(t)$ is also reached in this month as we will show in Section 4.4.

Months	Max/Min	$vol99/vol1$	$vol95/vol5$	$vol90/vol10$	$vol75/vol25$
Jul 2006	2.68	1.68	1.44	1.33	1.16
Aug 2006	2.50	1.59	1.39	1.29	1.15
Sep 2006	2.49	1.62	1.41	1.31	1.15
Oct 2006	2.67	1.66	1.43	1.32	1.16
Nov 2006	2.64	1.66	1.44	1.33	1.16
Dec 2006	2.77	1.69	1.45	1.33	1.16
Jan 2007	2.82	1.67	1.44	1.33	1.16
Feb 2007	2.76	1.78	1.52	1.39	1.19
Mar 2007	2.64	1.69	1.45	1.34	1.16
Apr 2007	2.78	1.71	1.47	1.36	1.17
May 2007	2.64	1.69	1.46	1.34	1.17
Jun 2007	2.77	1.70	1.46	1.35	1.17
Jul 2007	2.62	1.67	1.44	1.33	1.16
Aug 2007	2.44	1.66	1.43	1.33	1.16
Sep 2007	2.61	1.64	1.43	1.32	1.16
Oct 2007	2.70	1.68	1.45	1.33	1.16
Nov 2007	2.40	1.60	1.39	1.29	1.14
Dec 2007	2.77	1.70	1.47	1.35	1.17
Jan 2008	2.64	1.67	1.45	1.34	1.16
Feb 2008	2.60	1.66	1.43	1.33	1.16
Mar 2008	2.69	1.68	1.45	1.34	1.16
Apr 2008	2.64	1.68	1.45	1.34	1.17
May 2008	2.62	1.66	1.44	1.33	1.16
Jun 2008	2.71	1.67	1.44	1.33	1.16
Jul 2008	2.52	1.64	1.44	1.33	1.17
Aug 2008	2.61	1.64	1.42	1.31	1.16
Sep 2008	2.67	1.69	1.47	1.35	1.17
Oct 2008	2.12	1.49	1.32	1.24	1.12
Nov 2008	2.88	1.78	1.50	1.38	1.18
Dec 2008	2.74	1.68	1.45	1.34	1.17
Jan 2009	2.83	1.72	1.48	1.35	1.17
Feb 2009	3.37	1.87	1.56	1.41	1.20
Mar 2009	2.76	1.72	1.47	1.35	1.17
Apr 2009	2.50	1.62	1.41	1.31	1.15

Table 2: Time evolution of the VDPI percentiles ratios.
This table presents the median of the ratios between VDPI percentiles for all months of the sample period. $\frac{vol\alpha_1}{vol\alpha_2}$ is a time series of the median (calculated across the stocks) of the ratios between the α_1 and α_2 percentiles.

4.3 Open-to-open and close-to-close volatilities

In this subsection we analyze the location of the open-to-open and close-to-close volatilities on the VDPI. We aim to answer questions such as: Is the open-to-open volatility almost close to VDPI median? Is the close-to-close volatility located in the left-side of the VDPI? More specifically, our goal is to determine the frequency in which the open-to-open and the close-to-close volatilities are located in the 1%, 5%, 10%, 25%, 50%, 75%, 90%, 95% and 99% percentiles of the VDPI.

As we have 77.03 stocks on average in each month and we have 34 months in the sample, there are 2,619 observations of the open-to-open and the close-to-close volatilities. To study the location of these volatilities on the VDPI, we computed, in each month, for all stocks, how many observations of them are below the percentiles of the corresponding VDPI. Then we sum the number of observations for all months and, finally, divided this sum by the sample size (2,619).

For example, let's analyze the close-to-close volatility. We compute in July 2006 how many close-to-close volatilities of all stocks are below the 10% percentile of the corresponding VDPI (each stock in July 2006 has its own VDPI). We do the same procedure for all months. After this we compute the number of observations of the close-to-close volatility that are below the 10% percentile of the corresponding VDPIs and divide this value by 2,619. The result is 18.56%. Thus, the close-to-close volatility often lies in the left tail of the VDPI. Table 3 presents all results of this procedure. Note that both the close-to-close and the open-to-open volatilities frequently are located in the tails of the VDPI. Moreover we can see that, at average, the open-to-open volatility is in the right side of the VDPI while the close-to-close volatility is in the left side.

percentile	open-to-open	close-to-close
1%	2.14%	2.71%
5%	7.10%	11.00%
10%	11.76%	18.56%
25%	23.67%	36.50%
50%	42.73%	59.18%
75%	61.70%	78.50%
90%	76.94%	90.99%
95%	84.50%	95.00%
99%	93.01%	98.09%

Table 3: Location of the open-to-open and close-to-close volatilities. This table presents the frequency in which the open-to-open and the close-to-close volatilities are located in each percentile of the VDPI. Each entry in the table represents the number of times that a specific volatility (open-to-open or close-to-close) lies on a specific VDPI percentile (1%, 5%, 10%, 25%, 50%, 75%, 90%, 95% and 99%). The analysis covers all the month and all stocks jointly.

Let us assume that the median is a good measure to summarize the information contained in a probability distribution. Then, observing the 50% percentile, we note that the close-to-close volatility seems to underestimate the VDPI while the open-to-open volatility seems to overestimate it. To confirm these conclusions we implement two t-tests.

The first seeks to verify if the open-to-open volatility is greater than the median. The alternative hypothesis for each stock i is:

$$H_1 : \underset{t}{\texttt{Mean}} \quad vol_{\texttt{open}}(i,t)/vol_{\texttt{median}}(i,t) > 1.$$

The second one tests if the close-to-close volatility is less than the median. The alternative hypothesis for the stock i is:

$$H_1 : \underset{t}{\texttt{Mean}} \quad vol_{\texttt{close}}(i,t)/vol_{\texttt{median}}(i,t) < 1.$$

The results are remarkable. With a 5% significance level, in 38 of 84 stocks we can affirm that the mean of the open-to-open volatility is higher than the median volatility. With a 10% significance level, in 46 of these stocks, we conclude the same. For the close-to-close volatility, in 19 stocks we can infer that this volatility is lower than the median volatility on the basis of a 5% significance level. With a 10% significance level, this number increases to 25. Therefore, we conclude that these volatilities cannot be good proxies of the day-to-day volatility perceived by an investor who trades during the trading hours and not just in the opening or closing of the market.[11]

4.4 Discussion

Our results presented in the three previous subsections bring some issues about the modern theory of portfolio selection, risk management and option pricing. The uncertainty of the economy drives all the financial decision process. Although not immune to criticism, the asset volatility is the most famous risk measure.[12] However, we show that the usual practice of calculating asset volatility using opening or closing prices may not capture the uncertainty that an investor is exposed.[13]

From the portfolio selection point of view, an investor who buys or sells assets based, for example, on the mean-variance frontier built using closing prices can hamper her optimum return/risk allocation since her evaluation of risk (volatility) is misleading. On the risk management aspect, our results provide sound arguments to a strand of the literature which contests the traditional methodologies to gauge market risk (see, for instance, Danielsson, 2002, Taleb, 2007 and Vicente and Araujo, 2010). When a player uses a daily volatility measure which does not consider intraday prices, such as the close-to-close volatility, she may assess inaccurately the uncertainty. Consequently, she may bias the risk incurred. Let's look at an actual example. Consider a risk manager who estimates a 1-Day 95% VaR by the Delta-Normal model. The portfolio includes only VALE5 shares and its market value is \$1 (long position). If she uses the standard deviation of closing returns of January 2009 (3.56%), the estimated VaR turns out to be \$0.0586. In contrast, if the risk manager uses the median of the VDPI (4.05%) the VaR will be \$0.0666 (13.76% higher).

The inability of the close-to-close or the open-to-open volatilities to assess the degree of uncertainty of an asset can also yield mispricing problems. For example, a trader can input a wrong volatility in an option pricing model. Suppose that an investor wish to price a call of VALE5 on March 1st, 2007 with 6 months to maturity and exercise price R\$66.[14] Assume that the investor uses the Black & Scholes formula and estimate the volatility of VALE5 by the standard deviation of the returns of the last month (February 2007). The 6-month risk-free compound interest rate is 11.71% and the stock price is R\$ 62.46. The call price provided by the Black & Scholes model is R\$7.70, R\$6.29, R\$ 6.84, R\$ 5,71 or R\$8.88 if she uses the close-to-close volatility, the open-to-open volatility, the VDPI median, the VDPI 5% percentile or the VDPI 95% percentile, respectively.

[11] We did another t-test to verify if the open-to open volatility is greater than the close-to-close volatility. The alternative hypothesis for the stock i is:

$$H_1: \underset{t}{\texttt{Mean}} \quad vol_{\texttt{open}}(i,t)/vol_{\texttt{close}}(i,t) > 1.$$

With 5% significance level, in 54 of 84 stocks the H_0 is rejected. With 10% significance level, in 62 of 84 stocks the H_0 is rejected.

[12] Among others drawbacks, it is well-known that in a consumption-based model the riskiness of a payoff depends on its covariance with the stochastic discount factor rather than its variance (see, for instance, Cochrane, 2005). Another problem with variance stems from the fact that it can fail to assess the downside risk (see Markowitz, 1991).

[13] For an amazing discussion of the important role of the volatility assessment, we refer to Poon and Granger (2003).

[14] The Brazilian Real/US Dollar exchange rate was around 1.80 in 2009.

Policy makers are also interested in the evaluation of volatility. As point out by Poon and Granger (2003), they rely on market estimates of volatility as a measure of the vulnerability of the economy. Thus, according to our results, policy makers who assess the volatility using close-to-close prices can underestimate the vulnerability while the one who uses open-to-open prices can superestimate it.

The impact of those issues are more pronounced in time of financial crisis. Figure 3 shows the time series of the median (in the stock dimension) of the difference between the VDPI 99% and 1% percentiles. The maximun distance of the VDPI 99% percentile to the VDPI 1% percentile occurs in the epicenter of the subprime crises (October 2008). The average of the median of the difference between the VDPI 99% and 1% percentiles is 1.40% while in October 2008 it reaches the value of 3.36%.

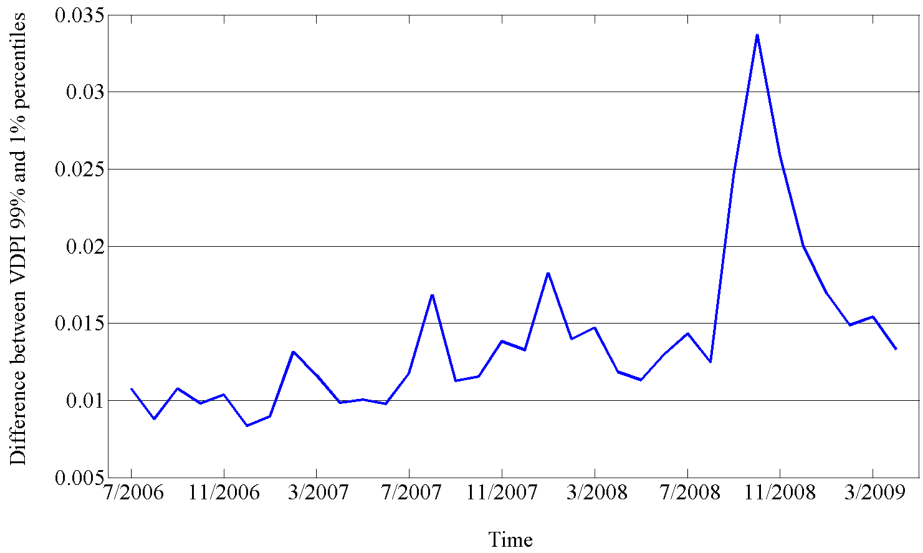

Figure 3: Difference between the VDPI 99% and 1% percentiles

This figure contains the time evolution of the median (in the stock dimension) of the difference between the VDPI 99% and 1% percentiles.

The poor performance of the close-to-close volatility in the assessement of daily risk points to the realized volatility models. However, the pratical use of these models is very costly.[15] For some applications, such as risk regulation, clarity and parsimonious are fundamental. The VDPI can be very helpful in this case. For example, in order to taken into account the variability of the volatility perceived by investors, a regulator can adjust the capital requirement by a factor. This factor can be set as a mean value of the ratio between a high VDPI percentile (e.g. 95%) and the close-to-close volatility.

Our findings also imply that studies which use the close-to-close volatility as the day-to-day realized volatility may contain a bias, as this volatility tends to be on the left side of the VDPI. For example, works as Day and Lewis (1992), Lamoureux and Lastrapes (1993) and Christensen and Prabhala (1998), which investigate the relation between implied and realized volatility, may have different conclusions if the VDPI was taken into account.

[15] The main caveat regarding the realized volatility models is the severe problems caused by microstructure noise (see, for instance, McAleer and Medeiros, 2008).

5 Conclusion

The purpose of this paper is to verify whether investors who trade daily but at different times have distinct perceptions about the risk of an asset. For this objective we propose a simple procedure to assess the volatility perceived by an investor who trades shares at random times of a day. The methodology consiste in calculating the realized daily volatility using prices that are obtained through a random drawing among all the negotiations occurred on each day. The set of volatilities estimated by this procedure is what we call the volatility distribution perceived by investors (VDPI). We find that the dispersion of the VDPI can be very high. We show that for the same share the volatility perceived by an investor may be twice the volatility perceived by others in the same period.

It is common practice (for both market participants and academics) to calculate the daily volatility using opening or closing prices. Comparing these realized volatility with the VDPI allows us to assess whether they actually capture the volatility perceived by an investor. The results show that this practice can provide volatilities that do not represent the level of uncertainty perceived by an investor. The open-to-open and close-to-close volatilities are often far from the median of the VDPI. Therefore, an investor who measures the risk of her portfolio using these volatilities can often bias it.

The findings of this study raise issues regarding the way risk management, portfolio allocation and pricing are traditionally performed in practice. In general, market professionals and regulators elect volatility calculated from the opening and closing prices as a metric for the dispersion of the daily returns of an asset. However, as shown in this study, these volatilities can be quite different from the volatility perceived by investors.

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Stocks	Min	1%	5%	10%	25%	50%	75%	90%	95%	99%	Max
ALLL11	1.64%	2.21%	2.38%	2.48%	2.66%	2.90%	3.13%	3.34%	3.46%	3.70%	4.35%
AMBV4	1.06%	1.36%	1.51%	1.57%	1.69%	1.83%	1.96%	2.07%	2.14%	2.30%	2.75%
ARCZ6	1.08%	1.63%	1.77%	1.85%	1.99%	2.14%	2.29%	2.42%	2.51%	2.68%	3.21%
BBAS3	1.40%	1.85%	2.03%	2.15%	2.32%	2.54%	2.75%	2.91%	3.01%	3.19%	3.74%
BBDC4	1.33%	1.72%	1.85%	1.93%	2.07%	2.24%	2.39%	2.54%	2.65%	2.83%	3.42%
BNCA3	1.33%	1.75%	1.90%	1.98%	2.13%	2.31%	2.47%	2.66%	2.75%	2.92%	3.47%
BRAP4	1.61%	1.94%	2.06%	2.14%	2.28%	2.44%	2.65%	2.81%	2.90%	3.07%	3.64%
BRKM5	1.48%	1.98%	2.11%	2.19%	2.32%	2.47%	2.65%	2.81%	2.91%	3.10%	3.69%
BRSR6	1.53%	2.15%	2.33%	2.45%	2.65%	2.91%	3.17%	3.40%	3.53%	3.78%	4.52%
BRTO4	1.48%	1.86%	2.02%	2.12%	2.30%	2.50%	2.70%	2.90%	3.07%	3.35%	4.05%
BRTP4	1.44%	1.95%	2.14%	2.23%	2.38%	2.57%	2.76%	2.93%	3.06%	3.28%	4.10%
BTOW3	1.76%	2.28%	2.44%	2.53%	2.68%	2.87%	3.12%	3.46%	3.68%	3.95%	4.60%
CCRO3	1.35%	1.78%	1.96%	2.06%	2.23%	2.42%	2.64%	2.84%	2.97%	3.18%	4.00%
CESP6	1.50%	1.92%	2.10%	2.19%	2.38%	2.61%	2.89%	3.08%	3.20%	3.44%	4.35%
CGAS5	0.82%	1.17%	1.31%	1.40%	1.53%	1.68%	1.83%	1.98%	2.06%	2.19%	2.73%
CLSC6	1.01%	1.31%	1.43%	1.49%	1.64%	1.80%	1.95%	2.12%	2.22%	2.37%	2.88%
CMIG4	1.20%	1.54%	1.69%	1.77%	1.93%	2.08%	2.24%	2.42%	2.54%	2.76%	3.17%
CNFB4	1.07%	1.48%	1.60%	1.67%	1.80%	1.93%	2.12%	2.38%	2.58%	2.90%	3.53%
CPFE3	1.09%	1.48%	1.59%	1.65%	1.76%	1.90%	2.06%	2.21%	2.31%	2.51%	3.05%
CPLE6	1.24%	1.63%	1.79%	1.88%	2.04%	2.21%	2.36%	2.51%	2.60%	2.75%	3.27%
CRUZ3	1.33%	1.82%	1.97%	2.05%	2.18%	2.36%	2.55%	2.74%	2.84%	3.03%	3.74%
CSAN3	1.98%	2.57%	2.80%	2.93%	3.14%	3.37%	3.63%	3.90%	4.01%	4.22%	4.92%
CSMG3	1.34%	1.74%	1.87%	1.94%	2.07%	2.22%	2.43%	2.65%	2.78%	2.96%	3.48%
CSNA3	1.51%	1.96%	2.12%	2.23%	2.43%	2.61%	2.79%	2.96%	3.08%	3.27%	3.79%
CYRE3	1.94%	2.56%	2.78%	2.88%	3.04%	3.24%	3.48%	3.71%	3.82%	4.09%	4.94%
DASA3	1.26%	1.69%	1.84%	1.92%	2.04%	2.27%	2.46%	2.60%	2.70%	2.93%	3.54%
DURA4	1.59%	2.06%	2.28%	2.41%	2.63%	2.85%	3.07%	3.28%	3.39%	3.61%	4.44%
ECOD3	2.20%	3.25%	3.52%	3.63%	3.88%	4.18%	4.50%	4.78%	4.94%	5.26%	6.12%
ELET6	1.31%	1.72%	1.87%	1.96%	2.13%	2.30%	2.46%	2.62%	2.72%	2.91%	3.42%
ELPL6	1.16%	1.63%	1.80%	1.89%	2.07%	2.30%	2.56%	2.74%	2.85%	3.04%	3.81%
EMBR3	1.19%	1.54%	1.66%	1.72%	1.83%	1.95%	2.08%	2.29%	2.37%	2.51%	3.04%
ENBR3	1.24%	1.69%	1.86%	1.93%	2.06%	2.21%	2.38%	2.54%	2.63%	2.82%	3.42%
ETER3	1.25%	1.68%	1.85%	1.93%	2.07%	2.24%	2.44%	2.62%	2.74%	2.96%	3.52%
FFTL4	1.11%	1.46%	1.59%	1.66%	1.79%	1.95%	2.11%	2.38%	2.54%	2.69%	3.17%
FHER3	1.91%	2.58%	2.82%	2.93%	3.12%	3.36%	3.60%	3.85%	4.00%	4.28%	4.87%
GETI4	0.86%	1.25%	1.38%	1.45%	1.58%	1.71%	1.85%	2.03%	2.13%	2.34%	2.79%
GFSA3	1.96%	2.50%	2.69%	2.79%	2.97%	3.17%	3.37%	3.61%	3.75%	3.98%	4.70%
GGBR4	1.66%	1.99%	2.11%	2.17%	2.30%	2.45%	2.66%	2.84%	2.94%	3.14%	3.71%
GOAU4	1.51%	2.01%	2.17%	2.26%	2.40%	2.57%	2.75%	2.88%	2.95%	3.10%	3.60%
GOLL4	1.73%	2.14%	2.28%	2.37%	2.51%	2.68%	2.86%	3.11%	3.22%	3.43%	4.08%
GVTT3	1.93%	2.57%	2.82%	2.93%	3.12%	3.33%	3.54%	3.72%	3.84%	4.05%	4.86%
ITAU4	1.31%	1.71%	1.84%	1.92%	2.07%	2.25%	2.42%	2.57%	2.66%	2.82%	3.31%
ITSA4	1.35%	1.74%	1.89%	1.97%	2.10%	2.24%	2.38%	2.56%	2.66%	2.82%	3.34%
JBSS3	1.87%	2.49%	2.72%	2.86%	3.09%	3.34%	3.59%	3.82%	3.95%	4.20%	4.98%
KLBN4	1.31%	1.81%	1.97%	2.06%	2.20%	2.38%	2.55%	2.73%	2.82%	3.04%	3.68%
LAME4	1.45%	1.93%	2.12%	2.22%	2.40%	2.64%	2.88%	3.06%	3.16%	3.35%	3.98%
LIGT3	1.33%	1.83%	1.99%	2.09%	2.27%	2.44%	2.61%	2.77%	2.89%	3.10%	3.86%
LOGN3	1.38%	1.95%	2.09%	2.16%	2.31%	2.51%	2.72%	2.97%	3.13%	3.41%	4.19%
LREN3	1.73%	2.27%	2.45%	2.55%	2.75%	2.96%	3.24%	3.54%	3.69%	3.98%	4.79%
LUPA3	1.63%	2.17%	2.34%	2.43%	2.60%	2.81%	3.04%	3.28%	3.39%	3.60%	4.30%
MMXM3	1.49%	2.13%	2.32%	2.44%	2.63%	2.84%	3.07%	3.32%	3.48%	3.76%	4.40%
MRFG3	1.80%	2.30%	2.50%	2.63%	2.85%	3.15%	3.37%	3.61%	3.77%	4.03%	4.86%
MRVE3	2.49%	3.12%	3.39%	3.60%	3.93%	4.29%	4.61%	4.89%	5.04%	5.34%	6.28%
NATU3	1.49%	1.96%	2.09%	2.18%	2.39%	2.57%	2.79%	3.02%	3.16%	3.36%	3.88%
NETC4	1.58%	2.03%	2.18%	2.27%	2.42%	2.63%	2.83%	3.17%	3.29%	3.51%	4.05%
OHLB3	1.49%	1.88%	2.00%	2.07%	2.23%	2.41%	2.60%	2.84%	2.99%	3.25%	3.95%
PCAR4	1.27%	1.70%	1.85%	1.93%	2.05%	2.18%	2.37%	2.55%	2.65%	2.82%	3.36%
PDGR3	1.85%	2.63%	2.80%	2.92%	3.14%	3.38%	3.58%	3.84%	4.02%	4.31%	5.22%
PETR4	1.49%	1.88%	2.02%	2.09%	2.22%	2.38%	2.53%	2.64%	2.71%	2.86%	3.34%
POSI3	1.85%	2.44%	2.66%	2.78%	2.99%	3.21%	3.43%	3.63%	3.75%	3.98%	5.02%
PRGA3	1.56%	1.99%	2.20%	2.32%	2.50%	2.71%	2.92%	3.09%	3.19%	3.41%	4.09%
PSSA3	1.14%	1.47%	1.57%	1.62%	1.74%	1.85%	2.01%	2.24%	2.32%	2.46%	2.93%
RAPT4	1.30%	1.66%	1.80%	1.88%	2.02%	2.22%	2.48%	2.72%	2.87%	3.13%	3.90%
RDCD3	1.79%	2.23%	2.44%	2.57%	2.79%	3.03%	3.25%	3.47%	3.61%	3.85%	4.69%
RENT3	1.51%	1.98%	2.17%	2.27%	2.45%	2.70%	2.92%	3.15%	3.28%	3.53%	4.26%
RSID3	2.08%	2.70%	2.93%	3.05%	3.26%	3.51%	3.78%	4.01%	4.16%	4.43%	5.11%
SBSP3	1.32%	1.81%	1.97%	2.06%	2.22%	2.43%	2.63%	2.80%	2.90%	3.10%	3.71%
SDIA4	1.59%	2.02%	2.20%	2.29%	2.47%	2.67%	2.90%	3.11%	3.23%	3.44%	4.04%
SLCE3	2.07%	2.57%	2.82%	2.97%	3.17%	3.45%	3.74%	4.00%	4.17%	4.50%	5.31%
SUZB5	1.15%	1.54%	1.66%	1.73%	1.87%	2.02%	2.26%	2.42%	2.52%	2.70%	3.23%
TAMM4	1.78%	2.31%	2.49%	2.59%	2.76%	2.96%	3.18%	3.38%	3.51%	3.74%	4.42%
TBLE3	1.29%	1.65%	1.78%	1.86%	2.00%	2.17%	2.35%	2.53%	2.62%	2.81%	3.33%
TCSL4	1.51%	1.88%	2.03%	2.12%	2.29%	2.49%	2.68%	2.85%	2.99%	3.19%	3.82%
TLPP4	0.95%	1.26%	1.39%	1.47%	1.59%	1.74%	1.89%	2.12%	2.24%	2.38%	2.79%
TMAR5	1.20%	1.65%	1.82%	1.91%	2.05%	2.22%	2.40%	2.56%	2.66%	2.90%	3.55%
TNLP4	1.39%	1.74%	1.87%	1.94%	2.07%	2.23%	2.42%	2.60%	2.69%	2.85%	3.40%
TRPL4	1.15%	1.70%	1.89%	1.98%	2.11%	2.27%	2.54%	2.73%	2.84%	3.03%	3.65%
UBBR11	1.14%	1.59%	1.74%	1.82%	1.96%	2.12%	2.30%	2.46%	2.55%	2.74%	3.33%
UGPA4	1.24%	1.62%	1.72%	1.78%	1.95%	2.09%	2.24%	2.39%	2.46%	2.63%	3.15%
UNIP6	1.02%	1.43%	1.55%	1.61%	1.75%	1.93%	2.08%	2.22%	2.32%	2.50%	3.06%
USIM5	1.53%	1.89%	2.02%	2.10%	2.25%	2.46%	2.66%	2.85%	3.00%	3.25%	3.82%
VALE5	1.43%	1.78%	1.91%	1.98%	2.10%	2.24%	2.40%	2.54%	2.63%	2.79%	3.22%
VCPA4	1.36%	1.75%	1.95%	2.06%	2.20%	2.36%	2.54%	2.71%	2.80%	2.99%	3.52%
VIVO4	1.65%	2.11%	2.29%	2.41%	2.60%	2.82%	3.05%	3.27%	3.38%	3.57%	4.27%

Table A: VDPI - Spatial dimension

This table presents the medians of VDPI 1%, 5%, 10%, 25%, 50%, 75%, 90%, 95% and 99% percentiles of the VDPI for the 84 stocks in the sample. It also contains the median of the maximum and the minimun of the VDPI. The medians, the maximum and the minimun are calculated for each stock between July 2006 and April 2009.

Stocks	Max/Min	vol99/vol1	vol95/vol5	vol90/vol10	vol75/vol25
ALLL11	2.60	1.66	1.43	1.33	1.16
AMBV4	2.69	1.70	1.45	1.34	1.16
ARCZ6	2.78	1.69	1.45	1.33	1.16
BBAS3	2.64	1.70	1.45	1.34	1.16
BBDC4	2.50	1.61	1.41	1.31	1.15
BNCA3	2.60	1.65	1.43	1.32	1.16
BRAP4	2.35	1.55	1.37	1.28	1.14
BRKM5	2.52	1.62	1.41	1.31	1.15
BRSR6	2.82	1.72	1.47	1.35	1.17
BRTO4	2.57	1.69	1.46	1.35	1.17
BRTP4	2.70	1.68	1.45	1.34	1.17
BTOW3	2.67	1.68	1.45	1.34	1.17
CCRO3	2.87	1.68	1.45	1.34	1.17
CESP6	2.89	1.72	1.47	1.35	1.17
CGAS5	3.34	1.85	1.54	1.40	1.19
CLSC6	2.87	1.76	1.49	1.36	1.17
CMIG4	2.65	1.65	1.43	1.32	1.16
CNFB4	2.96	1.77	1.50	1.37	1.18
CPFE3	2.73	1.74	1.49	1.37	1.18
CPLE6	2.67	1.69	1.46	1.34	1.17
CRUZ3	2.91	1.78	1.50	1.38	1.19
CSAN3	2.44	1.58	1.39	1.29	1.15
CSMG3	2.67	1.75	1.48	1.36	1.16
CSNA3	2.36	1.59	1.39	1.30	1.15
CYRE3	2.58	1.62	1.41	1.31	1.15
DASA3	2.56	1.67	1.44	1.33	1.17
DURA4	2.65	1.66	1.43	1.32	1.16
ECOD3	2.80	1.81	1.53	1.40	1.20
ELET6	2.76	1.69	1.45	1.33	1.16
ELPL6	2.97	1.78	1.53	1.40	1.19
EMBR3	2.70	1.69	1.46	1.35	1.17
ENBR3	2.62	1.65	1.43	1.32	1.16
ETER3	2.73	1.69	1.47	1.35	1.17
FFTL4	2.82	1.75	1.47	1.35	1.17
FHER3	2.55	1.62	1.42	1.32	1.16
GETI4	3.12	1.86	1.54	1.39	1.19
GFSA3	2.49	1.60	1.40	1.30	1.15
GGBR4	2.36	1.60	1.40	1.30	1.15
GOAU4	2.38	1.58	1.39	1.29	1.14
GOLL4	2.41	1.59	1.40	1.30	1.15
GVTT3	2.52	1.61	1.41	1.31	1.15
ITAU4	2.35	1.58	1.40	1.30	1.15
ITSA4	2.52	1.63	1.42	1.31	1.15
JBSS3	2.50	1.61	1.40	1.30	1.15
KLBN4	2.91	1.75	1.49	1.36	1.18
LAME4	2.75	1.69	1.45	1.34	1.17
LIGT3	2.92	1.78	1.51	1.38	1.18
LOGN3	3.07	1.82	1.53	1.40	1.19
LREN3	2.95	1.78	1.51	1.38	1.18
LUPA3	2.67	1.71	1.46	1.35	1.17
MMXM3	2.76	1.78	1.49	1.36	1.18
MRFG3	2.70	1.71	1.48	1.36	1.18
MRVE3	2.36	1.58	1.38	1.29	1.14
NATU3	2.81	1.73	1.48	1.36	1.18
NETC4	2.63	1.65	1.43	1.32	1.16
OHLB3	2.78	1.72	1.49	1.37	1.18
PCAR4	2.52	1.61	1.40	1.30	1.15
PDGR3	2.62	1.61	1.40	1.30	1.15
PETR4	2.50	1.63	1.42	1.32	1.16
POSI3	2.56	1.63	1.43	1.32	1.16
PRGA3	2.66	1.66	1.43	1.33	1.16
PSSA3	2.57	1.63	1.41	1.31	1.16
RAPT4	2.89	1.76	1.49	1.37	1.18
RDCD3	2.74	1.72	1.47	1.36	1.18
RENT3	2.74	1.69	1.45	1.34	1.17
RSID3	2.54	1.66	1.44	1.33	1.16
SBSP3	2.76	1.68	1.45	1.33	1.16
SDIA4	2.65	1.69	1.45	1.34	1.17
SLCE3	2.61	1.70	1.46	1.34	1.17
SUZB5	2.80	1.70	1.46	1.34	1.17
TAMM4	2.47	1.58	1.38	1.29	1.14
TBLE3	2.87	1.72	1.47	1.35	1.17
TCSL4	2.54	1.65	1.43	1.32	1.16
TLPP4	2.96	1.75	1.49	1.37	1.18
TMAR5	2.86	1.76	1.51	1.38	1.18
TNLP4	2.53	1.62	1.41	1.31	1.16
TRPL4	3.13	1.80	1.53	1.40	1.19
UBBR11	2.79	1.74	1.49	1.37	1.18
UGPA4	2.60	1.68	1.45	1.33	1.16
UNIP6	3.02	1.81	1.52	1.38	1.18
USIM5	2.27	1.56	1.38	1.29	1.14
VALE5	2.34	1.59	1.39	1.29	1.14
VCPA4	2.59	1.65	1.43	1.32	1.15
VIVO4	2.63	1.68	1.46	1.35	1.17

Table B: Ratios between percentiles of VDPI.

This table presents the ratios of the VDPI percentiles presented in Table A. Each entry represents the median calculated over the sample period between the ratio of two VDPI percentiles.

Months	Min	1%	5%	10%	25%	50%	75%	90%	95%	99%	Max
Jul 2006	1.30%	1.77%	1.91%	2.00%	2.16%	2.32%	2.45%	2.59%	2.69%	2.84%	3.38%
Aug 2006	1.17%	1.51%	1.64%	1.71%	1.82%	1.94%	2.06%	2.18%	2.25%	2.39%	2.89%
Sep 2006	1.24%	1.60%	1.74%	1.82%	1.95%	2.09%	2.27%	2.44%	2.53%	2.67%	3.19%
Oct 2006	1.02%	1.38%	1.50%	1.57%	1.69%	1.84%	1.98%	2.13%	2.22%	2.36%	2.76%
Nov 2006	1.20%	1.65%	1.80%	1.88%	2.00%	2.16%	2.31%	2.44%	2.52%	2.69%	3.18%
Dec 2006	0.95%	1.30%	1.42%	1.47%	1.55%	1.68%	1.78%	1.91%	1.99%	2.13%	2.63%
Jan 2007	1.16%	1.48%	1.57%	1.62%	1.75%	1.86%	1.99%	2.11%	2.18%	2.37%	2.87%
Feb 2007	1.22%	1.57%	1.70%	1.77%	1.90%	2.06%	2.29%	2.44%	2.57%	2.89%	3.47%
Mar 2007	1.24%	1.66%	1.78%	1.87%	2.02%	2.18%	2.32%	2.47%	2.59%	2.82%	3.34%
Apr 2007	1.04%	1.32%	1.44%	1.51%	1.64%	1.82%	1.94%	2.07%	2.14%	2.30%	2.76%
May 2007	1.11%	1.46%	1.59%	1.65%	1.76%	1.89%	2.07%	2.23%	2.31%	2.46%	2.94%
Jun 2007	1.01%	1.35%	1.47%	1.54%	1.64%	1.79%	1.92%	2.08%	2.17%	2.33%	2.78%
Jul 2007	1.31%	1.65%	1.78%	1.85%	1.98%	2.12%	2.30%	2.48%	2.57%	2.82%	3.44%
Aug 2007	2.22%	2.72%	2.89%	3.00%	3.19%	3.46%	3.74%	3.98%	4.20%	4.40%	5.19%
Sep 2007	1.30%	1.73%	1.88%	1.97%	2.13%	2.27%	2.41%	2.55%	2.65%	2.86%	3.40%
Oct 2007	1.32%	1.77%	1.92%	2.01%	2.13%	2.33%	2.50%	2.67%	2.75%	2.93%	3.53%
Nov 2007	1.95%	2.54%	2.70%	2.81%	2.97%	3.16%	3.39%	3.59%	3.70%	3.92%	4.68%
Dec 2007	1.37%	1.87%	2.05%	2.14%	2.27%	2.47%	2.68%	2.85%	2.96%	3.19%	3.91%
Jan 2008	1.98%	2.62%	2.83%	2.96%	3.20%	3.45%	3.71%	3.98%	4.11%	4.45%	5.27%
Feb 2008	1.51%	2.02%	2.19%	2.30%	2.47%	2.67%	2.88%	3.10%	3.21%	3.41%	3.99%
Mar 2008	1.69%	2.19%	2.37%	2.51%	2.72%	2.88%	3.08%	3.30%	3.42%	3.67%	4.36%
Apr 2008	1.33%	1.75%	1.86%	1.95%	2.09%	2.29%	2.53%	2.65%	2.73%	2.93%	3.49%
May 2008	1.28%	1.68%	1.83%	1.91%	2.06%	2.22%	2.39%	2.53%	2.64%	2.82%	3.35%
Jun 2008	1.31%	1.74%	1.90%	2.00%	2.16%	2.32%	2.50%	2.70%	2.83%	3.04%	3.64%
Jul 2008	1.73%	2.19%	2.34%	2.45%	2.63%	2.82%	3.04%	3.23%	3.39%	3.62%	4.33%
Aug 2008	1.44%	1.91%	2.08%	2.15%	2.32%	2.48%	2.65%	2.83%	2.96%	3.16%	3.85%
Sep 2008	2.67%	3.56%	3.84%	4.00%	4.30%	4.67%	5.10%	5.41%	5.60%	6.02%	6.98%
Oct 2008	5.41%	6.76%	7.17%	7.38%	7.79%	8.29%	8.78%	9.35%	9.66%	10.12%	11.86%
Nov 2008	2.58%	3.55%	3.91%	4.10%	4.46%	4.80%	5.14%	5.50%	5.71%	6.12%	7.36%
Dec 2008	2.04%	2.71%	3.01%	3.18%	3.43%	3.73%	4.03%	4.30%	4.45%	4.72%	5.64%
Jan 2009	1.64%	2.25%	2.48%	2.62%	2.85%	3.09%	3.31%	3.49%	3.59%	3.94%	4.80%
Feb 2009	1.19%	1.67%	1.88%	1.99%	2.17%	2.38%	2.59%	2.79%	2.91%	3.16%	3.91%
Mar 2009	1.45%	1.95%	2.13%	2.26%	2.42%	2.65%	2.90%	3.13%	3.26%	3.49%	4.19%
Apr 2009	1.69%	2.19%	2.36%	2.46%	2.61%	2.81%	3.02%	3.23%	3.33%	3.52%	4.30%

Table C: VDPI - Time dimension

This table presents the medians of VDPI 1%, 5%, 10%, 25%, 50%, 75%, 90%, 95% and 99% percentiles for all months in the sample period. It also contains the median of the maximum and the minimun of the VDPI. The medians, the maximum and the minimun are calculated across stocks for each month between July 2006 and April 2009.