Área 3 - Macroeconomia, Economia Monetária e Finanças

Resumo
Esse artigo apresenta um modelo de gerações sobrepostas com rigidez no mercado de trabalho e três diferentes escolhas ocupacionais: administrador em tempo integral, autônomo e trabalhador. O modelo mostra que falhas no mercado de trabalho podem explicar diferenças na estrutura do emprego e tamanho de firma entre países. Países em desenvolvimento apresentam uma alta proporção de empreendedores por trabalhador mas tamanho de firma pequeno. O ponto central deste trabalho é que por excluir os trabalhadores menos produtivos do mercado de trabalho, políticas como salário mínimo criam “maus” empreendedores, no sentido de que esses agentes estariam melhor se empregados como trabalhadores. A diminuição da oferta de trabalho e da demanda por capital têm efeitos de equilíbrio geral que impedem a acumulação de capital, um menor número de empreendedores com firmas maiores e mais produtivas.

Palavras-Chave: Crescimento, Desigualdade, Escolha Ocupacional

Abstract
This paper presents an overlapping generation model with rigidities in the labor market due to labor policies and three different occupational choices: full-time manager, self-employed and worker. The model shows that labor market frictions can explain differences in employment structures and firm sizes across countries. Developing countries have a high entrepreneur to workforce ratio but low firm size. The main idea of this work is that by excluding less productive workers from the labor market, policies such as minimum wages create “bad” entrepreneurs, in the sense that these individuals would be better off employed as workers. The decreased supply of labor and demand for capital have competitive equilibrium effects which prevents capital accumulation with a lower number of entrepreneurs but much larger and more productive firms.

Keywords: Growth, Inequality, Occupational Choice

Classificação JEL: E60, G38, O11
1.1 Introduction

Firm size is an important determinant of growth. Larger firms have higher levels of physical capital and higher investment in R&D (Freeman, 1982; Mowery and Rosenberg, 1989; Chandler, 1990). Therefore, there is a positive and robust relationship between average firm size and growth because larger firms can take advantage of the increasing returns on investment in human capital. Evidence also suggests that the direction of causality is from firm size to growth (Pagano and Shivard (2003)). In this sense, it is important to remember the seminar paper of Lucas (1978). Through a general equilibrium effect, Lucas could explain the differences of firm size across countries. The main idea is that capital accumulation leads to an increase in the marginal productivity of labor, which increases wages. This mechanism generates an increase in the labor force, fewer entrepreneurs and larger firms. In the long run, small firms and self-employment should decrease, concentrating production.

Gollin (2008) found that in most developing countries, small firms and self-employment are the dominant firm forms. On the other hand, in the developed countries fewer entrepreneurs run much larger firms, on average. For instance, in the United States the entrepreneur-workforce ratio is 7.02%, while in Brazil the ratio is about 31%. Gollin also notes that there are some outliers such Italy, Spain and Greece with the entrepreneur-workforce ratio of 23.32%, 17.65% and 35.08%, respectively. These countries are known for their rigid and highly centralized wage-setting system. Therefore, two important questions arise: why do countries have such different firm size distributions and why is the structure of production so rigid?

Hurst and Lusardi (2004) found that financial wealth is important for the choice to become an entrepreneur only for the richest households. Their result changed the common view that initial wealth is an important feature for explaining occupational choice. Quadrini (2008) emphasizes that even though financial constraints are not important for the decision to become an entrepreneur, they are an important determinant of firm size and other entrepreneurial decisions. Hence, frictions in the financial market help to explain differences in the firm size and, consequently, in economic development (Antunes et al, 2008).

The model in this chapter compliments capital market explanations of differences in firm size across countries by focusing on labor force frictions. The idea is that labor market frictions create unemployment, causing unemployed agents to become "bad" entrepreneurs in the sense that they would be better off as workers. Since they cannot be workers, they become self-employed entrepreneurs who operate small firms. An indirect impact is that frictions in the labor force also decrease the share of "good" entrepreneurs, i.e., full time managers with a higher demand for capital, which decreases the overall demand for capital and employment, impeding the competitive equilibrium adjustment mechanism described in Lucas (1978). Therefore, labor market rigidity can help explain why small businesses and self-employment are so persistent in some countries.

This work is related to two main literatures. The first literature focuses on occupational choice between work and being an entrepreneur and on firm size (Lucas, 1978; Quadrini, 2000; Antunes et al, 2008, Hurst and Lusardi, 2004, Cagetti and DeNardi, 2006). We add a third occupational choice: self-employment (see Gollin, 2008). The key difference is that we focus on labor market frictions instead of financial market frictions. The second literature is related to how labor market rigidities affect employment, firm size and growth (Fajgelbaum, 2011; Chor, 2006; Helpman and Itskhoki, 2007, Jovanovich, 1979 and McCall 1970). These papers differ from this work because they focus on trade to explain the effect of labor market frictions on firm size, using search and matching models where the frictions come from the cost of firing/hiring workers and bargaining power. As explained below, the frictions we consider capture differences in government policies such as minimum wages and social security benefits, as well as costs of hiring and firing workers.

The model is based on Antunes, Cavalcanti and Villamil (2008). Agents choose between being a worker or an entrepreneur. The difference is there are two types of entrepreneurs: full-time managers and self-
employed as in Gollin (2008). Self-employed agents divide time between managing a firm and working. The remuneration of both types of entrepreneurs will be strictly increasing and convex with respect to ability, but full time managers' profit is more convex. That is, it is strictly better for higher ability agents to be full time managers instead of self-employed. The other important assumption of the model is that we use a more general definition of entrepreneurial ability than the one considered in Lucas (1978). Ability increases profit as an entrepreneur but also increases labor productivity:

It is tempting to argue that the most talented people become entrepreneurs because they have the skills required to engage in creative activity. Perhaps so, but this flies in the face of some facts. The man who opens up a small dry-cleaning shop with two employees might be termed an entrepreneur, whereas the half-million-dollar-per-year executive whose suit he cleans is someone else's employee. It is unlikely that the shop owner is more able than the typical executive.

The reverse might be true. As necessity is the mother of invention, perhaps entrepreneurs are created when a worker has no alternatives. Rather than coming from the top of the ability distribution, they are what is left over. This argument also flies in the face of some facts. Any ability measure that classifies John D. Rockefeller, Andrew Carnegie, or, more recently, Bill Gates near the bottom of the distribution needs to be questioned. (Poschke, 2008, apud Lazear, 2005).

Evidence corroborates the assumption that more productive workers are also better entrepreneurs if educational attainment is used as a proxy for entrepreneur ability. Moreover, an important share of agents decides to become entrepreneurs “out of necessity”, i.e., because there is no opportunity to work. Poschke (2008) shows that the probability of being an entrepreneur is higher at extreme levels of the productivity distribution.

Finally, it is important to note that many studies have highlighted the role of small business mainly in developing countries, but they also attest to the importance of changes in productive structures as central to development and growth (see, for instance, Liedholm and Meade, 1999; Tybout, 2000; Fafchamps, 1994; Hirschman, 1958; Rostow, 1960; Lewis, 1965). The main view is that small family and self-employment businesses would decrease in importance if market rigidities were removed.

The chapter proceeds as follows. Section 1.2 provides some empirical evidence to support our theory. Section 1.3 presents the model. Section 1.4 derives the results and analyzes different government labor market policies. Section 1.5 provides concluding remarks.

1.2 Empirical Evidence

Countries have very different policies with respect to labor. For instance, most countries have a federal minimum wage but for some countries in Europe, such as Italy and Iceland, minimum wages are negotiated in various collective bargaining agreements that are industry or sector based. In this section, an employment rigidity index built by Botero et al. 2004 is used to measure the difference in labor regulations across countries. The employment rigidity index is an average of three different indexes: the difficulty of hiring index, rigidity of hours index and the difficulty of redundancy index. They used data for 85 countries. “Our measures of labor regulation deal with three broad areas: (i) employment laws, (ii) collective relations laws, and (iii) social security laws. In addition, we assembled some data on civil rights laws in different countries” (Botero et al., 2004).

The index runs from 0 (less rigid) to 100 (more rigid) and, as expected, varies greatly across countries. The United States has the lowest index, 0, while Spain, Greece, Brazil, Italy, Argentina and New Zealand have 49, 47, 46, 38, 21 and 7, respectively. The rigidity index considers all types of costs firms incur to hire and fire, such as contracts and notification requirements. The index also considers restrictions on
minimum and maximum workweek and the existence of a minimum wage by law or by a collective bargaining agreement, the size of the minimum wage in different sectors, public or private, and in different industries. Our model will use two different policies to capture labor market rigidity. The first is a minimum wage policy and the second is a deadweight labor cost for firms. This second policy captures all obligatory benefits not paid directly for workers and costs due to contracts.

Figure 1a below shows the positive relationship between the employment rigidity index and the entrepreneur-workforce ratio. A simple correlation analysis shows that labor market rigidity alone is responsible for 65% of the variability in the number of entrepreneurs across countries. Figure 1b shows the positive relationship between the employment rigidity index and the percentage of entrepreneurs who are involved in entrepreneurship because they had no other option for work. Labor market rigidity is responsible for more than 20% of the variability of entrepreneurial activity driven by necessity.

Figure 2a below shows the well-documented inverse relationship between the number of entrepreneurs and average firm size. Firm size varies greatly across countries and is measured by the total number of employees. For instance, in Brazil, 79.9% of entrepreneurial firms in 2008 had no employee. The average size of an entrepreneurial firm in Brazil is 0.9 while in US, for the same period, is 2.7. Firm size also varies greatly across European countries, for Italy, an entrepreneurial firm had 1.3 employees on average in 2008 while the United Kingdom had 2.2. Figure 2b compares firm size and the employment rigidity index. There is a negative relationship between frictions in the labor market and firm size. Rigidities account for more than 68% of variability in the average firm size across countries, indicating that labor market frictions affect not just the decision to become an entrepreneur but also the firm size.

1.3 Model
There is a continuum of measure one individuals each period. There is no population growth. An agent chooses consumption, bequest for her child and occupational choice. The difference in this paper is that agents can choose among three occupational choices: worker, full time manager or self-employed. Agents differ in their ability, \( x \in (1,0) \), which is drawn from a continuous cumulative probability distribution function, \( \Gamma(x) \), and initial wealth/bequest, \( b \). We consider a more general definition of ability \( x \), where an agent with higher ability is a more lucrative manager if she is an entrepreneur but also more productive if she chooses to be a worker.

1.3.1 Preferences

An agent with ability \( x \) and initial wealth/bequest \( b_t \) maximizes:

\[
u = c_t^\gamma b_{t+1}^{1-\gamma}
\] (1)

An agent with ability \( x^i \) will make an occupational choice by comparing the profits if she is a full time manager, \( (fm) \), self-employed, \( (se) \), or the wage as a worker, \( (l) \). Her lifetime wealth is:

\[
Y^i_t = \max_{fm,se,l}\left[\pi^i_{fm}(b_t; \text{w}_t, r_t), \pi^i_{se}(b_t; \text{w}_t, r_t), x^i \text{w}_t\right] + (1 + r_t)b_t
\] if \( x^i \text{w} \geq w_{min} \).

\[
Y^i_t = \max_{fm,se,l}\left[\pi^i_{fm}(b_t; \text{w}_t, r_t), \pi^i_{se}(b_t; \text{w}_t, r_t)\right] + (1 + r_t)b_t
\] if \( x^i \text{w} < w_{min} \).

Let \( \text{w}_t \) denote the wage rate per unit of efficiency per hour, \( r_t \) be the interest rate and \( w_{min} \) be the minimum wage. Hence, the wage rate per hour, \( x^i \text{w} \), cannot be less than \( w_{min} \). The first impact of the minimum wage policy is that it rules out less productive workers in the formal labor market. Section 1.3 shows that this work force will be self-employed entrepreneurs that work and manage their own firms. They demand just the minimum capital required to start the business. We call them "bad" entrepreneurs in the model because they would be better off as workers, but their low productivity precludes this option.

1.3.2 Technologies

We abstract from financial market frictions to focus on the effects of labor market frictions on occupational choice.

1.3.2.1 Full Time Managers

A full time manager with ability \( x \) maximizes:

\[
\Pi_{fm}^i = x^i k^\alpha n^\beta - wn(1 + \tau) - (1 + r)k
\] (3)

where \( n \) is the total efficiency units of labor and \( \tau \) is the additional cost of labor that firms pay due to obligatory benefits and contract costs that are not paid directly to workers. Notice that since the wage per hour cannot be less than \( w_{min} \), then full time managers will not employ any worker \( j \) with \( x^j < \frac{w_{min}}{w} \).

\[\text{For simplicity, } \tau \text{ is treated in the model as a deadweight loss. Social security benefits and old agents could be added to the model but this would increase complexity but not change the main results.}\]

\[\text{Minimum wage policy distorts the relative price between units of efficiency of labor as long there exists } x^i > 0 \text{ such that } x^i \text{w} < w_{min}, \text{i.e., the minimum wage policy is effective in the economy. However, the production function for full-time managers (equation 3) treats units of efficiency of labor as perfect substitutes. Therefore, in equilibrium, full time managers}\]
1.3.2.2 Self Employed Managers

A self-employed manager will divide her time as a manager and worker in her firm. Following Gollin\(^3\) (2008), the time constraint faced by a self-employed manager is given. Let \(\theta\) be the time the manager works in her firm as a worker; therefore \((1 - \theta)\) is the time that she works as a manager. This entrepreneur with ability \(x\) maximizes:

\[
\Pi_{se}^i = (1 - \theta)x^1k^\alpha(\theta x)^\beta - (1 + r)k
\]

(4)

1.4 Competitive Equilibrium

1.4.1 Household's Problem

Given the utility function defined in equation (1), the optimal choice for households is to consume proportion \(\gamma\) of their lifetime wealth given by equation (2) and leave bequest \((1 - \gamma)\) of \(Y^i_t\). Therefore, given prices \((w_t, r_t)\), household occupational choice solves (2) and \(c^i_t = \gamma Y^i_t\) and \(b^i_t = (1 - \gamma)Y^i_t\) (Antunes et al, 2008)\(^4\).

Before we consider the market clearing conditions, define \(\Omega = (0, \infty)\). We assume that bequests might be near zero but not equal to zero. This guarantees positive lifetime wealth and therefore, positive bequests for all agents even in the presence of a minimum wage policy. Finally, let the measure of households in each occupational choice be given by:

\[
E_{fm}(w, r) = \{(b, x) \in \Omega\}: fm = \text{argmax}\{Y^i(b_t; w_t, r_t)\},
\]

\[
E_{se}(w, r) = \{(b, x) \in \Omega\}: se = \text{argmax}\{Y^i(b_t; w_t, r_t)\},
\]

\[
E_w(w, r) = \{(b, x) \in \Omega\}: l = \text{argmax}\{Y^i(b_t; w_t, r_t)\}.
\]

maximize profit by choosing zero units of efficiency of labor for all \(x^i w < w_{min}\). Let \(x^i \in (0,1)\) such that \(x^i w = w_{min}\). Mathematically, for given \(w, r\) and \(w_{min}\):

\[
\max_{k[L_t]|x|\in(0,1)} x^\alpha \left(\int_0^1 x_t L_t dx\right)^\beta - w(1 + \tau) \int_0^1 x_t L_t dx - w_{min}(1 + \tau) \int_0^1 L_t dx - (1 + r)k
\]

From the first order conditions, it is straightforward to show that \(L_t > 0\) if and only if \(x_t \in [l, 1]\). This means a competitive equilibrium with no unemployment exists if and only if the wage rate increases until \(x_t = w_{min} w\) where \(x_t\) is the lowest bound for \(x\). Since \(x \in (0,1)\), there is no \(w \in (0, \infty)\) and no such competitive equilibrium exists.

\(^3\) Gollin (2008) allows self-employed managers to choose additional labor in the market. For simplicity, we assume that self-employed agents choose only capital. This assumption will not change the analytical results, but it might decrease profit and capital demand for some levels of ability. On the other hand, with the imposition of labor market rigidities, this assumption decreases in importance since if labor market frictions are high enough the optimal labor demand for self-employed managers will be zero.

\(^4\) It is possible to define the competitive equilibrium with labor frictions \((w_{min}, \tau)\) for a general production function, where the elasticity of substitution among different labor productivity is finite. In this case, equation (2) for \(x^i w < w_{min}\) is: \(Y^i = Y^i(b_t; w_t, r_t) = \max_{x, y, z} \left[\left(\int_{x_t}^{1} x_t L_t dx\right)^\beta - w(1 + \tau) \int_{x_t}^{1} x_t L_t dx - w_{min}(1 + \tau) \int_{x_t}^{1} L_t dx - (1 + r)k\right]\) where \(u(x^i; w_t, r_t)\) is the unemployment function rate for each \(x_t \in (0,1)\). In the definition of the competitive equilibrium, \(u(x^i; w_t, r_t)\) is such that, for given \(w_{min}\), total demand of labor for type \(x_t \in (0,1)\) equals supply: \(w_{min}(1 - u(x^i; w_t, r_t))L_t\).
1.4.2 Market Clearing Conditions

Following Antunes et. al (2008), let $Y_0$ be the initial distribution of wealth. Then the market clearing conditions for the labor and capital markets are:

$$\int\int_{(b,x) \in E_{fm}(w_t, r_t)} n(x; w_t, r_t) Y_t(db) \Gamma(dx) = \int\int_{(b,x) \in E_{w}(w_t, r_t)} x Y_t(db) \Gamma(dx)$$

$$\int\int_{(b,x) \in E_{fm}(w_t, r_t) \cup E_{w}(w_t, r_t)} k(b, x; w_t, r_t) Y_t(db) \Gamma(dx) = \int\int b Y_t(db) \Gamma(dx)$$

The law of motion for wealth distribution is:

$$Y_{t+1} = \int P_t(b, A) Y_t(db) \text{ where } P_t(b_t, A) = \Pr(b_{t+1} \in A/b_t).$$

1.5 Qualitative Results

1.5.1 Model without Labor Market Frictions

First, let's consider the efficient case where $w_{min}$ equals zero. In the appendix, we show that the income for both types of entrepreneurs is strictly increasing and strictly convex in the ability level. Moreover, profits for full-time managers are “more convex” than the total remuneration of self-employed agents.

**Assumption 1.**

$$\theta^\beta (1 - \theta) (1 - \alpha)^{1-\alpha} \leq \beta^\beta (1 - \alpha - \beta)^{1-\alpha-\beta}. \quad (5)$$

**Proposition 1.** Under assumption 1, there is no self-employment. Furthermore, there is a unique $x^*$ such that for all $x \leq x^*$, the agent prefers to work and for all $x > x^*$ the agent prefers to be a full-time manager.

Assumption 1 guarantees that there are no self-employed managers in this economy, i.e., the parameterization is such that the economy is in the case shown by figure 1.3. This happens because in the equilibrium, if it is optimal for an agent to be a self-employed manager instead of a worker, then it will be optimal to be a full-time manager instead a self-employed manager (see the Appendix for a complete proof). Note that assumption 1 does not depend on prices: interest rate and wage. This result is consistent with theory: in a long-run equilibrium without frictions, the productive structure will be concentrated, with small family business disappearing and self-employed workers being wage workers (Lucas, 1978, Kuznets, 1966, Schumpeter, 1934). Therefore, without any frictions, in the labor market the inclusion of a self-employment sector is redundant as figure 1.3 below shows:

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5 If agent is indifferent, assume that she chooses to work.
1.5.2 Model with a Minimum Wage Policy

Despite the simplicity of the labor market frictions incorporated in the model, the occupational choices for different levels of ability differ significantly with respect to the model without frictions in the labor market. Recall that we do not consider any financial market frictions.

Proposition 2: for any \( w_{min} > 0 \), \( \exists x^{**} > 0 \) and \( x^* > 0 \), such that

i) \( \forall x < x^{**} \), agents will be self-employed.

ii) \( \forall x \in [x^{**}, x^*] \), agents will work.

iii) \( \forall x > x^* \), agents will be full-time managers.

Figure 1.4 below shows that, under assumption 1, there are self-employed managers, the "bad" entrepreneurs. With a minimum wage policy, there is a level of ability, \( x^{**} \), such that for all \( x < x^{**} \), agents will be ruled out from the formal labor market. Since their ability is low enough, they would be better off working but since they are unemployed, they will become "bad" entrepreneurs in order to assure a positive income. Even without any borrowing constraints or spread between interest rates, it is not optimal for them to be full-time managers because their projects will not be sufficiently profitable.

Another interesting implication of introducing a minimum wage policy in the labor market is the general equilibrium effect. Notice that the number of "good" entrepreneurs decreases in figure 1.4. This happens because the less productive workers are not in the labor market, and hence the supply of labor decreases, increasing the equilibrium wage and interest rate. Therefore, the threshold ability \( x^* \) is greater in this model. The higher the minimum wage, the higher is this general equilibrium effect.
Figure 1.4: Occupational Choice with Frictions in the Labor Market

Figure 1.5: Demand of Capital for Different Levels of Ability with Frictions in the Labor Market

Figure 1.5 above shows the capital demand by self-employed and full-time managers. The levels of capital demand vary significantly for these two types of entrepreneurs. In figure 1.5, firm size for self-employed entrepreneurs is slightly greater than zero; they invest just the amount necessary to keep the business open. For $x \in [x^*, x^{**}]$, optimal occupational decision is to be a worker and, therefore, their demand of capital is zero. For $x > x^*$, agents will be full-time managers and the demand for capital jumps. These are the "good" entrepreneurs, in the sense that they are responsible for capital accumulation, higher wages and higher production.

1.5.3 Model with a Deadweight Cost

Now consider another friction in the labor market. Suppose there is no minimum wage policy but there is a distortion between the cost of labor for full time managers and the wage received by workers. This distortion reflects different policies that increase the cost of labor for firms such as hiring and firing costs and penalties, advance notice requirements, contract length and cost, legal workweek constraints, etc. Let, as before, $wx$ be the total wage received if the agent is a worker but full time managers pay the wage rate: $(1 + \tau)w$.

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6 The demand for capital by both types of entrepreneurs is strictly increasing and convex in the ability level.
Assumption 2. \( (1 + \tau) > \frac{\beta}{\theta} \left( \frac{1}{1-\theta} - \frac{1}{1-\alpha} \right) \frac{1}{\beta} \)

Proposition 3. Under assumption 2, there exists a unique \( x^* > 0 \) and \( x^{**} > 0 \) such that:

i. For all \( x < x^{**} \), the agent will be worker.

ii. For all \( x \in [x^{**}, x^*] \), the agent will be self-employed.

iii. For all \( x > x^* \), the agent will be a full-time manager.

With the introduction of protective labor laws, threshold ability \( x^* \) increases, i.e., the number of full time managers decrease due to the decrease in profits. If the increase in labor costs perceived by full-time managers is high enough, proposition 3 shows that, at an optimum, agents with intermediate levels of ability will be self-employed instead of full-time managers, therefore, decreasing the total demand for capital in the economy, as figure 1.7 below shows.

1.6 Concluding Remarks

Despite the simplicity of the model and the labor policies considered, we show that labor market frictions have important effects on occupational choice and firm size. This result is consistent with the empirical facts presented and offers an additional explanation for the differences and rigidity of productive structures across countries. The other main finding of this model is the trade-off between working and being a full-time manager for high ability agents. This trade-off comes directly from the assumption that more able managers will also be more productive workers. In the presence of labor market rigidities, the trade-off is reinforced by a general equilibrium effect that decreases the share of “good” entrepreneurs in
the economy, decreasing the total demand for capital and total output. This prevents the economy from shifting to another productive structure with higher a concentration of capital and larger firms.

Finally, it is important to highlight that we focused solely on the cost of labor market rigidities for different agents and how this cost affects their occupational choice and firm size. One could also consider the benefits that labor policies such as social security have on the decision to become worker. Modeling and measuring the cost versus the benefit of different protective labor laws is an important extension of this work, however, that is beyond the goal of this chapter. Policies that increase the benefit of working (net of their cost), would increase the supply of labor and, therefore, in a competitive equilibrium, the wage rate should decrease, offsetting the impact of these benefits in the occupational choice model studied here.
APPENDIX A

A.1. Self-Employed Managers Problem

Consider the problem of an agent with ability $x$ that chooses to be self-employed, he maximizes capital for given $\theta$:

$$\Pi_{se}(x) = \max_{(k)} (1 - \theta) x k^\alpha (\theta x)^\beta - (1 + r)k$$

(6)

The necessary first order conditions are given by:

$$\frac{\partial \Pi_{se}}{\partial k} = (1 - \theta) \alpha x k^{(\alpha - 1)}(\theta x)^\beta - (1 + r) = 0$$

(7)

Therefore:

$$k_{se}^*(x) = \left[ \frac{\alpha}{1+\theta} (1-\theta) \theta^\beta x^{1+\beta} \right]^{\frac{1}{1-\alpha}}$$

(8)

Therefore,

$$\Pi_{se}^*(x) = x^{\frac{1+\beta}{1-\alpha}} \left( \frac{(1-\theta)\theta^\alpha x^{\alpha}}{(1+r)^\alpha} \right)^{\frac{1}{1-\alpha}} (1 - \alpha)$$

(9)

Note that $k_{se}^*(x)$ and $\Pi_{se}^*(x)$ are strictly increasing and strictly convex functions in ability, $x$, since $\frac{dk_{se}}{dx}, \frac{\partial^2 k_{se}}{dx^2} > 0$ and $\frac{d\Pi_{se}}{dx}, \frac{\partial^2 \Pi_{se}}{dx^2} > 0$, for all $x > 0$.

A.2. Full Time Managers Problem

Consider the problem of an agent that chooses to be a full time manager, with $\alpha + \beta < 1$:

$$\Pi_{fm}(x) = \max_{(k,n)} x k^\alpha n^\beta - wn - (1 + r)k$$

(10)

The necessary first order conditions are:

$$\frac{\partial \Pi_{fm}}{\partial k} = \alpha x k^{(\alpha - 1)}(n)^\beta - (1 + r) = 0$$

(11)

$$\frac{\partial \Pi_{fm}}{\partial n} = \beta x k^\alpha (n)^{(\beta - 1)} - w = 0$$

(12)

Therefore:

$$n^*(x) = x^{(1-\alpha-\beta)} \left( \frac{\beta}{\alpha} \right)^{\frac{1-\alpha-\beta}{1+\theta}} \left( \frac{\alpha}{1+\theta} \right)^{\frac{\alpha}{1-\alpha-\beta}}$$

(13)

$$k_{fm}^*(x) = \left[ \left( \frac{\alpha}{1+\theta} \right)^{(1-\beta)} \left( \frac{\beta}{\alpha} \right)^\beta x \right]^{\frac{1}{1-\alpha-\beta}}$$

(14)

and
\[ \Pi_{fm}^*(x) = x^{\frac{1}{1-\alpha-\beta}} \left( \frac{\beta}{w} \right)^{\frac{\beta}{1-\alpha-\beta}} \left( \frac{\alpha}{1+r} \right)^{\frac{\alpha}{1-\alpha-\beta}} (1 - \alpha - \beta) \]  

(15)

Note that \( k_{fm}^*(x) \) and \( \Pi_{fm}^*(x) \) are strictly increasing and strictly convex functions in ability \( x \), since, \( \frac{dk_{fm}}{dx} > 0, \frac{d^2k_{fm}}{dx^2} > 0 \) and \( \frac{d\Pi_{fm}}{dx} > 0, \frac{d^2\Pi_{fm}}{dx^2} > 0 \), for all \( x > 0 \).
APPENDIX B

Let $x^*$ be such that: $\Pi_{fm}^*(x^*) = wx^*$. Since $\Pi_{fm}^*(x)$ is strictly increasing and convex and labor income, $wx$, is linear in the ability level and $\Pi_{fm}^*(0) = 0$, then, there exists a unique, single cross-point: $x^* > 0$ such that for all $x>x^*$, agent strictly prefer to be full-time manager instead of worker. Equation 10 can be used to explicitly solve for the threshold ability level $x^*$:

$$x^* = \left[ \frac{w(1-\alpha)}{\alpha} \left( \frac{1}{\beta} \right)^\alpha \left( \frac{1}{1-\alpha-\beta} \right)^{1-\alpha-\beta} \right]^{\frac{1}{\alpha+\beta}} \tag{16}$$

In the same way, we can compute $x^{**}$ such that $\Pi_{se}^*(x^{**}) = wx^{**}$:

$$x^{**} = \left[ \frac{w}{1-\alpha} \left( \frac{1-\alpha-\beta}{\alpha} \right)^\alpha \left( \frac{1}{1-\theta-\beta} \right) \right]^{\frac{1}{\alpha+\beta}} \tag{17}$$

Therefore, there exists a unique $x^{**} > 0$ such that for all $x>x^{**}$, agent strictly prefer to be self-employed than worker.

Since $\Pi_{fm}^*(x)$ and $\Pi_{se}^*(x)$ are both strictly convex it is necessary to show that:

(i) $\exists$ a unique $\bar{x}$ s.t $\Pi_{se}^*(\bar{x})= \Pi_{fm}^*(\bar{x})$,
(ii) $\forall \; x < \bar{x}$, $\Pi_{se}^*(x) > \Pi_{fm}^*(x)$,
(iii) $\forall \; x > \bar{x}$, $\Pi_{fm}^*(x) > \Pi_{se}^*(x)$

Let $\bar{x}$ be such that $\Pi_{se}^*(\bar{x})= \Pi_{fm}^*(\bar{x})$, using equations (9) and 10:

$$\bar{x} = \left( \frac{w}{\beta} \right)^{(1-\alpha)/(\alpha+\beta)} \left( \frac{\alpha}{1+\theta} \right)^{\alpha/(\alpha+\beta)} (\beta(\alpha+\beta))^{(1-\alpha-\beta)/(\alpha+\beta)} \left( \frac{1-\alpha}{1-\alpha-\beta} \right)^{(1-\alpha-\beta)/(\alpha+\beta)} \tag{18}$$

It is clear than $\bar{x} > 0$. Moreover $\Pi_{fm}^*(x) > \Pi_{se}^*(x)$ if and only if $x > \bar{x}$ and $\Pi_{fm}^*(x) < \Pi_{se}^*(x)$ if only if $x < \bar{x}$.

Therefore, note that $x^{**} > x^*$ is a sufficient condition for the no-existence of a self-employment using equations (16) and (17):

$$x^{**} > x^* \text{ implies:}$$

$$(1-\theta)\beta(1-\alpha)^{1-\alpha} < \beta^\alpha (1-\alpha-\beta)^{1-\alpha-\beta}$$

Condition above shows assumption 1. As long as the full time entrepreneur share, $(1-\alpha-\beta)$, is high enough, in the optimal, there is no self-employment agent in the economy.

Finally note that the demand for capital for both types of entrepreneurs is strictly increasing and convex in the ability level. Comparing equations (8) and (14):

$$k_{fm}^*(x) \geq k_{se}^*(x)$$

$$x \geq \left( \frac{1+r}{\alpha} \right)^{\alpha/(\alpha+\beta)} \left( \frac{w}{\beta} \right)^{(1-\alpha)/(\alpha+\beta)} \left( \frac{1-\alpha-\beta}{\beta(\alpha+\beta)} \right)^{(1-\alpha-\beta)/(\alpha+\beta)} \tag{19}$$
Let $\tilde{x}$ be such that $k_{m}^{\ast}(\tilde{x}) = k_{se}^{\ast}(\tilde{x})$. Using equation (18), one can show that $\tilde{x} < \tilde{x}$, therefore, if it is optional for agent to be full-time manager in the model without frictions, his demand for capital is higher than if self-employed, i.e, any policy that changes his occupational choice result in a lower demand for capital.
REFERENCES


