DISTRIBUTION, STRUCTURAL CHANGE AND ECONOMIC EXPANSION IN A TWO-SECTOR MODEL: AN APPLICATION TO BRAZIL

Henrique Morrone[1]

1 INTRODUCTION

One of the increasing concerns in economic development is the assessment of the interrelations between the formal and informal sectors along the cumulative process of growth. Economists agree that in many low- and middle-income countries, the informal sector is a key player as a provider of jobs and a source of labour surplus in periods of rapid output expansion. Moreover, in the 1990s, stylized facts highlight an increase in the share of the informal sector during economic expansion in many developing countries (Rada, 2010). This phenomenon, known as jobless growth, is present in some developing countries such as India, China and in parts of South America.

In this context, in the 2000s, the Brazilian economy adopted a new economic model that has a focus on job creation and the redistribution of income toward the poor segments of society. A key interrogation of this study is whether redistribution of income to labour in Brazil is expansionary. If aggregate demand is wage-led, the redistribution of income to labour creates the conditions to output expansion. The fact that some economies are wage-led, while others are profit-led is entirely an empirical question.

The redistribution of income toward labour may positively impact the economy through several channels. First, the rise in labour's income generates a significant increase in demand because workers consume a large fraction of their income. This process of rise in consumption stimulates industrial production, causing a surge in labour productivity and structural change. It occurs because workers move from low productivity sectors to high productivity activities. Second, with the rise in labour income workers start to spend a higher fraction of their incomes in the consumption of the industrial or modern good, further stimulating the industrial production. This shift in consumption pattern occurs because industry pays higher wages. Moreover, since the industrial sector presents considerable economies of scale, the gains in labour productivity are substantial and might lead the economy into a virtuous circle. In Kaldorian fashion, the boost in industrial demand is fundamental to drive domestic labour productivity and output expansion.

Alas, the robust increase in productivity may hurt job creation and stop the fundamental process of labour transfer to industry, setting the economy into a low equilibrium position. To prevent this negative outcome, the government should promote policies with focus on both sectors.

Because of the critical role of the informal sector in Brazil, it is important to evaluate its relationship with the formal sector as a tool to understand the complexities of the process of economic expansion and to support future economic policy in the Brazilian economy. Policies that try to reduce poverty and promote economic growth must be based on a profound understanding of the economic structure. In this context, the government should prevent situations of sectoral isolation, i.e, prevent a situation where sectors maintain loose forward and backward structural linkages.

The purpose of this paper is to present a dual model that attempts to describe the Brazilian economy after 1994. The dual model must be capable of shedding light on the interaction between the two sectors during economic expansion. In this way, the current paper attempts to fill a gap in the literature on growth and distribution and its application to Brazil. A central thesis in the present study is that redistribution of income in favor of workers is expansionary. One aim of this study is to evaluate whether exchange rate depreciation is expansionary and if a policy that combines exchange depreciation and redistribution is feasible for the Brazilian economy.

The Structuralist model presented in this paper is standard (Taylor, 1983). The model describes an open, developing economy with two sectors, two commodities, and three classes. The model assumes no financial sector. We use the dual model in the short run to compare the effects of five experiments: an

[1] Economista INCRA/MDA. Universidade de Utah. E-mail: hmorrone@hotmail.com.

income transfer toward workers in the modern sector, an income transfer toward workers from the subsistence sector, an investment shock, an exchange rate shock, and the policy mix; i.e., the combination of exchange rate depreciation and income transfer toward modern sector workers. Additionally, sensitivity analysis experiments are applied to test the sensitivity of model results with respect to different economic scenarios.

This paper is organized as follows. Following this introduction, we present a brief background on the Brazilian economy. In the following section, the two-sector model is presented. The remainder two sections present simulation results and conclusions for the Brazilian economy. The complete description of the mathematical model appears in the appendix.

2 BACKGROUND

The Brazilian economy has performed rather poorly in the 1980s and 1990s: the average annual real growth rate was 3.02 and 1.64 per cent, respectively. Starting in the 1980s, many economists and policy makers believed that South American countries had previously chosen the wrong development model. The well-known model of industrialization through substitution of imports was adopted by many South American countries during the 1930s. The strategy was to protect the domestic market against external competition. The main goal was to promote industrialization through exchange rate controls and subsidies for key economic sectors. This model worked relatively well until the 1970s; however, the oil shocks of 1973 and 1979, the debt crisis of the 1980s, and the lack of resources necessary to produce industrial goods were some factors responsible for the abandonment of this development model.

The Industrialization for Substitution of Imports model (ISI) was replaced by a heterogeneous model in the early 1980s, and around the mid-80s Brazil started its process of economic liberalization. The abandonment of the ISI model was followed by profound criticism. The main shortcoming emphasized by the critics of the ISI model was that industrialization would solve the historical problem of income inequality.

In 2003, following a change in government, Brazil adopted a new economic model with a focus on economic growth with inclusion. The policy shift toward job creation marked a new phase in the Brazilian economy.

Tables 1, 2, 3, and 4 document these changes. Almost all the economic indicators show that the economic situation in the 1990s was the worst Brazil had experienced in 30 years. Table 1 shows that average GDP growth rate during the 1990s was even worse than the previous decade (1980-89). The Gini index of concentration revealed a slight improvement in the 1990s. The period was characterized by the profound intensification of neoliberal policies.

Table 1. Average GDP growth rate for Brazil during 1970-2009

	1970-79	1980-89	1990-99	2000-09
Brazil	8.789	3.022	1.645	3.33

Source: Instituto de Pesquisa Econômica Aplicada (IPEA).

Table 2. Inequality in Brazil (Gini index), 1985-2009

	1990	1995	2001	2005	2009
Brazil	0.614	0.601	0.596	0.569	0.543

Source: Instituto de Pesquisa Econômica Aplicada (IPEA).

Table 3. Land distribution in Brazil, 1967-2000

	1967	1972	1978	1992	1998	2000
Brazil	0.836	0.837	0.854	0.831	0.843	0.802

Source: Instituto Nacional de Colonização e Reforma Agraria.

Table 4. Informality in Brazil (%), 2003-2007

	2003	2004	2005	2006	2007
Brazil	54.3	53.7	52.9	52	50.7

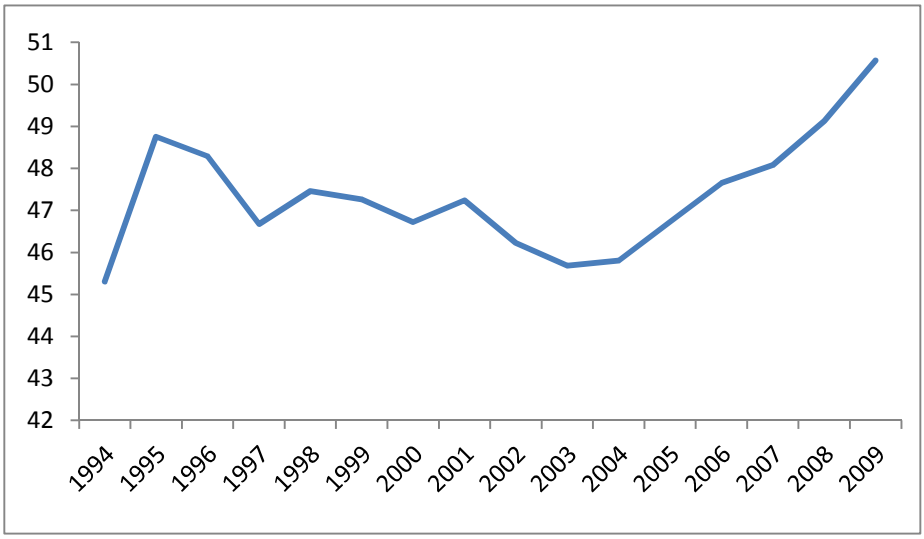
Source: Instituto de Pesquisa Econômica Aplicada (IPEA).

The next decade presented some improvements in the economic indicators. Table 4 shows that there was a decline in the degree of informality in the Brazilian economy. It was 54.3 per cent in 2000 and dropped to 50.7 per cent in 2007 (IPEA, 2010), a modest but important recovery. One of the possible reasons for the equality improvement might be the conditional cash transfer program called Bolsa Familia that provides financial support for poor families. This program redistributes income for people situated in the informal sector may have contributed to the improvement in the Gini coefficient. In 2009, this program benefited 12,370,915 families, which compared to the 2004 numbers (6,571,839 families) represents an increase of 14.38 per cent (IPEA, 2010). Furthermore, Table 2 shows better numbers for income inequality, as measured by the Gini index, after 2000. It seems that redistributive policy combined with economic expansion in the period may have positively impacted the economy.

Figures 1 and 2 exhibit the wage share and social spending as a percentage of GDP, respectively. Figure 1 shows a downward trend in the wage share [2] between 1995 and the mid-2000s. This persistent decline is interrupted in 2004, the turning point where the wage share starts to grow again. The intensification of distributive policies in an attempt to fight poverty and inequality, together with an economic model that has focused on job creation, are the defining features of the Brazilian economy during the 2000s.

Turning now to Figure 2, we detect that social spending increases during the 2000s. Social expenditures include government spending in social transfers, eduction, culture, health, social security, and housing. This positive trend on social spending with respect to GDP revels a possible positive effect on output expansion.

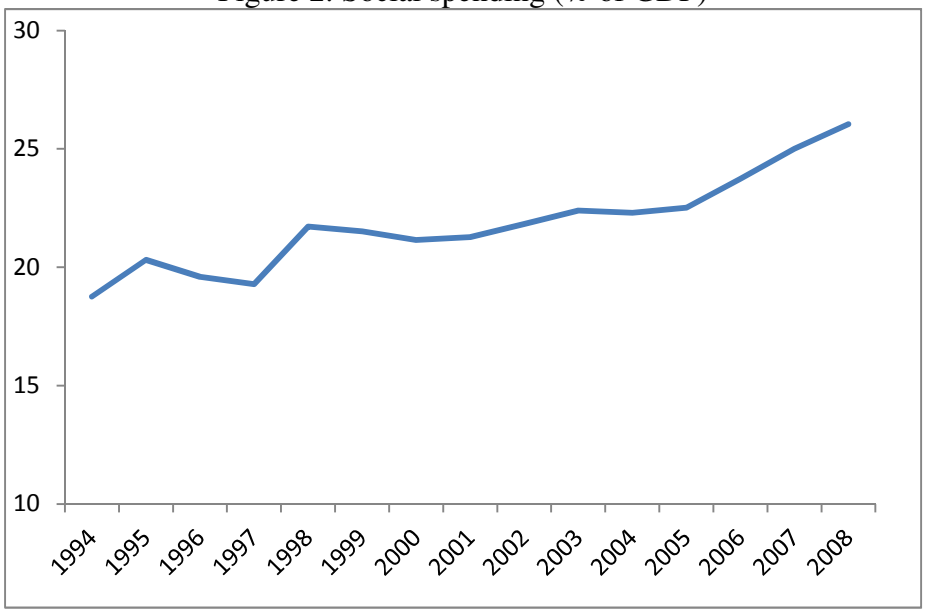
Figure 1. Wage share (% of GDP)



Source: Instituto Brasileiro de Geografia e Estatística (IBGE).

[2]The wage share was calculated as the wage bill over value added measured at factor costs. The estimation does not adjust for the informal sector.

Figure 2. Social spending (% of GDP)



Source: Economic Commission for Latin America & the Caribbean.

To summarise, the Brazilian economy's performance in the 1990s was a result of the adoption of a neoliberal economic model and negative external shocks. Redistribution of income may foster short-term economic expansion if the accelerator is the most important factor to explain the increase in the investment level. In other words, income redistribution is a sufficient condition for development if and only if the Brazilian economy is wage-led. The fact that some economies are wage-led and others are profit-led is entirely an empirical question. A progressive policy that combines exchange depreciation, and creation of jobs with social policies, through its impact in aggregate demand, may be an important source of economic growth. Indeed, the positive economic statistics in the 2000s are the result of an implemented policy with a focus on social policies and generation of formal jobs (International Labour Organization Report, 2011). Therefore, as the Great Recession (2007-2009) unfolded, the Brazilian economy was capable to present a fast recovery.

3 THE MODEL

The application of Structuralist Computable General Equilibrium (CGE) models for Brazil started in the 1980s. Taylor and Lysy (1980) have a model for the Brazilian economy that investigates the economic results of an exogenous change in the distribution of income. Subsequent research attempted to address questions related to balance of payments constraints and policies of stabilization.

The model presented in this section is straightforward. It represents a surplus labour open economy with two sectors, two commodities, and three economic classes - a capitalist, a modern, and a subsistence household, respectively. The model can be considered structuralist because it takes into account the structural features of the economy as important determinants of its evolution. The antecedents of the model are the Taylor (1983) and Rada (2007) two-sector models.

The Social Accounting Matrix (SAM) in Table 5 provides a schematic description of the economy. It describes the circular flow of income for the economy. The data comes from the System of National Accounts (SNA-IBGE, 2011) and Morrone (2012). Table 6 presents the SAM for Brazil for 2006. The Social Accounting Matrix consists of a union of both the input-output (I-O) table, which describes the inter-industry transactions in the economy, and a flow of funds table, which shows the income transfers between institutions. In addition, the SAM is a square matrix, which is a necessary condition to the existence of one solution. The columns of the matrix represent purchases and the rows

represent sales.

Table 5. A schematic social accounting matrix for a two-sector economy

	Costs		Use of income						
	(T)	(N)	Ywt	Yπt	Ywn	Yf	G	Accumulation	Totals
(T) Modern	Pt att Xt	Pt atn Xn	Pt Ctwt Lt		Pt Ctwn Ln	Pt Et		Pt It	Pt Xt
(N) Subsistence	Pn ant Xt	Pn ann Xn	Pn Cnwt Lt		Pn Cnwn Ln				Pn Xn
Income									
Labor (t)	(1-π)Zt Yt						TRt		Ywt
Profit (t)	πZt Yt								Yπt
Labor (n)		Zn Yn					TRn		Ywn
Foreign	M								Yf
Government				t π Zt Yt					G
Savings				Sπt		Sf		-Pt It	0
Totals	Pt Xt	Pn Xn	Ywt	Yπt	Ywn	Yf	G	0	

The two sectors that are important in the analysis are the subsistence sector (n) and the modern sector (t).[3] The former produces a nontradable (N) good while the latter produces a tradable (T) good. They are not perfect substitutes. Private income is distributed among three classes: capitalists in the modern sector, workers in the modern sector, and workers in the subsistence sector. Capitalists do not consume. Workers spend all of their income on the consumption of both tradable and nontradable goods, which is in agreement with the classical approach. The modern sector produces its own tradable commodity that can be exported, consumed, or invested. In this way, the foreign sector supplies intermediate inputs to the modern sector. It is important to emphasize that the subsistence sector[4] presents a low labour productivity level, whereas the modern sector does not.

Table 6. A social accounting matrix for Brazil 2006

Costs		Use of Income					
Formal	Informal	Formal HH	Business	Informal HH	Exports	Investment	Totals
1334.10	169.53	737.21		222.48	407.60	448.62	3319.52
226.63	30.97	165.37		57.26			480.23
902.58							902.58
702.34							702.34
	279.73						279.73
153.87							153.87
		0.00	702.34	0.00	-253.73	-448.62	0.00
3319.52	480.23	902.58	702.34	279.73	153.87		

Note: SNA and author's calculations.

A central assumption of the model is that there is no unemployment in the economy. Workers are assumed to always find a job in the subsistence sector. As in the extensions of the Lewis model, there is a difference in wages; that is, wages in the modern sector tend to be higher than in the subsistence sector. The equation below formalizes the labour market assumption.

$$L=L_t+L_n \qquad (1)$$

The labour remuneration in the subsistence sector is $w_n=\varepsilon_n Z_n$; hence, there is no clear distinction between capital and labour income in this sector. The transfer of workers from the subsistence sector, a

[3]Throughout this study the label `n' is used to refer to the subsistence (informal) or low productivity sector; the term `modern' and the subscript `t' represent formal activities.

[4]There is no division between labour and capital income in the subsistence sector.

low labour productivity sector, to the modern sector, a high labour productivity sector, leads to a rise in average labour productivity in the whole economy. With a higher capital-labour ratio and access to capital, the industry can make any transferred worker more productive.[5]

The subsistence sector is supply-constrained; that is, the price level of the subsistence sector adjusts to achieve the new equilibrium in the short run. There is no excess capacity in this sector. The output equation of the subsistence sector is presented below. Considering that labour productivity is equal to the subsistence sector value-added divided by the subsistence sector labour, or $\varepsilon_n = Y_n / L_n$, we can rewrite the equation as:

$$Y_n = \varepsilon_n L_n \tag{2}$$

In contrast to the subsistence sector, the modern sector operates with excess capacity. It is a quantity-clearing sector and, hence, demand-constrained. Output in the modern sector is supposed to change to accommodate disturbances in other variables. Notice that capital stock is present only in the modern sector. The variable investment is endogenous. It is a function that incorporates the value-added of the modern sector and profit as explicative variables. The investment function below includes the accelerator and the effect of profits on investment.

$$I_t = z_0 + z_1 \Pi + z_2 Y_t \tag{3}$$

Before we start to analyse the short-term adjustment of the model, some additional variables must be introduced. Let us begin with some important variables. First, the value-added of the two sectors is proportional to their respective supply. The shares of value-added in supply are presented below:

$$\nu_t = \frac{Y_t}{X_t} = 1 - a_{tt} - a_{nt} - fe \tag{4}$$

$$\nu_n = \frac{Y_n}{X_n} = 1 - a_{nn} - a_{tn} \tag{5}$$

where f, $f = \frac{M}{X_t}$, and e, stand, respectively, for the share of imports in supply and nominal exchange rate. The element a_{ij} (i,j=n,t) represents a technical coefficient; the term input-output coefficient is also used. For instance, the element a_{tt} measures a fixed relationships between the formal sector's output and its own produced inputs. In this sense, $a_{tt} X_t$ represents intermediate sales of the formal sector to itself.

Moreover, the model has exports and imports as endogenous variables that respond to price and output changes. The two equations are presented below:

$$Et = \chi^0 (\rho)^{\chi} X_f \tag{6}$$

$$M = \phi^0 (\rho)^{-\phi} X_t \tag{7}$$

where ρ, $\rho = \frac{eP^*}{P_t}$, is the real exchange rate and X_f is the foreign demand for the modern sector goods. The parameters ϕ and χ stand, respectively, for exports and imports' trade elasticities.

Turning now to sectoral prices, some considerations are important. The modern sector price, P_t, is

[5]Empirical evidence shows that the Kaldor-Verdoorn (KV) Law is important in explaining the process of economic growth and the standing of industry as the engine of growth. For instance, Marinho et al. (1998) estimated the KV Law for the manufacturing sector of the Brazilian economy during 1985-1997, and they found a statistically significant verdoorn coefficient. Their Error Correction Model (ECM) shows that an increase in manufacturing output causes a rise in its labour productivity of .88 per cent in the short run and .33 per cent in the long run.

established by an accounting relationship. Different from the subsistence sector, this price is cost-determined. It might be mathematically expressed as a weighted average of cost components (von Arnim and Rada, 2011). Using the variables introduced previously, the function is presented below:

$$P_t = \frac{a_{nt} P_n + \nu_t Z_t + fe}{1 - a_{tt}} \tag{8}$$

To include intermediates into the model, we need to incorporate value-added prices of the two sectors. In this sense, Z_n stands for the value-added price of the subsistence sector and the variable Z_t represents the value added price for the traditional or modern sector. The former is an accounting equation to clear the cost decomposition while the latter is a behavior function. However, since we consider the modern sector labour-output ratio, b_t, fix, the net price Z_t responds to changes in the formal sector wage and profit share. The respective equations are presented below:

$$Z_n = \frac{(1 - a_{nn}) P_n - a_{tn} P_t}{\nu_n} \tag{9}$$

$$Z_t = \frac{1}{(1 - \pi)} w_t b_t \tag{10}$$

In conclusion, P_t responds to costs, Z_n reacts to changes in the excess of P_n over sectoral costs, and Z_t reacts on changes in the functional distribution of income.

Finally, the aggregate price for the whole economy, the GDP-deflator, is calculated as a Fisher index[6] of the two sectoral prices. It is estimated as the square root of the multiplication of Laspeyres and Paashe indexes, considering sectoral value added prices and quantities pre- and post-shocks.

To analyse the short-term adjustment of the model, we need to consider the excess demand function for both markets. The macro equilibrium condition is achieved when the excess demand equations for both subsistence and modern sectors are zero, $ED_n = ED_t = 0$, which describes a situation when the Social Accounting Matrix balances. The excess demand equation for the subsistence sector, ED_n, is the difference between aggregate demand and aggregate supply (X_n). The excess demand equation for the subsistence sector is presented below:

$$ED_n = a_{nn} X_n + a_{nt} X_t + C_{wn}^{\ n} L_n + C_{wt}^{\ n} L_t - X_n = 0 \tag{11}$$

where $C_{wn}^{\ n}$ and $C_{wt}^{\ n}$ stand, respectively, for the consumption of the subsistence good by workers in the subsistence and modern sectors.

We assume that both workers consume the nontradable good. We use the Linear Expenditure System (LES) to incorporate the consumer choice into the analysis, which is derived mathematically in Appendix A. Notice that both workers consume a minimum amount, θ, defined as the floor-level consumption of the subsistence good. A Positive θ [7] implies an income-inelastic subsistence good demand and an income-elastic modern sector's good demand. The rest of the income is divided between the two goods, in this case, $(1-\alpha)$ and $(1-\beta)$. We can include the demand equations from Appendix A to rewrite the equation as:

$$ED_n = a_{nn} X_n + a_{nt} X_t + \left(\frac{(1-\beta)(w_n + TR_n)}{P_n} + \beta\theta_n\right) L_n + \left(\frac{(1-\alpha)(Z_t w_t + TR_t)}{P_n} + \alpha\theta_t\right) L_t = 0 \tag{12}$$

where TR_n, b_t, TR_t, α, and β, stand for the income transfer from profit to labour in the subsistence

[6] The procedure to estimate the GDP-deflator is based on von Arnim and Rada (2011).

[7] See Taylor (1979: 219-22) for more details.

sector, the labour-output ratio of the modern sector, the transfer to workers in the modern sector, the share of worker's income in the modern sector spent on the consumption of the modern good and the share of income spent by the subsistence sector on the consumption of the modern good, respectively. If we use the variables previously introduced and make some extra manipulation to solve the equation for P_n, we can get:

$$P_n=\frac{(1-\alpha)(Z_t\nu_t X_t(1-\pi)+TR_t)-\frac{(1-\beta)((\nu_t Z_t+fe))a_{tn}X_n}{(1-a_{tt})}+(1-\beta)TR_n}{\beta(X_n-a_{nn}X_n-\theta_n L+\theta_n b_t\nu_t X_t)-\alpha\theta_t b_t\nu_t X_t-a_{nt}X_t+\frac{(1-\beta)a_{nt}a_{tn}X_n}{1-a_{tt}}} \quad (13)$$

Following similar procedure for the modern sector, we can get the results below. Notice that this is a demand-constrained sector; hence, we should solve the equation for gross output (X_t).

$$ED_t=a_{tt}X_t+a_{tn}X_n+C_{wt}^{\ t}L_t+C_{wn}^{\ t}L_n+E_t+I_t-X_t=0 \quad (14)$$

where E_t and I_t stand, respectively, for tradable good exports and investment. Using the Linear Expenditure System (LES) demand functions described in the Appendix A, we can rewrite the above equation as follows:

$$ED_t=a_{tt}X_t+a_{tn}X_n+\frac{\alpha(Z_t w_t+TR_t-P_n\theta_t)}{Pt}L_t+\frac{\beta(w_n+TR_n-P_n\theta_n)}{P_t}L_n+E_t+I_t-X_t=0 \quad (15)$$

Then, setting the equilibrium condition that $ED_t=ED_n=0$ and solving for the endogenous variable X_t, we can get:

$$X_t=\frac{P_t(a_{tn}X_n+z_0+\chi^0(\rho)^{\chi}X_f-\beta a_{tn}X_n)+\alpha TR_t+\beta TR_n-\beta P_n(\theta_n L-X_n+a_{nn}X_n}{P_t(1-z_1\pi\nu_t-z_2\nu_t-a_{tt})+\alpha P_n\theta_t b_t\nu_t-\alpha(1-\pi)Z_t\nu_t-\beta P_n\theta_n b_t\nu_t} \quad (16)$$

The above equations (see the glossary in the appendix) might be arranged in blocks as follows:

1. Sectoral Balances

$$a_{nn}X_n+a_{nt}X_t+C_{wn}^{\ n}L_n+C_{wt}^{\ n}L_t-X_n=0 \quad (17)$$

$$a_{tt}X_t+a_{tn}X_n+C_{wt}^{\ t}L_t+C_{wn}^{\ t}L_n+E_t+I_t-X_t=0 \quad (18)$$

2. Price Equations

$$Z_n=\frac{(1-a_{nn})P_n-a_{tn}P_t}{\nu_n} \quad (19)$$

$$Z_t=\frac{1}{(1-\pi)}w_t b_t \quad (20)$$

$$P_n\propto a_{nn}X_n+a_{nt}X_t+C_{wn}^{\ n}L_n+C_{wt}^{\ n}L_t-X_n \quad (21)$$

$$P_t=\frac{a_{nt}P_n+\nu_t Z_t+fe}{1-a_{tt}} \quad (22)$$

3. Total Disposable Income by Classes

$$DY_{wt}=(P_t C_{wt}^{\ t}+P_n C_{wt}^{\ n})L_t=Z_t w_t b_t Y_t+TR_t L_t \quad (23)$$

$$DY_{wn}=w_n b_n Y_n+TR_n L_n \quad (24)$$

$$DY_{\pi}=(1-t_t-t_n)(\pi Z_t Y_t) \tag{25}$$

$$DY_f=eP^* fX_t \tag{26}$$

4.Consumer Demand Equations

$$D(c_{wt}^{\ n})=\frac{(1-\alpha)(Z_t w_t+TR_t)}{P_n}+\alpha\theta_t \tag{27}$$

$$D(c_{wt}^{\ t})=\frac{\alpha(Z_t w_t+TR_t-P_n\theta_t)}{Pt} \tag{28}$$

$$D(c_{wn}^{\ t})=\frac{\beta(w_n+TR_n-P_n\theta_n)}{P_t} \tag{29}$$

$$D(c_{wn}^{\ n})=\frac{(1-\beta)(w_n+TR_n)}{P_n}+\beta\theta_n \tag{30}$$

5. Investment Function

$$I_t=z_0+z_1\Pi+z_2 Y_t \tag{31}$$

6. Saving-Investment Balance

$$S_{\pi}+S_f=P_t I_t \tag{32}$$

$$(1-t_n-t_t)\pi\frac{Z_t}{P_t}Y_t+e(P^*/P_t)fX_t-E_t-z_0-z_1\pi Y_t-z_2 Y_t=0 \tag{33}$$

Equation 33 exhibits the macroeconomic balance between saving and investment. If we include equations 27, 43 and 44 into equation 32 and divide by Pt, we get the saving-investment balance equation, 33. Equations 43 and 44 reveal the saving supply process; in other words, the total saving is the sum of saving out of profits and foreign saving. The closure of the model is Keynesian; that is, investment is triggered by changes in output. In other words, the investment level rises in response to a change in aggregate demand.

The system is locally stable if and only if the trace of the Jacobian matrix is negative and the determinant of the Jacobian matrix is positive. Also, the two eigenvalues of the Jacobian of partial derivatives should be positive to imply a system with local stability. See Appendix A for more details.

The macroadjustment process can be explained using the excess demand equations (equations 12 and 14). From equation 12, a surge in the modern sector's gross output, X_t, generates an excess demand in the subsistence sector. This disequilibrium is solved through an increase in the price of the subsistence sector good. The variables P_n and X_t, therefore, are positively related, and the excess demand curve, ED_n, is positively sloped.

In the modern sector, equation 14 shows that an increase in P_n leads to an expansion in the gross production of the modern sector, X_t. Consequently, the modern sector excess demand curve, ED_t, is positively sloped.

The magnitude of the increase in X_t, however, is not clear. The magnitude of the increase in X_t will depend on Engel's law.[8] An increase in P_n can affect X_t through two channels. First, an increase in P_n leads to an increase in wages of the subsistence sector that will cause a rise in demand of the modern sector goods. Second, a rise in P_n will cause a decrease in real wages in the modern sector because

[8] Empirical evidence shows that food consumption is income-inelastic; that is, a 1 per cent increase in income will produce less than a 1 per cent rise in food consumption.

workers consume the subsistence sector's good. As a result, there is a reduction in the demand for the modern sector's goods. The final result will depend on the magnitude of these two effects. A strong Engel's effect, therefore, will lead to a lower demand from the modern sector, which will cause X_t to grow less. In some cases, Engel's effect may be so strong that a rise in P_n leads to a decline in X_t. Conversely, a weak Engel's effect will cause X_t to grow faster.

4 EMPIRICAL RESULTS

In this section, five simulation experiments are analysed: an income transfer toward modern sector workers, an income transfer to subsistence sector labour, an investment shock, an exchange rate depreciation, and a shock that combines depreciation and income transfer to modern workers together. Although, to estimate the post-shock results of the third simulation (investment shock), we need to employ a slightly different model. Here, the investment level is considered an exogenous variable. For the remaining experiments, an independent (endogenous) investment function, à la Kalecki and its colleague Steindl, is applied. The three components of the independent investment function are: animal spirits, a coefficient measuring the effect of profits on investment, and an accelerator. The animal spirits component of the investment function represents the part of investment that is not explained by changes in profits and output. In other words, it is simply the autonomous part of the investment.

Before we explore the results, it is important to highlight some assumptions of the model. First, it is assumed that only modern sector imports exist; there are no subsistence sector or final goods imports. Second, the parameters of the investment function come broadly from econometrics. The effect of profit changes on the investment level is supposed to be small. It is assumed to be about 3 per cent, since econometric estimations of investment functions usually do not include any measure of profits as an explanatory variable. In contrast, the accelerator is supposed to be about 10 percentage points. Lastly, animal spirits are calculated as a residual. However, one difficulty appears in the process of estimation of these parameters. It occurs because the Brazilian economy has a low level of investment relative to GDP compared to other fast-growing, emerging economies. The solution is to make the relative value of the parameters with respect to each other stable and proportional. In other words, if, for instance, econometric estimations suggest that the accelerator is five times higher than the effect of profits on investment, we used this estimation to set our two parameters. In this sense, the parameters are, on average, in line with empirical evidence.

The fraction of income spent in the modern good, α, from modern sector workers and the fraction of income spent in the modern good, β, by workers in the subsistence sector depend on budget shares and Engel elasticities. Floor consumption of subsistence goods depends on the same variables and marginal budget shares. We assume the ratio of floor consumption with respect to total consumption of the modern sector households, $\frac{\alpha\theta_t L_t}{(c_{wt}^{\ t} L_t + c_{wt}^{\ n} L_t)}$, to be 4 per cent, while the ratio to the informal sector is 12 per cent. It means that only a small part of worker's demand is invariable to changes in their real income.

In Table 7, two calibrations are considered: a scenario where trade price elasticities are set to zero, and an alternative scenario where price elasticities of exports and imports are relatively high (χ, ϕ=0.75). For every shock, Table 7 shows two columns. Column (1) reveals the results for the first calibration, while Column (2) shows the results for the second. The top block of the table describes the results of macroeconomic indicators, such as inflation and real GDP growth. All the statistics are shown in percentage points. Lastly, the bottom block shows meso-economic indicators.

The remainder of this section is organized as follows. Below we explore the empirical results of the first two simulations. Next, we analyse the model results for the exchange depreciation shock, the investment shock and the policy mix shock. Finally, the final part discusses the sensitivity analysis experiments and summarises the results.

4.1 Income Transfer Shocks

In this experiment, the income transfer toward formal workers is raised by an amount equivalent to 1 per cent of GDP. This transfer is financed by an income tax on capitalists' income.

At the macroeconomic level, calibration (1) presents the following results. Real GDP grows at 4.9 per cent, and GDP-deflator grows at 5.1 per cent. Additionally, the private balance ($\frac{S-I}{GDP}$) improves by 0.074 percentage points. Because imports are proportional to output and exports respond to price changes, the external balance with respect to GDP, ($\frac{E-M}{GDP}$), deteriorates following the expansion. It deteriorates by 1.082 percentage points. The economic expansion is triggered by a surge in aggregate demand. In Kaleckian fashion, a redistribution of income from capitalists to workers leads to a rise in aggregate demand and output. This increase in output further increases investment, through the accelerator, which feeds back into higher output expansion. In some special cases, this may lead to a virtuous cycle. The rise in real activity generates enough savings to match the rises in investment. Structuralist CGE models applied to other countries presented similar macroeconomic results. Arnim and Rada (2011), and Cuesta (1990) found similar macroeconomic results for exogenous changes in nominal wages for Egypt and Colombia, respectively.

At the meso-economic level, the initial redistribution promotes structural change. The modern sector employment share rises; it improves by 2.45 percentage points. The modern sector employment share grows at 5.78 per cent, the same growth rate of the modern sector GDP so long as labour-output ratio remains constant and overall labour supply is exogenous. The transfer of workers from the subsistence sector, a low labour productivity sector, to the modern sector, a high labour productivity sector, leads to a rise in average labour productivity in the whole economy. With a higher capital-labour ratio and access to capital, the formal sector can make any transferred worker more productive. Labour productivity, ε_n, grows at 4.44 per cent. Because of the presence of a steep supply for the subsistence sector good, inflationary pressures emerge. The price of the informal good increases 23.11 per cent. The inflationary process hurts further expansion. In sum, the redistribution creates the sufficient conditions for economic expansion.

Comparing the results of calibration (1) and (2), we see that the sign pattern does not change. Both simulations have economic expansion as their main outcome. GDP grows at 2.89 per cent. Price, GDP-deflator, grows at 3.15 per cent. As expected, calibration (1) promotes a stronger economic expansion since leakages of the system are reduced. In conclusion, the simulation results suggest that more progressive redistributive policies, back in 2006, could stimulate a stronger economic expansion.

A similar interpretation is behind the results of the second experiment. An income transfer toward subsistence sector labour generates a boost in economic activity. Real GDP grows at 4.63 per cent; inflation is 4.64 per cent. As before, the external balance deteriorates by 1.022 percentage points while the private balance presents a small improvement, it improves by 0.022 percentage points.

An in-depth sectoral analysis allows us to verify that the consumption expansion promotes a smaller structural change. The modern sector employment share improves, going from 42.4 per cent (base year) to 44.71 percentage points. As expected, the migration of workers from the subsistence sector to the modern sector creates higher productivity in the whole economy. Because of a higher capital-labour ratio and easier access to capital, the formal sector is able to make any additional employed worker more productive. Labour productivity in the subsistence sector grows at 4.18 per cent since labour in this sector is assumed to be redundant.

Although this sector has not been recognized as strategic in the standard literature, according to the simulation result, it has a high capacity to stimulate economic activity. In a certain degree, the positive result occurs because informal goods inflation is translated into higher informal wages, leading to further expansion; whereas, the opposite occurs in the case of income redistribution toward modern sector

workers. Moreover, the simulation result relies on specific parameter values.[9]

To summarise, it is interesting to detect that both sectors have strategic roles. The formal sector is important and governmental policies should focus on ways to improve this activity without losing sight of the subsistence sector. The government should consider policies that positively impact both sectors, since they are strongly connected. Improvements in the labour productivity of the subsistence sector are required to achieve a sustainable expansion. The simulation results, therefore, suggest that more progressive redistributive policies, both back in 2006 and now, could stimulate a stronger economic expansion.

4.2 Investment, Exchange Rate and Policy Mix Shocks

Real investment is raised by an amount equivalent to one percentage point of GDP in this experiment. Columns 5 and 6 of Table 7 reveal the detailed numbers. Let us begin with the first calibration that turns off trade price elasticities.

At the macroeconomic level, real GDP grows at about 2.7 per cent and price grows at 2.36 percentage points. The private and external balances with respect to GDP deteriorate by 0.051 and 0.592 percentage points, respectively. An increase in investment leads the economy into a demand-driven expansion. This demand expansion leads to labour transfer, output expansion, and inflation.

The exogenous shock causes structural change. Higher demand for the modern sector good stimulates production and labour demand. The modern sector employment share improves by 1.313 percentage points. The labour transfer from the subsistence sector to the modern sector creates higher productivity in the whole economy. Labour productivity, ε_n, grows at 2.33 per cent. Because of the presence of a steep supply for the subsistence sector good, inflationary pressures emerge. In structuralist fashion, it is mainly caused by a relatively unresponsive supply of subsistence sector good.

Comparing the results of the two calibrations, we detect that the sign pattern does not change. As before, calibration (1) generates a stronger economic expansion because leakages in the system are reduced. In short, the simulation result suggests that a more progressive policy that promotes investment, such as industrial policy, back in 2006, could lead to a robust process of economic expansion.

Now let us turn the attention toward exchange rate depreciation. According to calibration (1), an exchange depreciation shock causes output contraction. A contractionary depreciation might be caused by many factors; for instance, capital goods imports might be price inelastic in some developing countries. This is a tight constraint for many low and mid-income countries. Another possible factor is that depreciation may cause a considerable reduction on real wages, consequently, reducing consumption. Krugman and Taylor (1978) present a detailed discussion of these factors. Moreover, Taylor (1983) applied a five sector Structuralist CGE model for India. He detected that exchange depreciations might generate contractionary results. In the Indian context, the depreciation rises imports, which drives up the intermediate costs of the industrial sector. This increase in costs triggers higher final prices of the modern sector good, reducing both the real wages of the modern sector workers and aggregate demand. Since the industrial sector clears by changes in quantities, the modern sector output falls.

Table 7 shows that post-shock the economy goes into a situation characterized by output contraction, deflation, and real depreciation. GDP grows at a negative rate of 1.695 per cent. At the same time, prices present a negative growth rate of 1.204 per cent. Private and external balances have a small deterioration; they decline by 0.325 and 0.051 percentage points, respectively. Following the exchange depreciation, real income and overall savings fall, which leads to a labour transfer from the formal to the informal sector. This labour transfer causes a reduction in the labour productivity of the whole economy. In this context, labour productivity in the subsistence sector declines by 1.445 per cent. Since wages in the subsistence sector, w_n, are a function of the subsistence sector's labour productivity, there is a real wage fall for subsistence workers. This wage reduction has a negative impact on consumption, leading to

[9] Depending on different parameter values, the model may present instability problems. For income transfers and investment shocks above 5 per cent of GDP, the system becomes unstable.

further decreases in output.

Using the results of calibration (2), we detect that exchange depreciation leads to economic expansion and inflation. Real GDP grows at 2.223 per cent and inflation grows at 2.079 per cent; the modern sector employment share improves by 1.106 percentage points. Labour productivity grows at 1.959 per cent. Comparing the two calibrations, there is a clear change in pattern. It seems that there is a threshold, in terms of trade price elasticities, beyond which further depreciations become expansionary. A similar sign change was found in Egypt by von Arnim and Rada (2011). Furthermore, Cuesta (1990) applied a Structuralist CGE model for Colombia that shows the same pattern; that is, exchange depreciation up to certain point is contractionary.

In conclusion, the lesson is that exchange rate policies should be implemented in a cautious way since real wage reductions might lead the economy into a recession. The fact that the Brazilian economy is relatively open, suggests that active exchange rate policies may be an important tool to achieve fast growth. These results are in line with the structuralist literature (See Krugman and Taylor, 1978; Taylor, 1983; and Taylor, 1990).

Finally, the last experiment combines an income transfer to formal workers in the amount of 1 per cent of GDP and 5 per cent exchange depreciation. The last column of Table 7 reveals the detailed numbers. At the macroeconomic level, real GDP grows at about 5.275 per cent and price grows at 5.743 per cent. The private balance with respect to GDP improves by 0.945 percentage points. The external balance with respect to GDP deteriorates by 1.161 percentage points. The implementation of the policy mix strategy leads the economy into a demand-driven expansion.

Again, the exogenous shock promotes structural change. Higher demand for the modern sector good stimulates production and labour demand. The modern sector employment share improves by 2.626 percentage points. The transfer of labour to the formal sector causes the labour productivity, ε_n, to grow at 4.778 per cent.

In this sense, a progressive police that focus on output expansion and creation of formal jobs might be crucial for the Brazilian economy. A post-Keynesian policy, and at least in a certain degree a revival of the ISI model (with focus on industrial policy and exports), may be important to foster economic development. This policy is probably viable for countries that have a significant size of the domestic market.

The exchange rate and redistributive policies employed together, therefore, might be a suitable strategy to boost economic activity, avoiding the current persistent pressures on the exchange rate. Currently an overvalued exchange rate has engendered the slow down in economic activity. At least partially, this is related to high domestic interest rate, which leads to carry trade further appreciating the exchange rate. This process hurts the industrial sector, reducing its exports. Back in the 1990s, the same pressures on the exchange rate caused a sluggish economic performance. The policy mix suggested here might alleviate these pressures, fostering economic expansion and providing additional resources to the segments of society that loses with the inflationary process.

Table 7. Simulation results

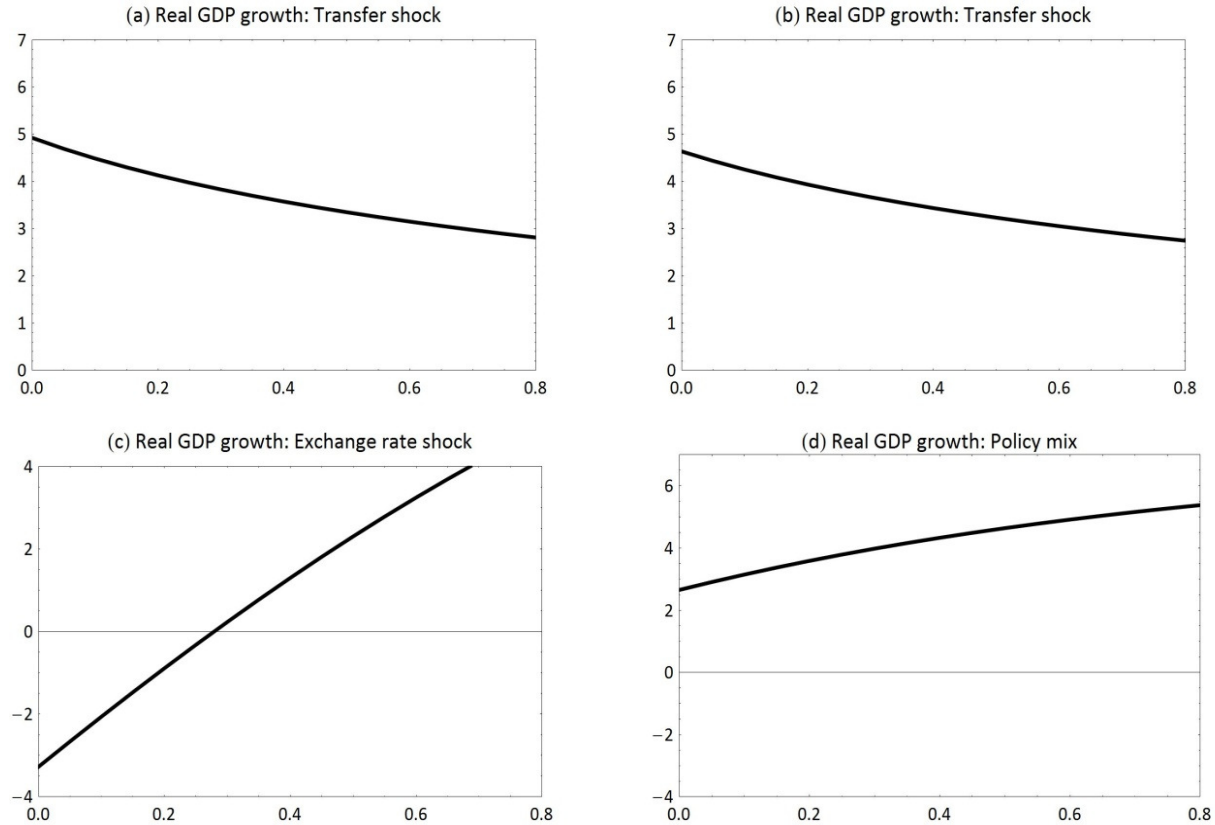
	Transfer to formal workers (1% of GDP)		Transfer to informal workers (1% of GDP)		Investment shock (1% of GDP)		Exchange rate depreciation (5%)		Policy mix
	1	2	1	2	1	2	1	2	2
Macroeconomic statistics									
Real GDP growth (%)	4.922	2.893	4.638	2.820	2.636	1.990	-1.695	2.223	5.275
Inflation (%)	5.150	3.152	4.640	2.884	2.366	1.795	-1.204	2.079	5.743
Real exchange rate (Δ in % pts)	-4.895	-3.055	-4.439	-2.803	-2.311	-1.763	6.279	2.862	-0.7022
Private balance (Δ in % points of GDP)	0.074	-0.297	0.022	-0.311	-0.051	-0.244	-0.325	0.411	0.945
External balance (Δ in % points of GDP)	-1.082	-1.005	-1.022	-0.958	-0.592	-0.654	-0.051	-0.066	-1.161
Mesoeconomic statistics									
Real GDP growth (Yt) (%)	5.780	3.397	5.446	3.311	3.096	2.337	-1.991	2.609	6.194
Inflation (informal good) (%)	23.110	14.014	20.820	12.817	10.505	7.948	-5.232	9.212	25.822
Inflation (formal good)	2.637	1.599	2.377	1.463	1.199	0.907	-0.597	1.051	2.947
Employment share (formal sector) (Δ in % pts)	2.453	1.441	2.309	1.404	1.313	0.991	-0.844	1.106	2.626
Labor productivity growth (informal sector)	4.440	2.565	4.180	2.498	2.333	1.751	-1.445	1.959	4.778

Note: Column (5-6) considers investment as an exogenous variable while in the other columns we have the investment endogenous. Police mix includes the combination of 5% depreciation of exchange rate and a transfer toward modern sector workers in the amount of 1% of GDP.

4.3 Sensitivity Analysis Experiments and Summary

Figure 3 presents the results. There are four panels. Panels (a) and (b) show, respectively, the results for an income transfer toward formal labour and an income transfer to informal workers. Beside panels (a) and (b), panel (c) and (d) reveal the results for 5 per cent exchange rate depreciation and the policy mix (the combination of 5 per cent exchange rate depreciation and income transfer to formal workers), respectively. The vertical axis reveals the real GDP growth, the horizontal axis shows the trade price elasticity range ($0 \leq \phi, \chi \geq 0.8$) .

Figure 3. Sensitivity of model results



Note: trade elasticity varying for the interval [0, 0.8]. Panel (a) shows the results for an income transfer toward formal workers while Panel (b) shows the result for an income transfer to informal workers. Finally, Panels (c) and (d) show the result of an exchange rate depreciation of 5% and policy mix (transfer of income of 1% of GDP to modern workers combined with 5% of exchange rate depreciation), respectively.

The results for panels (a) and (b) suggest that the higher the trade price elasticities, the lower is the growth rate of output. These results make perfect sense since the demand shock is translated into inflation, lower exports and higher amount of imports. Although the expansion loses part of its force, the result remains relevant.

As expected, the results for panel (c), show that, after the shock, there is a clear sign pattern change. There is a clear threshold, $\chi, \phi \approx 0.23$ per cent, beyond which depreciation becomes expansionary. Since it seems more likely that the Brazilian trade elasticities assume values above 0.23 per cent, the exchange rate policy might be important to foster economic expansion. In conclusion, the model's results suggest that exchange rate depreciation might be a viable way to boost economic activity.

Panel (d) exhibits the result of a shock that combines both an income transfer to moder workers (1 per cent of GDP) and an exchange rate depreciation (5 per cent). The higher the trade price elasticity, the more the economic output expands. The model's results suggest that this policy mix might be an important expansionary policy. If this policy had been adopted during the 90s and 2000s, the Brazilian economy could have grown faster, creating jobs and reducing poverty.

In 1994, following the stabilization process, the Brazilian economy suffered with an overvalued exchange rate, situation that could have been avoided if an active exchange rate policy had been adopted. Moreover, currently the Brazilian economy presents an overvalued exchange rate, which contributes negatively to output expansion. A high domestic interest rate to fight possible inflationary pressures leads to capital inflow and further exchange appreciation. At least partially, carry trade is behind the constant appreciation of the exchange rate during the 90s and 2011-2012. The model suggests that these negative

scenarios could have been avoided by an active exchange rate policy combined with income transfers.

In summary, the model's results reveal that redistributive policies may generate the initial conditions for economic progress. Overall, the higher the trade price elasticity, the more the economy expands at decreasing rates. An investment shock causes output expansion and inflation. Furthermore, the model points out that under certain conditions depreciation leads to output expansion and inflation. The simulation results, therefore, suggest that more progressive policies could foster economic activity.

5 CONCLUDING REMARKS

This paper has introduced an alternative model to investigate whether redistributive policies have the capacity to stimulate the economy. The model also attempts to explain the relationship between the two sectors during expansion.

The empirical results of the five simulations have economic expansion as their main outcome. These results, combined with the fact that the Brazilian government has more policy space to implement different policies, suggest that the economy could grow faster if redistributive policies and industrial policies are applied together. In this sense, the model's results reveal that redistributive policies may generate the initial conditions for economic progress.

Although the model can shed some light on the important structural linkages of the economy, some limitations are presented. The major drawback is that it does not include a financial sector. The financial dimension must be included in further research to accomplish a better understanding of the process of economic expansion.

The empirical results, therefore, suggest that income redistribution policies can boost economic activity. Counterfactual experiments suggest that the Brazilian economy could have presented a strongest process of economic expansion during 1990s and thereafter. In sum, an exogenous shock that destroys the perverse relationship between concentration of income and economic stagnation may foster economic expansion.

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7 APPENDIX A: DETAILED DESCRIPTION OF THE MODEL

7.1 Endogenous Variables

X_t : gross production (supply) of the high productivity sector;

Y_i : real GDP, i=n,t;

ρ : real exchange rate;

ν_i : share of domestic value added in supply, i=n,t.

ε_t : labour productivity in the subsistence sector;

P_n : price of subsistence good;

Zn : informal sector value added price;

Zt : formal sector value added price;

DY_i : disposable income, i=w_t,w_n.

C_i : consumption, i=n,t.

S_g : governments savings;

S_f : foreign savings;

S_π : capitalist savings;

I_t : investment in the modern sector;

κ : sectoral import propensity;

P_y : GDP-deflator;

Π : profit.

7.2 Exogenous Variables

a_{ij} : technical coefficients, i=n,t, j=n,t;

X_n : gross production of the low productivity sector;

X_f : foreign demand;

w_t : wage in the high productivity sector;

P_t : price of tradable good;

w_n : wages in the low productive sector;

b_t : labour-output ratio in the modern sector;

b_n : labour-output ratio in the subsistence sector;
L : total labour, i = t,n;
π : profit share;
γ : markup rate;
TR_t : income transfer to workers in the high productivity sector;
TR_n : income transfer to workers in the low productivity sector;
t_i : capitalist income tax or proportion of capitalist income that goes to workers, i=n,t;
E_t : exports of the high productivity sector.

7.3 Parameters

a : share of imported inputs in the output;
s : marginal propensity to save;
z_0 : autonomous investment or animal spirits;
z_1 : coefficient that measures the effect of profit on investment;
z_2 : accelerator coefficient;
e : nominal exchange rate;
α : fraction of income spent in the modern good from modern sector workers;
β : fraction of income spent in the modern good by workers in the subsistence sector;
p^* : price in terms of foreign currency;
ϕ : price elasticity of imports;
χ : price elasticity of exports;
θ : floor-level consumption of the subsistence good.

7.4 Modern Sector Price

Below we have the equation that reveals the price in the modern sector. The price depends on inputs, a_{tt} and a_{nt} ; subsistence price, P_n ; value-added price of the modern sector, Z_t ; share of value-added in supply, ν_t ; and imported inputs, $eP^* f$.

$$P_t = \frac{a_{nt} P_n + \nu_t Z_t + fe}{1 - a_{tt}} \tag{34}$$

7.5 Total Disposable Income by Class

As we indicated before, we have in our model three classes plus foreign income. Workers do not save while capitalists do not consume. The equations below show that workers' incomes are a positive function of wages and transfers. The transfer is financed by an income tax on capitalists' income, t .

$$DY_{wt} = (P_t C_{wt}^{\ t} + P_n C_{wt}^{\ n}) L_t = (1-\pi) Z_t Y_t + TR_t L_t \tag{35}$$

$$DY_{wn} = w_n b_n Y_n + TR_n L_n \tag{36}$$

$$DY_{\pi} = (1 - t_t - t_n)(\pi Z_t Y_t) \tag{37}$$

$$DY_f = eP^* f X_t \tag{38}$$

7.6 Consumer Demand Equations

Consumer demand functions for both goods are derived from the utility maximization process at the individual level. Workers consume a minimum floor-level, θ, which is insensitive to income and prices.

$$D(c_{wt}^{\ n}) = \frac{(1-\alpha)(Z_t w_t + TR_t)}{P_n} + \alpha\theta \tag{39}$$

$$D(c_{wt}^{\ t})=\frac{\alpha(Z_t w_t+TR_t-P_n\theta)}{Pt} \quad (40)$$

$$D(c_{wn}^{\ t})=\frac{\beta(w_n+TR_n-P_n\theta)}{P_t} \quad (41)$$

$$D(c_{wn}^{\ n})=\frac{(1-\beta)(w_n+TR_n)}{P_n}+\beta\theta \quad (42)$$

Foreign and Domestic Savings

$$S_f=eP^* fX_t-P_t E_t \quad (43)$$

Capitalist Saving

$$S_\pi=DY_\pi=(1-t_t-t_n)(\pi Z_t Y_t) \quad (44)$$

Saving-Investment Balance

The equation below shows the equilibrium between saving and investment.

$$S_\pi+S_f=P_t I_t \quad (45)$$

If we include eq. 23, 36 and 37 into equation 38 and divide by Ph, we get:

$$(1-t_t-t_n)\pi\frac{Z_t}{P_t}Y_t+e(P^*/P_t)aX_t-E_t-z_0-z_1\pi Y_t-z_2 Y_t=0 \quad (46)$$

The closure of the model is Keynesian, that is, investment is triggered by changes in output.

7.7 Government Savings

Government is always supposed to be in a balanced budget position. The only governmental task is to impose a tax on profits that will be transferred to workers in the form of income transfers. The government savings equation, equation 47, is presented below.

$$S_g=t_t\pi Z_t Y_t+t_n\pi Z_t Y_t-TR_t L_t-TR_n L_n=0 \quad (47)$$

7.8 The Demand Functions

The equation below reveals the maximization process to find the individual demands for each good. We extend the individual demand to the aggregate level as it is usually treated in microeconomics textbooks.

$$U=\sum_{i=h,l}\beta_i log(c_{wn}-\theta) \quad (48)$$

The parameter θ is the autonomous consumption for the subsistence good, that is, the consumption that is insensitive to changes in income. The first condition is that $\sum\beta_i=\beta_t+\beta_n=1$. In this way, $\beta_t=\beta$ is the fraction of income spent on the modern sector good while $\beta_n=1-\beta$ is the share spent on the subsistence good.

Equation (43) is the budget constraint where c_w stands for consumption at the individual level.

$$TR_n+w_n=P_t c_{wn}^{\ t}+P_n c_{wn}^{\ n} \quad (49)$$

Workers maximize their utility subject to the budget constraint.

$$Lagrangian=(1-\beta)log(C_{wn}^{\ n}-\theta)+\beta log(C_{wn}^{\ t}-\theta)-\lambda(P_n C_{wn}^{\ n}+P_t C_{wn}^{\ t}-w_n-TR_n) \quad (50)$$

$$D(c_{wn}^{\ t})=\frac{\beta(w_n+TR_n-P_n\theta)}{P_t} \quad (51)$$

$$D(c_{wn}^{\ n})=\frac{(1-\beta)(w_n+TR_n)}{P_n}+\beta\theta \quad (52)$$

The same approach is used to find the demand functions of the modern sector. Instead of β we use α to show the fraction of income spent on goods.

7.9 The Jacobian Matrix

The matrix below, which is called the Jacobian matrix, is a matrix of partial derivatives of excess demand functions with respect to P_n and X_t. A sufficient condition for local stability is that the Jacobian matrix should have a negative trace and a positive determinant.

Local stability implies that the system converges to a stable equilibrium after an exogenous shock. If the two eigenvalues are negative it implies that the determinant is positive and the system is stable.

$$\begin{pmatrix} \dfrac{-(1-\alpha)(Z_t\nu_t X_t + TR_t) - (1-\beta)((-a_{tn}X_n)\dfrac{(\nu_t Z_t + fe)}{1-a_{tt}} + TR_h)}{P_n^2} & (a_{nt} - b_t\nu_t(\beta\theta_n - \alpha\theta_t) + \dfrac{(1-\alpha)(Z_t\nu_t)}{P_n}) \\ \dfrac{-\alpha\theta_t b_t\nu_t X_t + \beta(1-a_{nn})X_n - \beta\theta_n(L - b_t\nu_t X_t)}{\dfrac{(a_{nt}P_n + \nu_t Z_t + fe)}{1-a_{tt}}} & -1 + a_{tt} + \nu_t(z_2 + z_1\pi) + \dfrac{\alpha(Z_t\nu_t - Z_t\nu_t\pi - P_n\theta_t b_t\nu_t) + \beta P_n\theta_n b_t\nu_t}{\dfrac{(a_{nt}P_n + \nu_t Z_t + fe)}{1-a_{tt}}} \end{pmatrix}$$

We can visibly see that the system has a negative trace because the main diagonal of the Jacobian matrix is composed by two negative components. The effect of income transfers on economic activity and subsistence price, P_n is presented below.

$$\partial P_n/\partial TR_t = det\begin{pmatrix} \dfrac{-(1-\alpha)}{P_n} & (+) \\ \dfrac{-\alpha}{\dfrac{(antP_n + mtZ_t + fe)}{(1-att)}} & (-) \end{pmatrix}/detJ = \frac{((+)-(-))}{(+)} = (+)$$

$$\partial X_t/\partial TR_t = det\begin{pmatrix} (-) & -(1-\alpha)/P_n \\ (+) & \dfrac{-\alpha}{\dfrac{(antP_n + mtZ_t + fe)}{(1-att)}} \end{pmatrix}/detJ = \frac{(+)-(-)}{(+)} = (+)$$