Potential Output and Equilibrium Interest Rate in Brazil[1]

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Abstract: *this paper measures equilibrium real interest rate for Brazil during 1999-2010 using different methodologies. The results show great difference in the estimates of the natural interest rate in Brazil depending on the specification of the IS curve and its explanatory variables besides the real interest rate. Measurement of the output gap is not a source of divergence among our estimation of natural rate as different methodologies yields similar values for the output gap. Joint estimation of the inflation and output cycles leads only to small difference in the output gap estimates and hence on natural interest rate. Finally, our results indicate that the impact of monetary policy on output gap increased during the last years.*

Resumo: *Este trabalho estima a taxa real de juros de equilíbrio no Brasil durante o período 1999-2010 usando diversas metodologias. Os resultados mostram uma diferença significativa na taxa de equilíbrio dependendo da especificação utilizada, principalmente na modelagem da Curva IS. A mensuração do hiato do produto não é o principal responsável pelos resultados encontrados para a taxa de juros de equilíbrio. Os resultados indicam ainda que houve um aumento no impacto da política monetária sobre o hiato do produto no Brasil.*

Área 3 - Macroeconomia, Economia Monetária e Finanças

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1 Introduction

In the last ten years the conduct of monetary policy has been designed mainly by adjusting interest rates. This movement reflects the emergence of Taylor Rules as best way to conduct or at least describe the conduction of monetary policy. The adoption of inflation target regimes also favoured the use of monetary policy rules in the way described Taylor (1993). A key aspect of Taylor rules is to adjust interest rates to inflation and output gap. The reasoning behind this postulate is that real interest rates must be increased or decreased to make inflation converge to the target. Even when there is no explicit target, the monetary authority can use this framework. Indeed, Taylor (1993) shows that this kind of simple rule fits the conduct of monetary policy in US, a country known for its option to not adopt the inflation target framework.

A key variable in this framework for conduct of monetary policy is the so called natural or equilibrium real interest rate. Theoretically, the natural rate of interest is the rate at which real output equals potential output or, in other words, the output gap is zero. Note that a zeroed output gap is consistent with no inflation pressures according to the Phillips Curve theory.

These two concepts, natural interest rate and output gap, plays a major role in the conduct of monetary policy not only in Brazil, but in several other countries. These are part of what Blinder (1998) calls fundamental concepts in modern macroeconomics and have been widely studied and discussed in the last years. In this paper we will focus on natural interest rate determination. More specifically, we apply statistical method to extract the natural rate of interest in Brazil based on some measures of inflation and output gap.

In this paper we will treat output gap as given in most estimations. In other words, we will not try to joint determinate output gap (or potential output) and natural interest rate. We are aware of possible failure of our natural interest rate measure as a good proxy, but we judge the benefit of joint determination is small in the case of Brazil.

In theory, real equilibrium interest rate (REIR) is the real interest rate that makes aggregate supply equal to aggregate demand. In other words, its the level of interest rate that makes the output gap equal to zero. The concept of equilibrium interest rate is of great importance in the conduct of monetary policy: for instance, the policy maker aiming at reducing the level of aggregate demand to a non-inflationary level must increase interest rate above the level of equilibrium interest rate for some time. Over the last years, some monetary policy rules evolved in a way to bypass the need to know the level of REIR. One of the monetary policy rule is to increase real interest rate when one believes aggregate demand is greater than aggregate supply. This is known as first difference rules, as it doesn't use the level of interest rate as a the policy instrument, but its first difference. Even this being a way of conducting monetary policy, it is of course a second best choice. Knowing the level of REIR is the best way of conducting monetary policy.

In this paper we explore the links between output gap and natural interest rate. Our results indicate that the natural interest rate decreased in Brazil over the last years. However, there is a large degree of uncertainty about the level of the natural rate. Different specifications for the IS curve lead to different estimation results for the natural interest rate. Different methodologies used for extracting the output gap lead to very similar results and we see this as an indication that future extensions of this paper would envisage more structural determination of the IS curve rather than proposing new methods to extract the output gap. Our results also indicate that the impact of monetary policy on output gap increased over the last years. The coefficients obtained in this paper are close to the ones observed in other countries and also close to the ones estimated by other authors for Brazil.

2 Brief Literature Review

The literature on real interest rates is vast and here we will discuss just a small part of it. We will discuss three different kind of literature on real interest rates: i) theoretical, ii) empirical and iii) empirical with Brazilian data.

The most recent studies on real equilibrium interest rate rely on the work of Woodford (2003). Woodford (2003) uses a "wicksellian" approach to define the real equilibrium interest rate. According to him, the natural or equilibrium interest rate is one that balances aggregate demand and aggregate supply.

Many empirical works on real interest rates had been published in last years. Some of them offers some theoretical background, like Laubach and Williams (2003), Neiss and Nelson (2003) and Blanchard and Summers (1984), while others focuses on the statistical procedures aiming at extract the natural real interest rate from the data. In most cases, the estimation of the output gap and the natural interest rate are made jointly by estimating a small scale macroeconomic model. In most cases, the macroeconomic models include both the potential output and the natural interest rate as latent variables and are estimated using standard Kalman filter techniques. This is the approach followed by Laubach and Williams (2003). We will not name other empirical works here. In most cases, the Laubach and Williams (2003) procedures are simply replicated or slightly modified.

In Brazil, there are only a few papers on the subject. This small number of empirical work doesn't mean the subject is not important. References to the natural rate of interest are common in the minutes of Brazilian Central Bank meetings. It is of common knowledge that the natural real interest rate in Brazil hover around 8%. This number is the mode of several different model specifications in Muinhos and Nakane (2006). Barcelos Neto and Portugal (2009) also provides estimates for the equilibrium interest rate in Brazil using a methodology that is similar to the one presented in this paper. The natural rate estimated by Barcelos Neto and Portugal (2009) is close to 7% for the period jan-00 to dec-05 (see section 6.3 for our own results) and the close to 9.5% using a structural macroeconomic model (see section 6.1 for our own results). The results from Barcelos Neto and Portugal (2009) do not present any evidence of a trend (downward or upward) in the natural interest rate.

3 Potential Output Measures

As one can observe from the previous discussion in the introduction of this paper, the concept of potential output is deeply connected with the concept of natural real interest rate. Indeed, several authors use a positive theory to define the natural rate of interest: the REIR is the rate of interest that equalizes output and potential output. In this section we briefly discuss the evolution of Brazilian gross domestic product and also factors affecting potential GDP growth. The idea behind this session is to give more facts related to the Brazilian economy to improve the discussion of the results regarding our estimates of potential output and REIR in the next sections. We divide the discussion into two themes: structural measures of potential output and statistical measures.

3.1 Structural Measures of Potential Output

In the literature of growth accounting, GDP growth is usually decomposed into the contributions of labor, capital and productivity. In this sub-section we briefly discuss labor and capital stock data for Brazil.

Labor force growth has been diminishing in Brazil in the last years. This is a result of lower population growth over the last 15 to 20 years: according to IBGE (Brazilian Institute for Geography and Statistics), population growth was above 2% in the early 80's and diminished to close to 1% in the last years. Working age population growth (population between 15 and 64 years) diminished from close to 2,5% in the early 80's to less than 1% in the last years.

As a result of lower population growth and higher GDP growth, the unemployment rate in Brazil has fallen during the last ten years (this is clearer after 2002). Using data from PNAD, the unemployment rate in the late 90's was above 12% and diminished in the following years to reach values as low as 6%. This pattern is clear using other source of data. Using the data from Dieese/Seade (covering only São Paulo metropolitan area) the unemployment rate fell from close to 14% in the late 80's to 8% in 2010. Using the Dieese/Seade unemployment rate for metropolitan areas[2] the unemployment rate fell from close to 12% in the late 90's to close to 7% in 2010. Using data from the IBGE Employment and Unemployment Survey, the unemployment rate fell from close to 14% in 2003 to 6% in 2010.

The other variable usually included in the growth accounting literature is capital stock. The construction of capital stock data is more complicated than estimate population or working age population in a country. It is difficult to track investments and mainly determine the depreciation of capital. Also it is difficult to estimate the "initial" capital stock, the stock of capital at some point in the past from where we use a law of motion based on investments and depreciation to update the values of capital stock.

Besides that, it is difficult to compare capital stock among countries. Different authors can use different methods to track the stock of capital in the economy and this can lead to different results. The stock of physical capital in Brazil has been estimated by Morandi and Reis (2004) and also Gomes et all (2005). Morandi and Reis (2004) show that the Brazilian capital stock is not very different from other countries, particularly more developed countries like USA, Japan or the European countries. On the other side, Gomes et all (2005) include in their calculation a "wasting" factor. The implication of this factor is to diminish the capital stock, i.e., the capital stock is lower than you could infer using the investment data along with a depreciation factor. According to these authors, this adjustment factor is particularly important in period of great public investment share in total investments. In the Morandi and Reis (2004) study, the capital-output ratio is close to 3x in the recent period, while in Gomes et all (2005) it is only close to 2x. In advanced economies, this relation hovers between 3x to 4x according to Morandi and Reis (2004). It means that the Brazilian capital stock lies in the lower bound of more developed countries at best.

[2] The metropolitan areas are: São Paulo, Belo Horizonte, Porto Alegre …

We bypass the problem of "measuring capital stock right" using data on the rate of utilization of capital. This variable is considered a good proxy for inflation pressures as the work of Gordon (1998) indicates. This variable has also been used in Brazil and is always mentioned in inflation reports or minutes of the Brazilian Central Bank meetings.

The output gap derived from the production function approach can be written as: let Y_t be the output any some time interval *t*. This produced using a production function like:

$$Y_t = A_t F(K, L)$$

Where *A* is the productivity factor and *F* can be expressed as the traditional Cobb-Douglas function:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

Now, assume there is a *non-accelerating inflation rate of capital utilization,* $\overline{N}_t$, and also a non-accelerating rate of inflation rate unemployment (NAIRU), $\overline{U}_t$. So the potential output in this case is given by:

$$Y_t = A[K_t \overline{N}_t]^{\alpha}[L_t(1-\overline{U}_t)]^{1-\alpha}$$

and finally assume that productivity is constant during our sample period. Using this formulation and the these assumptions, the output gap can be written as:

$$y_t = \alpha(N_t - \overline{N}_t) + (1-\alpha)(\overline{U}_t - U_t)$$

The values of y_t is computed calibrating the value of α from data on national accounts and value of potential utilization rate of capital and NAIRU are estimated using statistical techniques like HP or BP filter or even simpler approaches like time trends.

3.2 Statistical Potential Output Measures: HP Filter and Unobserved Components Decompositions

3.2.1 Hodrik-Prescott Filter

There are several different forms to measure the potential output of an economy. The most common way is to use the Hodrick-Prescott filter. The basic procedure of the HP filter consists of decomposing any time series y_t into a growth component g_t and a cyclical component c_t:

$$y_t = g_t + c_t, \quad t = 1, ..., T$$

The cyclical component has average zero over long periods. The growth component is referred as the trend of the variable and cyclical component is the temporary deviation from this trend. The growth component is relatively "smooth" and the programming for decomposition of y_t is given by:

$$\underset{\{g_t\}}{Min}\left\{\sum_{t=1}^{T} c_t^2 + \lambda \sum_{t=1}^{T}\left[\left(g_t - g_{t-1}\right) - \left(g_{t-1} - g_{t-2}\right)\right]^2\right\}$$

Hodrick and Prescott (1997) proposes $\lambda = 1600$ for quarterly series. To set this value, they assume as a reference a 5% value for the cyclical component and a 0,125% value for the variation of the growth rate ($\Delta^2 g_t$). Also, they take into account the fact that if both c_t and $\Delta^2 g_t$ had normal distribution with standard deviations of σ_1 and σ_2,respectively, the solution to the problem would be $\sqrt{\lambda} = \sigma_1 / \sigma_2$. So, using the reference values just mentioned they set $\sqrt{\lambda} = 5/0{,}125$, or, $\sqrt{\lambda} = 40$.

3.2.2 Structural Time Series Model

The Hodrick-Precott procedure is somewhat "deterministic" in the sense that some parameters are not estimated but rather imposed into the model. Harvey and Jaeger (1993) propose a different way to decompose y_t into trend and growth components. A simplified version[3] of the Harvey and Jaeger unobserved components model (UCM) can be written as:

$$y_t = \mu_t + \psi_t + \varepsilon_t, \quad t = 1, \ldots, T$$

$$\mu_t = \mu_{t-1} + \beta_t + \eta_t, \quad \eta_t \sim NID(0, \sigma_\eta^2)$$

$$\beta_t = \beta_{t-1} + \zeta_t, \quad \zeta_t \sim NID(0, \sigma_\zeta^2)$$

$$\psi_t = \rho \cos \lambda_c \psi_{t-1} + \rho \sin \lambda_c \psi_{t-1}^* + \vartheta_t$$
$$\psi_t^* = -\rho \sin \lambda_c \psi_{t-1} + \rho \cos \lambda_c \psi_{t-1}^* + \vartheta_t^*$$

The stochastic cycle ψ_t is a described by *sine* and *cosine* functions, the frequency of the cycles, λ_c, and a damping factor, ρ.

The HP filter is a particular case of this more general formulation when $\sigma_\eta^2 = 0$ and $\sigma_\varepsilon / \sigma_\zeta = 1600$. Harvey and Jaeger (1993) argue that is a good approximation for the US, particularly in the period studied by Hodrick and Prescott, but are not good for some other countries. Figure 2 plot the cycle component of Brazilian GDP using both the HP filter and unobserved components method used by Harvey and Jaeger (1993).

[3] This model is also called *structural time series model* (STM).

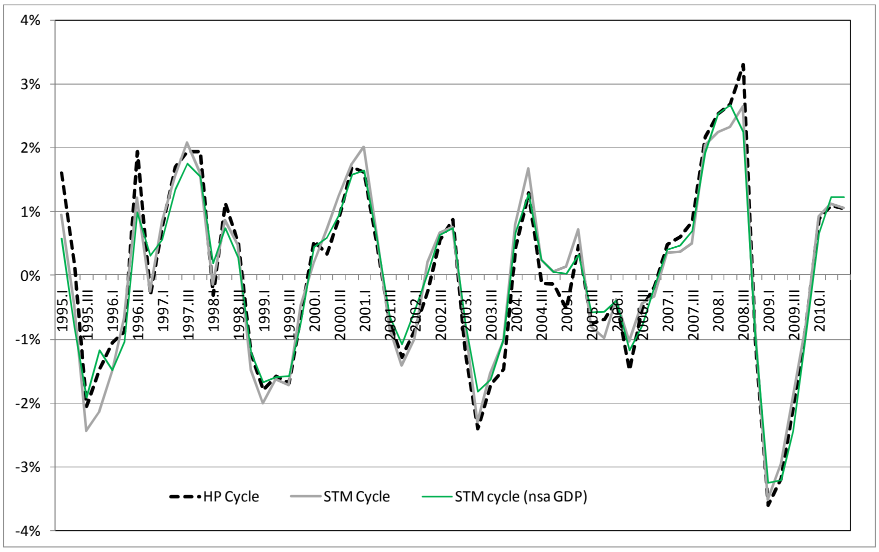
Table 1 – Brazilian GDP: Estimates of Structural Time Series (Unobserved Components) Model Paramaters*

	σ_{ζ}^{2}	σ_{η}^{2}	σ_{ε}^{2}	$\sigma_{\varepsilon} / \sigma_{\zeta}$	σ_{ϑ}^{2}	ρ	$2\pi / \lambda_{c}$
Brazilian GDP	0,061	0,000	17,082	279,421	7,924	0,815	11,1
Brazilian GDP** (not seas. adj.)	0,060	0,000	25,960	429,842	7,410	0,820	11,5

**Variances multiplied by 10[5]. $2\pi / \lambda_{c}$ is the frequency of the cycle in quarters.*

***Seasonal adjustment is part of the STM as already defined.*

Figure: GDP cycles – HP and UCM (STM) decomposition



3.2.3 Autoregressive Model

Finally, there is another way to calculate output gaps we will consider in this paper. The model is an univariate specification with permanent and transitory components. The permanent components is associated with GDP trend (or "growth" in the Hodrick-Prescott language) while transitory component is the cycle component. Despite being very simple, this specification is among the most tractable one that allow imposing consistency between output gap measures and inflation dynamics[4]. The model is described as:

$$y_t = \mu_t + z_t$$
$$\mu_t = \delta + \mu_{t-1} + e_t$$
$$z_t = a_1 z_{t-1} + a_2 z_{t-2} + \varepsilon_t$$

The consistency between output gaps and inflation is imposed by estimating this model jointly with the Phillips curve. The output gap in this model follows an AR(2) process. Multivariate versions of the HP filter have also been proposed in the literature as a way to improve the HP

[4] This model is attributable to Watson (1986) and Kuttner (1994). The link between inflation and this measure of output gap will be presented in the next sections.

filter as a measure of output gap consistent with inflation dynamics. The model described above however is simpler and easier to estimate and have been more widely used than the multivariate version of the Hodrick-Prescott filter.

4 A Simple Model for Natural Interest Rate Determination

In this section we sketch a baseline new Keynesian macroeconomic model. Our intent is just to explain the theory behind the econometric exercise we will present in the next pages. We will not get into the details of the model and particularly on its premises and assumptions. The discussion is based in Gali and Gertler (2007) but there are several good references for the new keynesian model including graduate textbooks like Romer (2011) and Woodford (2003). The traditional model, in its reduced form representation, is composed tree equations: an aggregate demand equation (IS curve), an aggregate supply equation (Phillips curve) and a monetary policy reaction function (Taylor rule).

In a very simple form, the model can be written as:

$$y_t = a_1 E_t y_{t+1} + a_2 (R_t - \overline{R}_t) + u_t \qquad \text{(IS equation)}$$
$$\pi_t = b_1 \pi_{t-1} + (1 - b_1) E_t \pi_{t+1} + b_2 y_{t-1} + e_t \qquad \text{(Phillips curve)}$$
$$R_t = \overline{R}_t + c_1 y_{t-1} + c_2 (E_t \pi_{t+1} - \overline{\pi}_t) + v_t \qquad \text{(Taylor rule)}$$

Despite this being a simple model, there are several complication we will treat in our empirical tests. First, there is lags between changes in the real interest rate gap, $R_t - \overline{R}_t$, and its impact on the output gap. Second, the model as expressed above considerer that only current interest rate gap is important for the determination of the output gap but a more general formulation includes all expected future interest rate gaps as being the determinant of current output (we will use long term rates as a proxy for expected future rates).

In the Phillips curve equation we are only including the output gap and past and expected inflation as explanatory variables for current inflation rate. In open economies, the external inflation is also an explanatory variable. And, as we mentioned in the IS equation case, there may be lags at which the output gaps affects inflation, so past output gaps could also be an explanatory variable.

In our specification for the Taylor rule we considered the real interest rate instead of the nominal interest rate. This is a minor difference since a nominal interest rate can be recovered using the current or expected inflation.

In our estimations, we will not rely on joint estimation of this model. Our main objective is to make inference about the level of $\overline{R}_t$, the equilibrium real interest rate. This variable appears in the Taylor rule equation and also in the IS equation. Note, however, that the Central Bank do not know this level but also need to make some inference about it. It can be the case that the Central Bank do not follow a Taylor rule just like the one specified above or that the level of the equilibrium interest rate inferred by the Central Bank is different from the one we can infer using our specification for the IS curve. Taking this into consideration, we will not include the Taylor rule in most of our models.

Also, the joint estimation can sometimes be misleading, particularly in cases where the Phillips curve cannot be easily estimated or in cases where the Phillips curve does not apply at all. This seems to be the case of Brazil at least in the first half of our sample. Besides being in a process of disinflation that the output gap alone could not be able to explain, there were several episodes of exchange rate devaluation with sensible effects on inflation and also supply shocks like the energy shortage of 2001. Note that these are some identifiable challenges to estimate a Phillips curve in Brazil. Several others, less easily identifiable can be also present in the data. Using a statistical language, when σ_e is large relative to σ_u, the joint estimation of these two equation can be misleading. This is reason we will rely on the univariate model in most of our specifications.

We incorporate the external output gap, y_t^*, into the model to capture the effects of a growing world economy into the Brazilian economy. Holland and Santos (2008) shows that this variable is statistically significant as a determinant of Brazilian output gap between 1996 and 2007. In the last years several studies analyzed the synchronization of business cycles around the world. We will not review this literature. It suffices to say that business cycles synchronization have been high for a long time and seems to have increased in the last decade. Among the possible explanations for this phenomena are the increasing commercial and financial integration. The debt and inflation crisis in Brazil in the 80’s and 90’s may have caused a decreased in the correlation between Brazilian GDP growth and GDP growth in other countries. During the late 90’s and 00’s this correlation increases again. Another explanation for the global business cycle synchronization is the occurrence of a common, global shock. The oil shocks of the 70’s is one example of such a shock. More recently, the subprime crisis can be considered another example of a global shock. Independent of the causes, the synchronization of the Brazilian and world GDP growth seems to be an important source of information in the evolution of output gap in Brazil. Taking this into account, we include a measure of world output gap in our model as explanatory variable.

5 Data Description

We construct our measure of output gap, *y*, using both the production function approach and statistical methods presented earlier. Capacity utilization is provided by FGV and unemployment rate by SEADE. The main advantage of using the Dieese/Seade unemployment rate is it longer data sample. The latest IBGE Employment and Unemployment survey starts in 2001 while the Dieese/Seade sample starts in 1998[5]. We also used the Brazilian GDP “quantity index” for the statistical measures of potential GDP. The index of consumption quantity was also provided by IBGE.

For the interest rate series we use the 6 month swap rate as a measure of nominal interest rate. The reason for using the 6 months swap instead of the 12 months swap is the higher liquidity of this rate at the first half of our sample period. This rate was also used by the Brazilian Central Bank in their simulations in the first part of our sample. More recently BCB has started to use the 12 months swap rate due to the good liquidity of this instrument. The difference between 6 months and 12 months swap rate was low during most part of our sample, including the second

[5] We use the Dieese/Seade survey for several metropolitan areas, similar to the survey performed by IBGE.

half (post 2005). The reason is that both rates include a term premium (or market premium) and also the forecast ability of market participants is low. The 12 months swap rate is more volatile in the first half of our sample but the level of both series are similar.

We use the 12 months ahead inflation expectation as a proxy for inflation expectations to form our measure of real interest rate. This is the simplest measure of inflation expectation and the one commonly used in empirical works in Brazil. It is also commonly used by the Brazilian Central Bank along with the current and next year inflation expectation.

The other variables used are the CRB commodity price index measured in R$. This is a measure of external price inflation and will be used in the specification of the Phillips curve. We also used a measure of external GDP growth (or external output gap) provided by OECD. The reason behind using this variable is to control the IS specification for other determinants of GDP growth like higher foreign demand and also higher liquidity in global capital markets. An alternative way to consider these forces into our model would be to include a measure of interest rate in USD available for Brazilian companies or a more detailed specification of demand, including an specific equation for exports. The problem with these alternatives is the small sample size: the more variables we include into the model, the lower the power of the statistical tests and the lower the confidence on the estimated coefficients. The inclusion of external GDP growth was also included in several other empirical work both for Brazil or other countries. Finally, the importance of external GDP growth precedes the 2008 financial crisis as the work of Holland and Santos (2008) shows.

At this point it is important to mention what variable we do not include. The main variables used in empirical work not incorporated in our models are the real exchange rate misalignment and also a measure of fiscal imbalance (or fiscal instance). Determining exchange rate misalignment is a difficult task and there are few works on the subject in Brazil. Despite this, this variable could have played an important role in determining the output gap in Brazil particularly in the period of fixed exchange rate regime. We consider the inclusion of exchange rate misalignment as a future refinement of this work.

We propose a simple measure of fiscal instance in the later section of this work. This can be important especially for the later part our sample, as fiscal benefits were widely used in the post-crisis period. But the fiscal instance may have been important in the previous years too: many post 1999-crisis measures undertaken by the Brazilian government were based on fiscal adjustments, especially higher taxes and lower expenditures[6]. We recognize this is a very naïve approach to deal to this subject and that further refinement is also needed in this regard.

6 Estimation Results

6.1 IS Curve and the Neutral Rate of Interest: a Simple Model

The simple model we want to estimate is given by just a IS equation and the random walk specification for the :

[6] Between the 1999-crisis and the 2008-crisis the Brazilian debt was reduced significantly and the measures taken to deal with crisis episodes changed from increasing primary surplus in the 1999 and 2003 crisis to reducing primary surplus in the 2008 crisis. The BCB refers to these changes as the breakdown of the crisis-amplification mechanism of public debt.

$$y_t = \alpha_0 + \alpha_2 y_{t-1} + \alpha_3 y_t^* + \alpha_4 (R_{t-1} - \overline{R}_{t-1}) + u_t \qquad (1)$$
$$\overline{R}_t = \overline{R}_{t-1} + v_t$$

In order to estimate this model using the Kalman Filter, we need some adjustments. First we drop out the constant term α_0 and rewrite the signal equation as:

$$y_t = -\alpha_4 \overline{R}_{t-1} + \alpha_2 y_{t-1} + \alpha_3 y_t^* + \alpha_4 R_{t-1} + u_t \qquad (2)$$

Dropping out the constant term do not seem to be a problem since the variable has a zero mean in almost all specification for the output gap. Indeed, in our model the sole variable with mean different from zero is the interest rate.

Defining $\alpha_4 \overline{R}_{t-1} = \overline{R}^*_{t-1}$, we get:

$$y_t = \overline{R}^*_{t-1} + \alpha_2 y_{t-1} + \alpha_3 y_t^* + \alpha_4 R_{t-1} + u_t \qquad (3)$$
$$\overline{R}^*_t = \overline{R}^*_{t-1} + v_t^*$$

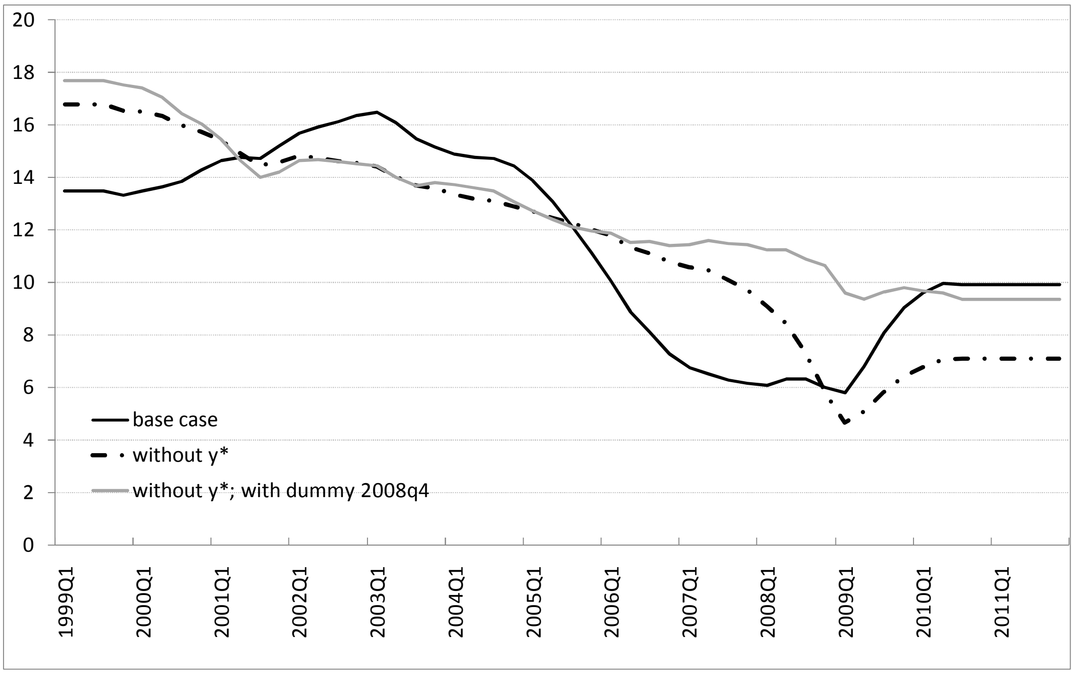
Written this way, the model can be estimated using the Kalman Filter procedure. It is important to note that the variance of the error term in the neutral interest rate equation has changed. Since , $\alpha_4 \overline{R}_{t-1} = \overline{R}^*_{t-1}$, $\sigma^2_{v*} = \alpha_4^2 \sigma_v^2$. It means that the variance of the error term is lower in this state variable.

The figure below presents the results for the estimation of the neutral real interest rate in Brazil. The results differs depending on the specification of the IS curve and shows a great diverge in the end of the sample. The inclusion of the foreign output gap makes the estimated natural interest rate more volatile. The inclusion of a dummy variable for the fourth quarter of 2008, the most acute moment of the sub-prime crisis, makes the natural interest rate less volatile.

Estimation Results: system (3)

	Coef.	Std. Error	z-Statistic	Prob.
α_2	0.364	0.091	3.994	0.0001
α_3	0.468	0.061	7.635	0.0000
α_4	-0.117	0.034	-3.370	0.0008
σ_u	-1.978	0.239	-8.277	0.0000
	Final State	Root MSE	z-Statistic	Prob.
R*	1.280	0.464	2.757	0.0058

Natural Real Interest Rate Estimates for Brazil ($\overline{R_t}$)



Our results favor the hypothesis of a reduction in the equilibrium real interest rate in Brazil. The estimated equilibrium real interest rate is as large as 18% in the beginning of the sample and diminishes to values close to 7% at the end of the sample. It is also worth mentioning the large dispersion of the estimation results at the beginning and at the end of the sample. This result seems to arise from the large variability of all variables at these points. Another important factor to consider is the exchange rate misalignment and also the fiscal instance at these moments. We did not include any of these variables in our estimation but we recognize that both variables are also important determinants of GDP growth and output gap. Both the exchange rate and the fiscal policy changed significantly during the quarters covered in our sample, particularly in the beginning and in the end (during the crisis period). We consider a more structural model for the IS curve as a possible future refinement of this paper.

At this point it is important to mention that the above estimates of the natural rate includes a term premium or a risk premium besides the expected future interest rates determined by the monetary authority. We didn't modeled the term premium in our estimation since we dealing with natural "effective" real interest rates instead of focusing on a real interest rate associated solely with the expected short term rates. The term premium diminished over the last years, making our estimation more acute in the final part of the sample.

6.2 Joint Estimation of Inflation and Output Cycles: Structural Time Series Model

A possible problem associated with the estimation of the model just described is that measurement of the output gap may not be corrected or may not be a good indicator of inflation pressure. Taking this into account, we now use a different specification for the output gap measurement. There are several different models for joint estimation of the output gap and inflation[7]. We will use a model proposed by Harvey (2008). In this model, both output and inflation are modeled using the structural time series model and we impose the restriction that these variables share a same common cycle. The model can be written as:

$$\begin{bmatrix} y_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} \mu_t^y \\ \mu_t^\pi \end{bmatrix} + \begin{bmatrix} \psi_t^y \\ \psi_t^\pi \end{bmatrix} + \begin{bmatrix} \varepsilon_t^y \\ \varepsilon_t^\pi \end{bmatrix} \quad (4)$$

In this model, both cycle components share the same frequency, λ_c, and the same damping factor, ρ. We include as other explanatory variables for the inflation process the commodity inflation in local currency and past inflation. This way, the inflation equation resembles the adaptive model, where explanatory variables are past inflation, a measure of supply shock (commodity prices) and a measure of demand pressure. The demand pressure is measured by the output gap and in this case we are "forcing" the output gap to be extracted from the inflation equation. The supply shock measure is given by:

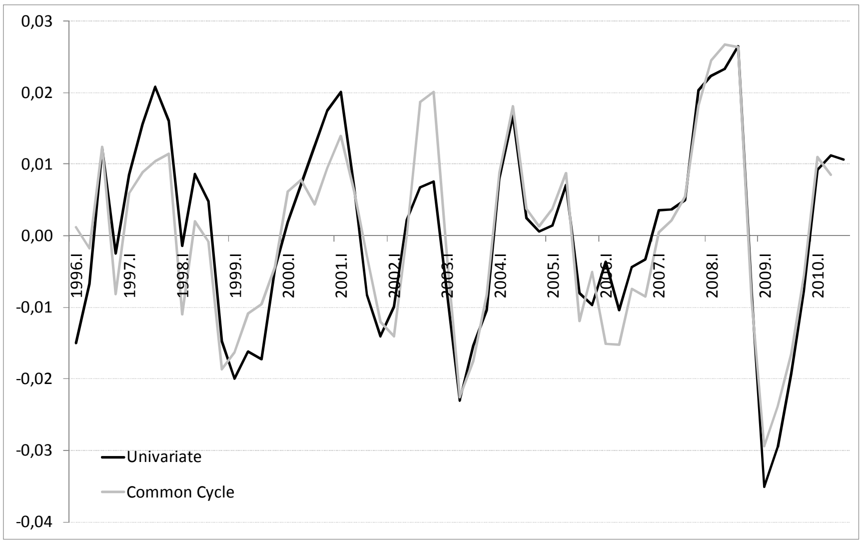
$$\begin{aligned} P_t^{*,\$} &= P_t^* e_t \\ \pi_t^{*,\$} &= \Delta \log(P_t^* e_t) \end{aligned} \quad (5)$$

Where P_t^* is the commodity price index (CRB) measured in American dollars and e_t is the Brazilian exchange rate measured in R$/US$. Note that this is a more general formulation than the usually applied in Brazil. Due to the large exchange rate depreciation in 1999 and 2002, several authors assumed $P_t^{*,\$} = \overline{P_t^{*,\$}}$, so that $\pi_t^{*,\$} \approx \Delta \log(e_t)$, in their empirical estimation of the Phillips curve. Despite being a good approximation for the period surrounding these two devaluations episodes, this approximation is not a good one for our whole sample. In particular, world recessions or world growth decelerations in 1997/98, 2000/01 and 2008/09 lead to an accommodation in world prices. Also, the world growth booms of 2003/04 and also 2007/08 lead to a sharp increase in commodity prices, $P_t^{*,\$}$, making the assumption of constancy in the external prices less accurate.

The result of such specification is presented in the figure bellow. As it is clear from the figure, the inclusion of the inflation process in the determination of output gap does not change the output gap estimate in a meaningful way. So we proceed to a third approach to estimate the natural rate.

[7] Osmani and Vasconcelos (2008) are a good reference for such models and applications to brazilian data.

Cyclical Component of GDP: univariate model and "common cycle" specification



6.3 Central Bank Reaction Function Approach

The last approach we use for estimating the natural rate of interest is to use the Central Bank reaction function (or Taylor rule). The reaction function can be written as:

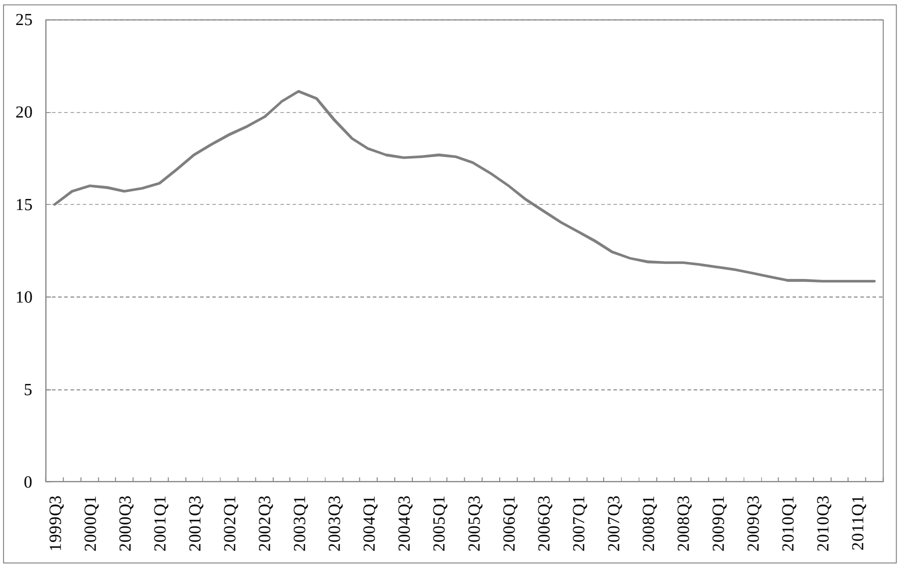
$$i_t = \alpha i_{t-1} + (1-\alpha)(i_t^* + \beta_\pi(\pi_t^e - \overline{\pi}_t) + \beta_y y_t) + \varepsilon_t \qquad (6)$$
$$i_t^* = i_{t-1}^* + u_t$$

In this case, we are modeling a nominal (i_t^*) instead of real natural interest rate. We will assume the neutral real interest rate is given by $\overline{R}_t = i_{t-1}^* - \overline{\pi}_t$. This approach to the Central Bank target to real interest rate is similar to the presented in the original paper of Taylor (1993). The estimation a nominal "natural" interest rate is more problematic when inflation target not constant or long run inflation expectation is volatile. This seems to be the case in Brazil in the first half of our sample. Considering this, we will focus our discussion on the results for the last part of our sample.

Estimation results: system (6)

	Coef.	Std. Error	z-Statistic	Prob.
α	0.531	0.046	11.338	0.0000
β_π	0.761	0.274	2.778	0.0029
β_y	1.211	0.946	1.28	0.2005
σ_ε	1.770	0.157	0.493	0.0061
	Final State	Root MSE	z-Statistic	Prob.
i_T*	10.43	2.756	3.784	0.0020

Natural Interest Rate Estimation (%): Central Bank Reaction Function Approach



The results of our estimation indicates that the "target" rate implicit in the estimated Taylor rule diminished in the last years form a maximum of more than 20% to close to 10.5%. Note that the inflation target in Brazil in the last years had been fixed at 4.5%. This indicates that the target "real" interest rate was close to 6% in the last years. This result is line with the indications presented in the Copom minutes: the minutes did not mention a specific value, but the minutes indicated sometimes when real rates were close to 6% that the real rate was close to the equilibrium levels. More important, the minutes indicated several times that equilibrium real interest rates in Brazil was trending lower which we judge is consistent with the Central Bank targeting a lower real interest rate.

7 Addendum: The Impact of Monetary Policy on Output Gap

There have been a lot of discussion during the last quarters on the impact of monetary policy on economic activity and particularly on output gap. The Brazilian Central Bank argued several times that the impact of monetary policy on output gap increased during the last years due to more credibility, increase in the credit/GDP ratio, among other factor[8]. In this section we perform the same "random coefficient approach" to estimate the impact of monetary policy on output gap and GDP growth.

To perform this calculation, we cannot use the estimated equilibrium interest rate estimated in the previous section since it was already extracted from another random coefficient estimation. Also, joint estimation of two random coefficients could not be performed since it would be difficult to reach convergence or the estimated standard deviations would be too large. To bypass these difficulties, we used a simple time trend as a proxy for natural interest rate.

[8] This discussion was present in most of the meeting minutes of 2010 and 2011 and also present in several Inflation Reports in 2010 and 2011.

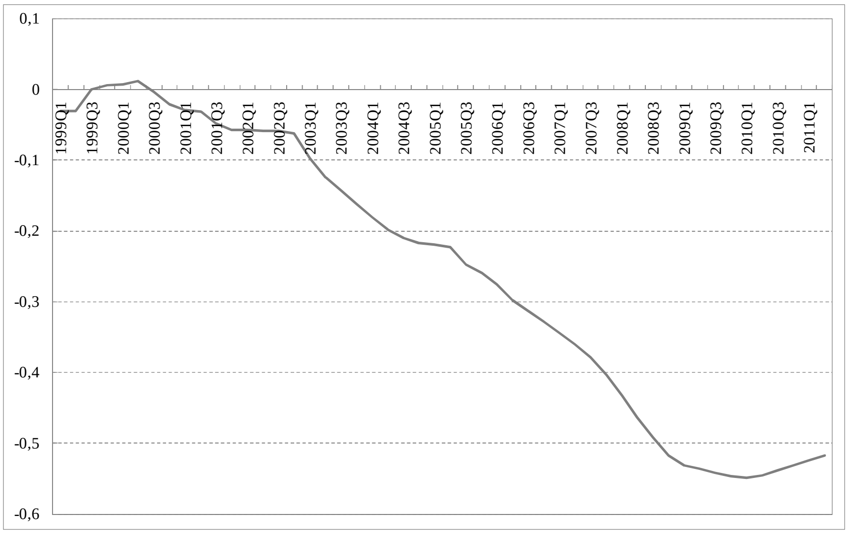
To measure the impact of monetary policy on the output gap we estimated a time-varying coefficient model for the output gap as follow:

$$y_t = \alpha_2 y_{t-1} + \alpha_3 y_t^* + \alpha_{4,t}(R_{t-1} - \overline{R}_{t-1}) + u_t \qquad (7)$$
$$\alpha_{4,t} = \alpha_{4,t-1} + v_t$$

Estimation results: system (7)

	Coef.	Std. Error	z-Statistic	Prob.
α_2	0,449	0,062	7,235	0,0000
α_3	0,517	0,137	3,769	0,0002
σ_u	-0,534	0,238	-2,238	0,0252
σ_v	-5,645	1,594	-3,540	0,0004
	Final State	Root MSE	z-Statistic	Prob.
$\alpha_{4,t}$	-0,517	0,201	-2,566	0,0103

Estimation result: impact of monetary policy on output gap ($\alpha_{4,t}$)



Our results indicate that the impact of monetary policy on output may have indeed increased in the last years. The estimated impact of monetary policy, measured as the "real interest rate gap", increased from close to zero in the beginning of our sample to close to 0.5 at the end of sample. Note that using a time-varying approach to natural rate and a fixed coefficient α_4, lead to a estimated impact of close to 0.1.

The estimation results are consistent with the evidence provided by other authors. Holland and Santos (2008) provided estimates for this coefficient close to 0.07 but using a sample that ended in 2008. If this coefficient increased in the last years as our results indicates, the 0.11 presented in the estimation of the system (3) is consistent with previous estimation. For United States, Rudebuch and Svensson (1999) also report a coefficient close to 0.10.

Most empirical work on the equilibrium interest rate are concerned with the estimation of the equilibrium rate and much less concerned with the estimated impact of “real interest rate gap”, $R_{t-1} - \overline{R}_{t-1}$, on the output gap. This make it difficult to compare our result with time-varying coefficient with other evidence. Boivin and Giannoni (2006) estimates a more structural model for United States and finds values for the impact of the real interest rate gap between 0.49 and 0.66, which is close to the values observed at the final part of our sample. Also in these estimation with US data, the impact of monetary policy measure as variations on the real interest rate gap is larger than the impact estimated using variation on the real interest rate.

8 Conclusion

In this paper we used different methodologies to estimate the output gap. Our results showed little difference between the output gap estimates from obtained from different methods. In particular, we call attention to the small difference from estimates obtained from purely statistical methods and more structural methods.

Our results indicate that the equilibrium real interest rate in Brazil decreased in the last years. However, the size of this reduction differs depending on the specification of the IS curve. The results also points to a high variability of the equilibrium real rates, particularly in the final part of our sample. This result can be attributed to the high volatility of output in Brazil and also in other countries following the financial crisis of 2008.

Our results indicate that the inclusion of a measure of “foreign output gap” changes the pattern of the equilibrium real interest rate. We attribute this result to other forces moving the output gap besides de local interest rate. We did not addressed in the paper the importance of the real exchange rate misalignment or the fiscal instance in Brazil as possible explanatory variables in the IS equation. We judge this could be a possible refinement for the present paper.

Finally, our results indicate that the impact of monetary policy on the output gap increased considerably in the last years. The estimated impact of monetary policy in this paper is close to the impact estimated by other authors without allowing for a time-varying coefficient. The estimated time-varying coefficient increased from close to zero in the first years of our sample to close to 0.5 in the final part of the sample.

9 References

BARCELOS NETO, P.; M. S. PORTUGAL (2009). The Natural Rate of Interest in Brazil between 1999 and 2005. Revista Brasileira de Economia, vol. 63 (2), pages 103-118.

BLINDER. AL. S. (1998). *Central Banking in Theory and Practice*. MIT Press, Cambridge.

BOIVIN, J.; M. GIANONNI (2006). Has Monetary Policy Become More Effective? The Review of Economic and Statistics, vol. 88 (3), pages 445-462.

GALI. J.; GERTLER. M. (2007). Macroeconomic Modeling for Monetary Policy Analysis. Journal of Economic Perspectives, vol. 21 (4), pages 25-45.

GORDON, R. (1998). Foundations of the Goldilocks Economy: Supply Shocks and the Time-Varying NAIRU. Brookings Papers on Economic Activity ,no.2 (1998), pp. 297-346.

HARVEY. A C and A JAEGER (1993). "Detrending. stylised facts and the business cycle." *Journal of Applied Econometrics,* 8: 231- 247.

HODRICK. R; E. PRESCOTT (1997). “Post-War Business Cycles: An Empirical Investigation”. Journal of Money, Credit and Banking, vol. 29.

HOLLAND. M.; SANTOS. F. S. (2008). Estimando a Demanda Agregada no Brasil – O Papel dos Fatores Externos. ANPEC, 2008.

MUINHOS. M. K.; NAKANE. M. (2006). Comparing equilibrium real interest rates: different approaches to measure Brazilian rates. Working Paper, Brazilian Central Bank, 2006.

KUTTNER. K. (1994). “Measuring Potential Output as a Latent Variable”. Journal of Business and Economic Statistics, vol. 12 (3), 1994.

LAUBACH. T. and J. C. WILLIAMS. (2003) “Measuring the Natural rate of Interest”. *The Review of Economics and Statistics,* 85 (4). November.

GUILLÉN, O.; C. H. VASCONCELOS (2008). Previsão de Inflação com Incerteza sobre o Hiato do Produto no Brasil. ANPEC, 2008.

NEISS. K. S.. and E. NELSON (2003). “The Real Interest Gap as an Inflation Indicator”. *Macroeconomics Dynamics. vol.* 7, pp. 239-262.

ROMER. D. (2011). Advanced Macroeconomics. 4th edition. McGraw Hill Irvin. New York.

RUDEBUSCH, G.; L. SVENSSON (1999). Policy Rules for Inflation Targeting. In J. Taylor (ed.) Monetary Policy Rules. Chicago University Press, 1999.

TAYLOR. J. B. (1993). “Discretion versus Policy Rules in Practice”. *Carnegie Rochester Conference on Public Policy. Vol* 39, pp. 195-214.

WATSON. M. (1986). “Univariate Detrending Methods with Stochastic Trends”. Journal of Monetary Economics. vol. 18, 1986.

WOODFORD. M. (2003). Interest and Prices: Foundations of a Theory of Monetary Policy. Princeton University Press