***Volatility Triggered Range Forward* (VTRF):**
An Instrument for Protection Against Volatility Fluctuations in the BRL/USD Pair

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Palavras-chave: câmbio, moedas, volatilidade, forward, opção, SWARCH, Switching Markov ARCH.

Abstract: This paper proposes an instrument able to absorb shocks in the BRL/USD rate, ensuring its holder the capability of doing foreign currency exchange at some immediate prevailing rate. The Volatility Triggered Range Forward resembles a plain-vanilla forward whose delivery price is unknown initially and will be set once a pre-determined level of volatility threshold is reached in the exchange rate along the instrument's life. Its payoff schedule can be set for any number of periods. Pricing and risk management is based on a trinomial lattice weighted between two possible regimes of volatility. These regimes are determined after a study of the BRL/USD series for the period between 2003 and 2009, based on a Switching Markov ARCH (SWARCH) model.

Keywords: foreign exchange, FX, volatility, forward, option, SWARCH, Switching Markov ARCH.

Área ANPEC: Área 7 - Microeconomia, Métodos Quantitativos e Finanças.

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Volatility Triggered Range Forward (VTRF): An Instrument for Protection Against Volatility Fluctuations in the BRL/USD Pair

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1. Introduction

Although volatility plays a major role in the financial market giving players a simple measure of assets uncertainty, few are the instruments able to protect these assets against periods of frequent changes in their prices. Instead, most derivatives assure only a one-time payoff to offset asset value changes, providing holders very short-term protection. This sort of protection may not be sufficient if there are exposures demanding mid or long-term protection.

Available instruments can individually price floating strikes (such as Lookback Options), volatility differences at maturity (Variance Swaps and Options), optionality and limits on forwards (Forward Options and Range Forwards), and multiple payoffs (Target Accrual Redemption Notes). None of them, however, price all these characteristics simultaneously. Products such as the Forward Volatility Agreement or the Variance Swap are examples of known financial products designed to deal with volatility. They are not able, however, to eliminate sudden and unpredictable changes in the price of most assets the way this paper proposes. Companies whose short-term receivables and payables are linked to the spot rate may incur in loses whenever these assets and liabilities have few days of mismatching and volatility breaks acceptable levels. In a controlled environment, paying for hedge would not sound adequate when there is a natural match in the notional of payables and receivables. But if there is a time mismatch between them, that is when protection against volatility comes into play. Creating a portfolio of instruments may be too complex for non-financial companies' treasury departments, so a single complete derivative instrument could better suit their needs.

Brazil has recently become a strong receiver of foreign investments, seeking its high interest rates and an attractive, developing stock market. This has turned the BRL vulnerable to this external flow. The 2002 election fear and the recent 2008 crisis have shown how much the BRL nominal value can swing with the scarce liquidity provided by international markets. This ends up leading financial controllers into losses for the portions of their exposures not fully hedged. Such "full hedge" may be, in many cases, impossible to be executed in advance as opportunities rise in unpredictable times.

This paper proposes a derivative instrument which acts like a forward, offering financial controllers with a shock-absorbing, mean-reverting-like device. Entities may carry it to use whenever volatility in the foreign currency reaches a predetermined level, moment at which the fixing rate is actually defined. Different from traditional hedging instruments, whose strikes are defined up-front for the underlying asset, this will only have its strike level set at the time of exercise (triggered by the volatility level), enabling the holder to count on current (or recent) market levels as a reference for the instrument payoff.

To reach a model capable of incorporating these characteristics to the Volatility Triggered Range Forward (VTRF), the BRL/USD[1] series behavior is analyzed. A number of derivative instruments are scrutinized to obtain parts that, together, will build the Volatility Triggered Range Forward. It ends up being the combination of a new type of option: volatility triggered calls and puts.

[1] BRL/USD is used along the text as the price quoted in Brazilian Reais for one American Dollar.

The work is divided in five sections, starting by this Introduction. In Section 2 the literature on the BRL/USD series and derivatives is presented. In Section 3, a model for the derivative instrument price and risk management is proposed. In Section 4 the BRL/USD series is analyzed and the low and high volatility stages are identified in order to estimate its variances behavior. This is incorporated to the VTRF parameters and various hypothetical scenarios are run to study the instrument's main characteristics, such as its Greeks. It is also tested in real scenarios in which the spot market volatility has increased. Section 5 concludes, discussing the instrument's effectiveness, limitations, and opportunities for enhancements.

2. Background

This section starts by covering the main aspects related to currencies exchange rates, especially that of the BRL/USD series, with its dynamics and volatility patterns. It is followed by a derivatives market explanation, including its current characteristics and limitations.

2.1. The BRL/USD Series Behavior

According to Andersen et al. (2002), innovations brought by macroeconomic events (that is, the difference between expectations and observations in such events) lead to jumps in the conditional mean of foreign exchange returns. These jumps tend to disappear quickly, while their variances do it in a much slower and asymmetric manner (positive or "good" innovations have a less intense effect than negative ones). The study analyzes 5 different currencies.

Dióǵenes (2007) has studied the effectiveness and duration of Brazilian Central Bank interventions in the BRL/USD onshore market, between 2003 and 2006. He concludes that for every US$ 1 billion, the purchase of US dollars leads to a decrease of 0.17% to 0.70% in the nominal value of the local currency, while the sale leads to an increase of 0.29% to 0.34%. The type of intervention (being in the spot or future markets) did not affect the results, and their effects were tested for up to 5 days. During this period, it was shown to last. In this same line of thought, Pereira (2010) analyzed the effects of such interventions for the BRL between 2002 and 2008. Results were not conclusive for the level, but very robust for the short-term volatility, with an increase during USD purchases and a decrease during USD sales.

Ventura and Garcia (2009) have estimated in 0.99% and 1.12% the variation in the future and spot exchange rates, respectively, for every US$ 1 billion orders dealt in the Brazilian interbank market between February 2006 and May 2007. Bergman and Hansson (2005) concluded that the real exchange rate for 6 currencies is best represented by a stationary 2-Stage Switching Markov model, with varying intercept between regimes. Regression was made using a 3-month average rate of these 6 currencies between 1973 and 1990. Conclusion was based on a comparison of this model with two others traditionally accepted for the years of 1991 to 1997, using projected and observed data. A similar study was made for the BRL/USD pair by Santana and Bueno (2008), when the authors compared the forecasting power of both implicit and conditional volatilities to the observed volatility. Conditional volatility was modeled with a Gaussian GARCH (1,1) model and compared to a SWARCH (Markov Switching ARCH), in which the conditional variance has a finite number of regimes, governed by a 2-stage Switching Markov chain. Adopting this last model showed to be most appropriate, as they demonstrate the use of a GARCH in regime switching series tends to overestimate parameters and lead to incorrect forecasts. The SWARCH also accepts a mixture of different probability densities and, therefore, is more flexible in adjusting to returns with unknown distribution. They conclude that SWARCH exceeds the forecasting power of both implicit and historical volatility models. The study is based on the PTAX rate (Central Bank of Brazil's local FX benchmark) from March of 1999 to April of 2004.

While most of the above mentioned papers seek the causes behind innovations and their quantitative impacts, this paper aims to analyze just the consequences of them to the stability of the local FX market. The innovations sought here are those with short-term persistence and direct impact to the short and medium-term planning of entities relying on the conversion of foreign to local currency (and vice-versa) as an important part of their daily activities.

2.2. The Use of Derivatives for Protection

Chowdhry (1995) shows that when the foreign currency cash flow is uncertain, firms attempt to borrow more than its debt financing needs. This may lead to an increase in their cost of capital. In Stulz (1996) one simple example is given by supposing a firm whose cash flow fluctuates randomly over time and, if holding debt, a cash drawn may obligate it to file bankruptcy, bringing extra costs to the firm and reducing its value.

Data from the Bank for International Settlements (2010) show that the FX Spot and OTC derivatives market grew from a daily average volume of around US$ 1.5 trillion in April, 1998 to US$ 4 trillion in April, 2010. From the same report, FX spot and OTC derivatives in Brazil totaled over US$ 14 billion in daily volume in April, 2010, with over 91% involving the American Dollar. This volume came from about 0, back in 1995. The average tenor for 45% of this volume range from 7 days to 1 year, showing the purpose of these protections are in line to that proposed in this paper. According to Saito and Schiozer (2007), from all companies that used derivatives for hedging, 100% of them did it for their Dollar exposure, and over 50% used OTC instruments.

2.3. Pricing Options

Since Black and Scholes' (1973) award-winning work, much has been done towards the precision of option pricing methods. Dropping some of their assumptions and developing complex formulae has been the way found by many authors to enhance the B&S framework. Others, with the same goal and similar results, developed more intuitive methods. This is the case of Cox, Ross and Rubinstein (1979) - CRR. Following a non-arbitrage approach, they weight the ratio of changes of the asset price in a lattice by their transition probabilities and make it equal to the risk-free interest rate. With the underlying asset value and risk-neutral probabilities set in every branch, a recursive method can be applied. The final nodes prices have their exercise prices calculated and, supposing an European option, they are discounted (and weighted by their risk-neutral transition probabilities) to the seed node, whose value is the option cost.

Clewlow and Strickland (1998) present this recursive method for pricing floating strike lookback options. Based on a trinomial lattice, they apply a variation of the above mentioned method of setting a final payoff in the terminal nodes and discounting them to the seed node. This is done considering a variable-strike value for the asset along the lattice, based on the path undertaken by this asset along the time. The asset price level in each node is derived from the initial asset price S_0, the time step size and the maximum standard deviation of the series. Risk-neutral transition probabilities are calculated based on the variance, the time-step size, and the risk-free interest rate.

Although the method is very straightforward and robust, it is based on a single-variance factor across all nodes. Given the instrument this paper aims to price is based on a switching-volatility series (the BRL/USD exchange rate), enrichments to this method were evaluated. A pentanomial lattice is proposed by Bollen (1998). Using two more branches, the author adds transition probabilities of switching regimes to the price structure. One major constraint is imposed: the higher volatility regime must be around twice as that of the lower volatility one. It will be clear in Section 4 that such 1:2 constraint in the volatility does not hold for the BRL/USD series, therefore requiring a solution which relaxes such need but keep the 2-state Markov switching characteristic in the pricing formulation.

An efficient approach to include in the model the switching nature of the BRL/USD series volatility without adding much complexity to it is that of Yuen and Yang (2010). Based on the CRR framework, they propose a model in which the *k*-state switching volatility is split into *k* trinomial lattices. Risk-neutral transition probabilities are calculated based on the lattice variance and risk-free interest rate. The option price is then found on both lattices considering the transition probabilities between states (in other words, between lattices).

Hedging has its costs. The trade-off between these costs and the level of protection obtained must be optimized in order to maximize the firm value. The VTRF aims to be a cheaper option compared to traditional FX hedging instruments available in the market. The next section explains how this can be achieved.

3. Defining the Series and Building the Instrument

This section starts with an overview of the SWARCH model and how it can be incorporated to lattices option pricing methods, giving them a more precise set of asset values and derivative prices. It then follows defining the model to price the VTRF and finishes modeling the risks associated to the instrument.

3.1. The SWARCH

Markov chain models describe time series processes in which the level of a random variable can assume different regimes (or states). Each regime of the series has its own variance and a factor defines how many times higher the variance of a regime compared to the lowest variance one is. One characteristic of such models is the transition probability, or the probability that the series will commute from one regime to another. Introduced by Hamilton (1989), the SWARTCH (*Switching Autoregressive Conditional Heteroskedasticity*) is a conditional variance model able to describe switching regime time series. The model is described in Hamilton (1994) as:

$$r_t = E[r_t|\Omega_{t-1}] + \varepsilon_t \tag{1}$$

with

$$\varepsilon_t = \mu_t \cdot \sqrt{g_{s_t}} \tag{2}$$

$$\mu_t = v_t \cdot \sqrt{h_t} \tag{3}$$

being

$$v_t \sim i.i.d. \tag{4}$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \cdot v_{t-i}^2 \tag{5}$$

in which:

υ_t may assume normal or *t*-student distributions,
r_t is the foreign currency exchange rate return,
h_t is the return's conditional variance,
$s_t = 1, 2, ..., k$ ($k \in N$) is an independent variable governed by a *k*-state switching Markov regime, that serves as an index of g ($g \in R^+$), which, in turn, alters the scale of the ARCH model variance according to the state of the series at time *t*.

The model may or may not have a leverage effect (denoted by a "-L" after the name "SWARCH"). This effect is incorporated to the autoregression residuals of the ARCH model, but will not be adopted in this study in order to keep it in line with that of Santana and Bueno (2008), whose conclusion was that the BRL/USD is best modeled without this effect.

Markov chains have their states described according to the probabilities below.

$$P(s_t = n \mid s_{t-1} = m, s_{t-2} = k, \ldots) = P(s_t = n \mid s_{t-1} = m) = \pi_{mn}, \text{ p/ } m, n = 1,2,\ldots,k$$

It is common to organize them in a $k \times k$ matrix where each column sums 1 and each element indicates the probability of a regime switch from state *m* to state *n*.

$$\Pi = \begin{bmatrix} \pi_{11} & \cdots & \pi_{k1} \\ \vdots & \ddots & \vdots \\ \pi_{1k} & \cdots & \pi_{kk} \end{bmatrix} \tag{6}$$

According to Tsay (2005), the expected duration of a process to remain in state *m* is the inverse of π_{mm}, so

$$t_{11} = \frac{1}{\pi_{11}} \tag{7}$$

is the estimated time duration of regime 1. Regime 1 is usually defined as the lowest variance one, which serves as the benchmark for the others' variance through the scale factor g_{s_t}.

3.2. Modeling the VTRF

The VTRF acts like a forward option, triggered by an observed volatility level in the asset and paying/receiving adjustments in consecutive days based on the asset level of the day prior to that when the option is exercised. The Volatility and Payoff Windows, tenors, and Threshold Level are to be defined and agreed upon between the writer and the holder.

The approach used here is to price two correlated American options, a call and a put, making a long position on one and a short on the other. These options are lookback ones created in such a way that strike and exercise decisions are based on the volatility of the underlying asset. This portfolio results in a synthetic forward, much like a Variance Swap, but with the optionality embedded to it. Another enrichment made to the common option pricing theory is the introduction of multiple payoffs. This enables the instrument to hold constant the asset value during the Payoff Window for its holder. The strike level of such options during the Payoff Window is taken as the asset value of the step prior to that of the exercise, in order to stabilize the exchange rate for the instrument holder during the following periods. The instrument's Greeks are also a sub-product of this options addition approach, turning risk management straightforward.

To price these options, the lattice structure is used. Following the non-arbitrage approach of Cox, Ross and Rubinstein (1979), Yuen and Yang (2010) develop a model adding a third branch (or moment) to each node (thus constructing a trinomial lattice). This intermediary branch enables the modeling of a non-changing asset value from one step to another. Using continuous interest rates, CRR's risk-neutral approach in this trinomial lattice can be represented as:

$$p_u \cdot e^{\lambda\sigma\sqrt{\Delta t}} + p_m + p_d \cdot e^{-\lambda\sigma\sqrt{\Delta t}} = e^{r\Delta t} \tag{8}$$

$$p_u . \lambda^2 \cdot \sigma^2 \cdot \Delta t + p_d . \lambda^2 \cdot \sigma^2 \cdot \Delta t = \sigma^2 \cdot \Delta t \tag{9}$$

in which p_u, p_m, and p_d are the risk-neutral probabilities of the asset to raise, maintain, or reduce its value, respectively.

The time-step length is defined as Δt; σ is the asset's return standard deviation; r is the continuous risk-free interest rate, and λ is defined as being greater than 1 to ensure the risk-neutral measure exists. According to Yuen and Yang (2010), common values in the literature for λ range between $\sqrt{1.5}$ and $\sqrt{3}$, with the former being used in this paper (explanation for this decision follows in Section 4). So far, variance is assumed constant along the option's life. To address the state switching nature of the series, Yuen and Yang (2010) change the risk-neutral probability measure for each regime. Assuming k Markov states, let $r_1, r_2, \ldots, r_k$ and $\sigma_1, \sigma_2, \ldots, \sigma_k$ be the risk-free interest rate and the asset price volatility, respectively, for each state. The price change ratio between time steps is defined as $e^{\sigma\sqrt{\Delta t}}$, in which $\sigma > \max(\sigma_n)$ for $1 \leq n \leq k$. Adding this switching condition to (8) and (9), the risk-neutral approach becomes:

$$p_u^n \cdot e^{\sigma\sqrt{\Delta t}} + p_m^n + p_d^n \cdot e^{-\sigma\sqrt{\Delta t}} = e^{r_n\Delta t} \tag{10}$$

$$p_u^n \cdot \sigma^2 \cdot \Delta t + p_d^n \cdot \sigma^2 \cdot \Delta t = \sigma_n^2 \cdot \Delta t \tag{11}$$

$$p_u^n + p_m^n + p_d^n = 1 \tag{12}$$

This is similar to creating k different lattices that run in parallel. Defining λ_n as σ/σ_n, the following is the solution for (10), (11), and (12):

$$p_m^n = 1 - \frac{\sigma_n^2}{\sigma} = 1 - \frac{1}{\lambda_n^2} \tag{13}$$

$$p_u^n = \frac{e^{r_n\Delta t} - e^{-\sigma\sqrt{\Delta t}} - p_m^n(1 - e^{-\sigma\Delta t})}{e^{\sigma\sqrt{\Delta t}} - e^{-\sigma\sqrt{\Delta t}}} \tag{14}$$

$$p_d^n = \frac{e^{\sigma\sqrt{\Delta t}} - e^{r_n\Delta t} - p_m^n(e^{\sigma\sqrt{\Delta t}} - 1)}{e^{\sigma\sqrt{\Delta t}} - e^{-\sigma\sqrt{\Delta t}}} \tag{15}$$

For σ, the following equation is suggested:

$$\sigma = \max_{1\le i\le n} \sigma_i + \left(\sqrt{1.5} - 1\right)\bar{\sigma} \tag{16}$$

,

in which $\bar{\sigma}$ can be the arithmetic mean or the root mean square of all σ_i (here, the former is adopted). This ensures σ will always exceed the highest volatility of all regimes and make λ_n close to what the literature finds appropriate.

With the lattice structure defined, the price of the option in each node is then found by weighting all k "parallel" nodes by the regime transition probabilities π_{mn} from (6) and the underlying asset value risk-neutral probabilities p_u, $p_{m,}$ and p_d. This is done recursively from the end nodes (t = T) to the seed node. Defining i as the time step ($0 \le i \le \frac{T}{N}$), j as the node level in time step i ($\frac{T}{N} - i \le j \le \frac{T}{N} + i$), and n as the Markov variance regime ($0 \le n \le k$), the option value $V_{i,j,n}$ at each (i,j,n) node is:

$$V_{i,j,n} = e^{-r_i^n \Delta t}\left\{\sum_{m=1}^{k} \pi_{nm}\left(p_u^n \cdot V_{i+1,j-1,m} + p_m^n \cdot V_{i+1,j,m} + p_d^n \cdot V_{i+1,j+1,m}\right)\right\} \tag{17}$$

For the sake of simplicity, and without losing much precision, the risk-free interest rate r_i^n will be considered constant along the option's life, for all volatility regimes, and will be denoted by r.

To construct the lattice, N time steps are defined. Each time step will have a length of $\Delta t = T/N$, in which T is the time to expiration of the option. The asset value at each node is, then:

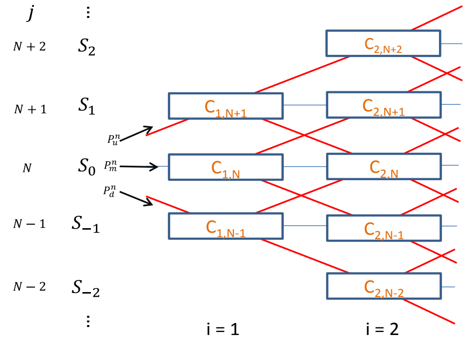
$$S_{i,j} = S_0 \cdot e^{(j-N)\sigma\sqrt{\Delta t}} \tag{18}$$

It is important to mention that, with this notation, the bottom node of the lattice (lower right) is that with index (N,0), the top one (higher right) is that of (N,2*N), and the seed one is that of (0,N). The option value can be calculated for all nodes recursively using (17), but it must also be considered the case of early exercise, which is discussed next.

3.2.1 Early Exercise

The above procedure suits well for traditional European and American options. In these cases, the underlying asset value at each node will serve either as a benchmark or as the option's strike. The option's holder will only exercise the option if this action maximizes its value against letting the option expire. In the case of traditional Floating Strike American lookback options, the decision is observed at each node, with the strike being the highest among all historical asset values since the option's initial date. The option value at the node is the greatest from (i) the payoff at the node and (ii) the option value in the following nodes weighted by risk-neutral probabilities and discounted by the risk-free interest rate. This is represented for an American call option in Figure 1 and formulas (19) and (20).

Figure 1 - Example of Pricing Through a Trinomial Tree



with

$$C_{i,j}^{n} = \max\left[(S_j - K); e^{-r\Delta t}\left(P_u^n \cdot C_{i+1,j+1} + P_m^n \cdot C_{i+1,j} + P_d^n \cdot C_{i+1,j-1}\right)\right] \tag{19}$$

$$K = \max\left(S_{i,j}\right) \tag{20}$$

,

in which K is the greatest asset value $S_{i,j}$ observed since the option's initial date.

For the VTRF, some modifications are necessary. First, the VTRF options exercise is not a maximization decision from the option's holder. It is, instead, triggered by some pre-defined volatility rate. According to Hull (2008), variance rate, the square of volatility, can be defined in step *i* for the last *b* observations as:

$$\sigma_i^2 = \frac{1}{b-1}\sum_{w=0}^{b}(u_{i-w} - \bar{u}_i)^2 \tag{21}$$

with u_i being given by:

$$u_i = ln\left(\frac{S_i}{S_{i-1}}\right) \tag{22}$$

is the continuously compounded return between step *i*-1 and step *i*, and

$$\bar{u}_i = \frac{1}{b}\sum_{w=0}^{b} u_{i-w} \tag{23}$$

is the mean of all u_i from the last *b* steps. When $i < w$ out-of-lattice observed values must be carried during calculation. They represent recent values of the underlying asset and must not be ignored, as setting them to any other value can prematurely, and incorrectly, trigger the instrument. For the VTRF *b* will serve as a parameter, to be defined between the writer and the holder of the instrument. Demeterfi *et al* (1999) remind that the procedure for calculating volatility should be clearly specified with respect to:

- the source and observation frequency of the asset price,
- the annualization factor for moving from daily observations, and
- the standard deviation calculation with sample mean subtraction or not.

Although it is a common market practice to consider $\bar{u}$ equal to zero (Windcliff *et al* (2006)) find that for daily sampling this makes minimal difference), this paper has not considered this assumption during its simulations.

Let ξ be the Volatility Threshold, or the Volatility Strike. This parameter, agreed upon between the holder and the writer, is the limit for the observed volatility during the past *b* steps (days) under which there are no adjustments for the instrument. If, at the end of any *b* days, this threshold is exceeded, both options will be exercised, triggering the instrument payoffs. This can be summarized as:

- While $\sigma_i^2 < \xi$, the instrument remains idle;

- When $\sigma_i^2 \geq \xi$, in which *i* is the lowest index for which σ_i^2 exceeds ξ, VTRF is triggered;

This concept is incorporated to the procedure proposed by Clewlow and Strickland (1998). Additional support lattices are also created to consider the possibility of early exercise of the option in each node. They contain:

- ${\sigma_{MAX}}_{i,j}$ – the maximum observed variance from the last *b* nodes,
- $\Psi_{i,j}$ – the highest observed variance path asset values,
- $\eta_{i,j,c}$ – the exercise action (yes/no) for each incoming connection *c* from last step's nodes,
- $p_{i,j,n}$ – the accumulated node reaching probability from seed node to any node (*i,j*),
- $\rho_{i,j,d,l,n}$ – the accumulated node reaching probability from any node (*i,j*) to the nodes encompassed by all possible asset paths during the exercise period (shaded area in Figure 2).

Variable *c* identifies the incoming path (from above - 0 , same level - 1 or below - 2). The highest observed variance $\sigma_{MAX,i,j}$ saves in each node the highest variance encountered along all possible paths reaching that node (considering the last *b* steps). It is found after applying (21) at each node's variance window (last *b* asset values from the highest variance path $\Psi_{i,j}$). The procedure in each node is to drop the first value of Ψ from the prior node and add, as the most recent (b^{th}) value, the current node asset value.

The volatility is then calculated for this new $\Psi_{i,j}$. This procedure is repeated for each of the 3 incoming paths of the node and the highest variance of them is saved under ${\sigma_{MAX}}_{i,j}$. The three variances are also square rooted and compared to the Volatility Threshold level ξ for exercise decision (adjusting this number, such as annualizing, may be necessary to meet the Volatility Threshold unit). Each incoming path is then marked with an "Exercise-Yes" or "Exercise-No" stamp.

The probability of reaching each node in the lattice is calculated for all possible paths considering accumulated p_u^n, p_m^n, and p_d^n. This is saved under $p_{i,j,n}$ and serves to weight the possibility of exercise from all incoming paths of a node with the payoff for each of these paths. With the above setup, it is possible to set the option's value in each node prior to the expiration nodes by finding the greatest between (i) the discounted option prices from the connecting nodes of the next step, and (ii) the expected payoff of the current node. This is represented by the following in the case of a call:

$$C_{i,j}^n = \max(E_{i,j}^n; M_{i,j}^n) \tag{24}$$

in which $E_{i,j}^n$ is calculated with the following formulas:

$$Payoff_{i,j,n} = \sum_{m=1}^{k} \sum_{d=i}^{max(i+D,N)} \sum_{l=j-d+i}^{j+d-i} \pi_{nm} \cdot \rho_{i,j,d,l,n} \cdot N_d \cdot Max(S_l - S_{j-1}, 0) \cdot e^{-(d-i)r\Delta t} \tag{25}$$

and

$$E_{i,j}^n = \sum_{m=1}^{k} \pi_{nm} \cdot Payoff_{i,j,n} \cdot \left(p_{i-1,j-1,n} \cdot p_u^n \cdot \eta_{i,j,2} + p_{i-1,j,n} \cdot p_m^n \cdot \eta_{i,j,1} + p_{i-1,j+1,n} \cdot p_d^n \cdot \eta_{i,j,0}\right) \tag{26}$$

While $M_{i,j}^n$ is simply the following step's discounted option values, given by:

$$M_{i,j}^n = e^{-r\Delta t} \cdot \left(p_u^n \cdot E_{i+1,j+1}^n + p_m^n \cdot E_{i+1,j}^n + p_d^n \cdot E_{i+1,j-1}^n\right) \tag{27}$$

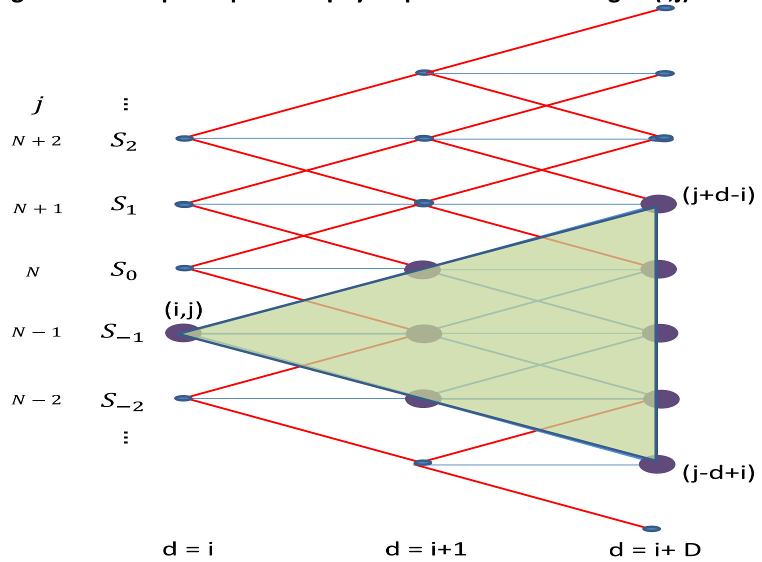
in which:

- *n* and *m* denote the Markov regime, with *n* being the initial state, and *m*, the final;
- *D* is the number of payoff steps;
- *d* denotes the payoff step (after exercise step *i*);
- N_d is the notional at payoff step *d*;
- *l* is the node in step *d* (within the exercise region shown in the shaded area of Figure 2);
- $\rho_{i,j,d,l,n}$ is the probability of node (*d,i*) being reached from node (*i,j*) under regime *m*;
- $\eta_{i,j,c}$ is a binary variable of node (*i,j*) with values 0 (in case the variance threshold could not have been reached for incoming path c) or 1 (in case path c can have a volatility higher than the Volatility Threshold).

In equation (25) the expected payoff in every node of the lattice is found by delimiting the payoff region, which is an inner lattice formed by all possible paths followed by the asset after exercise until the last payoff step. The region is illustrated in Figure 2. Every node in this inner lattice has a probability of being reached after the exercise (denoted by $\rho_{i,j,d,l,n}$). The adjustment value (difference from asset value at exercise step and at the evaluated step) is weighted by this probability and the regime transition probability π_{nm}. This is then discounted to node (*i,j*) for all nodes of this inner lattice to form an expected payoff for the option in that node. With this expected payoff set for all nodes, they are weighted in (26) by the probability of being reached from all paths, the regime transition probability and the exercise decision in each path. Puts follow the same process described by (24) - (27), except for the definition of the payoff, whose adjustment value is $Max(S_{j+1} - S_l, 0)$.

Figure 2 illustrates the concept behind the early exercise payoff and the relationship between the variables and nodes for a 3 period payoff VTRF, with exercise in node (*i*,*N*-1). The shaded triangle is the only possible region for the payoff during the exercise period. Purple nodes are those accounted in (25).

Figure 2 - Example of possible payoff paths for exercising in (i,j) with D=3



Starting at the seed node, options values in all nodes can be calculated by (26). After that, the recursive method can then be applied from the end to the seed node, where the VTRF cost is finally determined. In a nutshell, the following steps should be followed to find this cost:

1. Define the instrument parameters, including SWARCH's.
2. Calculate p_u^n, p_m^n, and p_d^n for both regimes.
3. Built the asset value lattice.
4. Calculate the probability of reaching every node from the seed node.
5. Calculate, for every node, the incoming path with highest variance.
6. Define, for every node and every incoming path, the existence of exercise.
7. Define the expected payoff for every node.
8. Weight the exercise decision, node reaching probability, regime transition probabilities and expected payoff for all paths and nodes from the seed to the last lattice step.
9. Discount the option value in each node, from last step to seed node, considering the highest between exercise value and holding value.

The seed node will have two different values, each corresponding to a regime. Adopting one or another will depend on the regime the asset is currently in.

3.2.2. Managing Risk

To manage the instrument risks, the traditional hedge sensitivities through Greeks can be employed. Delta, Gamma, Vega, Theta and Rho are computed according to Clewlow and Strickland's (1998) finite difference ratios in the lattice and options revaluation. Below each of these sensitivities is briefly explained with their calculation method. Calculation is defined at the initial node, but can be extended to any point in time. One new sensitivity factor is also proposed to account for the portfolio's volatility switching nature.

These data are presented in Section 4 with an example extracted from real market data. The instrument Greeks must be calculated as the sum or subtraction of both the call and the put sensitivities (observing long and short positions).

Delta

Defined as the rate of change in the portfolio value with respect to the price of the underlying asset, is the slope of the curve relating the portfolio value and the underlying asset price. In the case of a call,

$$\Delta = \frac{\partial C}{\partial S} \approx \frac{C_{0,N}(S_{N+1}) - C_{0,N}(S_{N-1})}{S_{N+1} - S_{N-1}} \tag{28}$$

with S_{N+1} and S_{N-1} being the asset values set as S_0 for the call pricing.

Gamma

Defined as the rate of change in the portfolio's delta with respect to the underlying asset price. It is the second derivative of the portfolio value with respect to the asset price. In the case of a call,

$$\Gamma = \frac{\partial^2 C}{\partial S^2} \approx \frac{\left(\frac{C_{0,N}(S_{N+1}) - C_{0,N}(S_N)}{S_{N+1} - S_N}\right) - \left(\frac{C_{0,N}(S_N) - C_{0,N}(S_{N-1})}{S_N - S_{N-1}}\right)}{\frac{1}{2}(S_{N+1} - S_{N-1})} \quad (29)$$

with S_{N+1} and S_{N-1} being the asset values set as S_0 for the call pricing.

Theta

Defined as the rate of change in the portfolio's value with respect to the time passage, *ceteris paribus*. In the case of a call,

$$\theta = \frac{\partial C}{\partial t} \approx \frac{C_{1,N} - C_{0,N}}{\Delta t} \quad (30)$$

Vega

Defined as the rate of change in the portfolio's value with respect to the volatility of the underlying asset, is not a letter in the traditional Greek alphabet, so no symbol will be used to represent it as a way of avoiding a grammatical error. In the case of a call,

$$Vega = \frac{\partial C}{\partial \sigma} \approx \frac{C_{0,N}(\sigma + \Delta\sigma) - C_{0,N}(\sigma - \Delta\sigma)}{2\Delta\sigma} \quad (31)$$

with Δσ being a small fraction of σ, such as 1 basis point.

$Vega_g$

As the instrument relies heavily on the volatility of the underlying asset and the SWARCH plays an important role in modeling the asset behavior, the variance scale factor g_{S_t} is introduced as a new measure of sensitivity for the instrument. The proposal of this factor is to measure changes in the portfolio's value with respect to the changes in the ratio between Markov regime variances. In the case of a call,

$$Vega_g = \frac{\partial C}{\partial g_{S_t}} \approx \frac{C_{0,N}(g_{S_t} + \Delta g_{S_t}) - C_{0,N}(g_{S_t} - \Delta g_{S_t})}{2 \cdot \Delta g_{S_t}} \quad (32)$$

with $\boldsymbol{g_{S_t}}$ being the ratio between the high and low variances, as defined in item 3.1, and $\Delta\boldsymbol{g_{S_t}}$ is a small variation in this ratio, such as 1%.

Rho

Defined as the rate of change in the portfolio's value with respect to the interest rate. In the case of a call,

$$\rho = \frac{\partial C}{\partial r} \approx \frac{C_{0,N}(r + \Delta r) - C_{0,N}(r - \Delta r)}{2\Delta r} \quad (33)$$

with Δr being a small fraction of σ, such as 1 basis point.

Having the series and the instrument defined, these definitions are applied and the results analyzed. Next section runs an econometric regression on the BRL/USD pair and applies its results to the pricing of the VTRF. It also applies market conditions to the instrument in order to test it in a real scenario, comparing its performance with a theoretical replicating portfolio of plain-vanilla options.

4. Results

In this section the BRL/USD series is analyzed through a SWARCH model and its parameters are used to price the VTRF under market conditions. The instrument is also applied in a real scenario and the results, compared to a theoretical portfolio of plain-vanilla options replicating the instrument.

4.1. Econometrical analysis

With the regression model defined, Hamilton and Susmel (1994) routines were used. Based on the GAUSS language and the optimization package OPTIMUM, Hamilton and Susmel estimated various SWARCH (*K, q*) for the United States weekly stock returns. For the BRL/USD series of this paper, a SWARCH (2, 3) with *t*-student distributed innovations was used, following the conclusions of Santana and Bueno (2008). The nominal spot rate is used to build the BRL/USD pair series, instead of the PTAX rate used by these authors. Regressions for the same period showed that such modification does not alter much the results found by Santana and Bueno (2008). By using the spot rate, this paper aims to avoid possible artificialities inherent to the PTAX during that period, such as valuation considering only part of the spot market deals (from the so-called “Central Bank dealers”), and low liquidity in the product serving as underlying for the rate calculation (interbank spot trades), among others. The PTAX is also a weighted average, which could smooth and hide some of the shocks this paper aims to identify. As expected, the same regression made by Santana and Bueno (2008) using the spot rate gave similar results, but zeroing the level factors and increasing g_{s_t}. The analysis period was chosen to be from January 2003 to December 2009. This interval captures moments of economical stability (from 2004 through early 2007) and stress (the 2008 financial crisis) during president Luis Ignacio Lula da Silva’ first and second mandates, and serves as a benchmark for testing the instrument during some shocks in 2010.

Loading the M@ximize v1.1 package from Laurent and Urbain (2003) into OxMetrics® (version 6), the regression described in Section 3 was run with OxGauss. The series used was that with the difference in the natural logarithm of the currency pair observations (log return of the BRL/USD), with a *t*-student distribution. The routine converges to the results summarized below (standard errors in parenthesis).

For the level:

Constant term in regression (r_0):	0.08243 (0.01562)
Autoregressive coefficient (r_1):	0.01975 (0.02391)

For the variance:

Constant term in ARCH process (α_0):	0.01586 (0.00614)
Coefficients in ARCH process (α_i):	
α_1:	0.24611 (0.04885)
α_2:	0.23086 (0.04749)
α_3:	0.30661 (0.04996)
Variance Factor (g_{s_t}):	18.716 (6.852)

Except for the autoregressive coefficient, all other terms are significant at a 5% confidence level (most at 1%). Thus, the model still fits into the conclusions of Santana and Bueno (2008), but some parameters show a difference for this recent period of the series. The pattern followed by the BRL/USD log return series is more random for the level, as the coefficient for the autoregressive term can now be considered 0 (against 0.099 for the 1999-2004 interval). The variance factor g_{s_t} is now 18.716, against 4.448 of almost ten years ago. This means the high volatility state (state 2) has now an 18.7 times higher variance - or 4.3 times higher volatility - than the low volatility state (state 1). This increase may be explained by the 2008 turbulences caused by the financial crisis. This factor is also higher than that used in the Variance Swap market, where the maximum realized volatility is typically set to 2.5 times the delivery price, according to Windcliff *et al* (2006). Another important information for the purposes of this paper, the Markov transition probability matrix, is shown below in Table 1.

Table 1 - Markov Transition Probability Matrix

	$P\ (s_{t=1})$	$P\ (s_{t=2})$
$s_{t-1=1}$	26.63%	9.33%
$s_{t-1=2}$	73.36%	90.67%

The above matrix is much different from that of Santana and Bueno (2008) and shows that there is a 26.63% probability that the regime will stay in state 1, while the chances of leaving state 2 and return to the lower variance state is 9.33%. From this data we can also extract an important information: the average period of time the series remains in the low and high volatility regimes. According to equation (7), this is equal to 3.7 days in the low volatility regime and 1.1 days in the high one, on average. Although this regression defines as around 1 day the period of high volatility, the VTRF was designed to be able to cover any number of payoff days, as empirically and according to Diógenes (2007), a larger duration for the higher volatility regime makes more sense. These results are assumed to be constant during the first semester of 2010 and no recalibration is considered during this period.

4.2. VTRF Behavior

Item 3.2 was coded using Microsoft Excel® 2010 VBA toolkit. Its flexibility, ease to use, and availability were key for the decision, which enables any PC with the MS Office package installed to run its code, price and get all possible information about the VTRF.

The standard VTRF used here is that applicable to USD sellers. It is composed by a short call and a long put under the VTRF concept. That means the holder of this instrument will be protected when volatility moves beyond the threshold level, receiving the payoff when the BRL appreciates, and paying when the BRL depreciates against the USD.

One important characteristic of this model is that it relies almost entirely on the volatility levels of the two SWARCH regimes. This determines σ, according to (16), which drives the asset values on the lattice for all nodes. Having a high σ means there will be jumps across the vertical axis of the lattice, making it less precise when empirical evidences show asset prices change with more parsimony from one step (day) to another. Also, the difference in price from one node to another is greater on the top nodes than on lower nodes. This is because the entire CRR-framework is based on continuous interest rates, as seen in (18). This exponential component turns the lattice prices unevenly distributed. This leads the model to two important behaviors:

- The asymmetry is also present in the payoff component of (25) and (26). Given there will only exist payoffs in the calls when the asset moves from one node to a higher one (the opposite applies to the puts), put payoffs in one node will always be lower than its equivalent call payoff. This is because the fixing rate used is the last asset price prior to exercise, so one will adopt the higher node and the other, the lower. Exercise node is not half-way between them, making one of the instrument´s components (the call) to benefit from a higher payoff. This makes the VTRF sometimes exhibit a negative cost, as the call premium surpasses the put premium under the same conditions.
- Variance tends to be higher, for the same Variance Window length, in the upper portions of the lattice. Considering the asset values swing more intensely on upper nodes, it is intuitive to think the same number of hops will produce a higher σ^2 on the top portion than on its lower equivalent of the lattice, according to (21).

One other characteristic to point out in the VTRF is its opposite behavior regarding early exercise under certain conditions. As opposed as to the conclusions of Hull (2008), which shows that early exercising of plain-vanilla American call options on non-dividend paying assets are never optimal (while early exercising of plain-vanilla American put options can be), the VTRF can be optimal when early exercised. Given the floating strike characteristic of the instrument, payoffs for exercising early in the lattice or far at the end only differ by the standard deviation applied in the lattice construction. These payoffs are both limited to the difference in the asset price from one node to its upper or lower neighbor (discounted to the seed node by the risk-free interest rate). If the standard deviation is not high but the interest rates is, for a given asset price level, exercising earlier or later will provide about the same payoff, but discounting the early exercised payoff can make it worth more than the latter when both are discounted to the origin. This explains why some threshold levels can trigger extreme events in price: they raise the chances of early exercise and fade away the participation of the rest of the lattice in the price construction process.

Backtesting the instrument during the first and second quarters of 2010 it was possible to find different situations and conditions when the instrument would have been exercised. The realized BRL/USD data was tested for the Variance Window at different Volatility Threshold levels. For each test, the first exercise day was found and the instrument payoff registered. An instrument contracted on January 4th, 2010, by a USD seller should have approximately those parameters in Table 2.

Table 2 - VTRF Instrument Parameters in January 4th, 2010.

Parameter	Value
Regime 1 Asset Log Return Volatility[2] (σ_1)	3% p.a.
Variance Scale Factor (g_{s_t})	18.72
Switching Probability (P_{11} = 1 - P_{12})	26.63%
Switching Probability (P_{22} = 1 - P_{21})	90.67%
Interest Rate[3] (r)	1.3% p.a.
Initial Asset Value (S_0)	1.7236 BRL/USD
Instrument Tenor (T)	60 days
Variance Window Size (b)	10 days
Payoff Window (D)	5 days
Daily Notional (N_d)	USD 1,000,000

Results are presented in Table 3, which contains VTRF fair price at inception, the exercise date, fixing rate, and payoffs for 10 different Volatility Thresholds.

Table 3 - Backtest of VTRF as of January 4th, 2010 (Values in BRL)

Volatility Strike	VTRF Price	Exercise Date	Fixing Rate	Payoff 1	Payoff 2	Payoff 3	Payoff 4	Payoff 5	Total
11%	(5,139.08)	5-Jan-2010	1.7236	19,400.00	7,500.00	(17,300.00)	(15,000.00)	(8,300.00)	(13,700.00)
12%	1,208.20	22-Jan-2010	1.7899	(28,400.00)	(29,100.00)	(46,400.00)	(61,000.00)	(65,700.00)	(230,600.00)
13%	(1,143.69)	25-Jan-2010	1.8183	(700.00)	(18,000.00)	(32,600.00)	(37,300.00)	(56,100.00)	(144,700.00)
14%	(446.23)	26-Jan-2010	1.8190	(17,300.00)	(31,900.00)	(36,600.00)	(55,400.00)	(57,900.00)	(199,100.00)
15%	463.64	27-Jan-2010	1.8363	(14,600.00)	(19,300.00)	(38,100.00)	(40,600.00)	400.00	(112,200.00)
16%	(47.56)	28-Jan-2010	1.8509	(4,700.00)	(23,500.00)	(26,000.00)	15,000.00	17,600.00	(21,600.00)
17%	(14.41)	29-Jan-2010	1.8556	(18,800.00)	(21,300.00)	19,700.00	22,300.00	(15,500.00)	(13,600.00)
18%	401.21	1-Feb-2010	1.8744	(2,500.00)	38,500.00	41,100.00	3,300.00	(500.00)	79,900.00
19%	627.31	No Exercise	-	-	-	-	-	-	-
20%	362.88	No Exercise	-	-	-	-	-	-	-

Taking the Volatility Threshold at 18% as an example and using a hypothetical scenario for which the holder had a "crystal ball" and knew, on January 4th, 2010, exactly the BRL/USD behavior for the following weeks, a price reasonability analysis can be drawn using market data. Suppose the holder could have formed a portfolio of BM&FBovespa[4] plain-vanilla options. Given that the Exchange only trades standard contracts, it is not possible to build a perfect replicating portfolio using its options. Ignoring the time value of money during the 5-day period, to replicate the VTRF payoff, one could have constructed a portfolio composed of USD 5,000,000 in long DOLGB8Z and short DOLGBDZ options (respectively, European USD call and put expiring on February 1st, both with strike at 1.875 BRL/USD). The USD 5,000,000 call option was priced at BRL 20,990 while the put, at BRL 737,990, according to BM&FBovespa reference prices[5]. The portfolio, thus, would have costed BRL 717,000 (approximately USD 415,990). This is many times more expensive than the VTRF, which would actually pay the client a premium. The same

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2 It is assumed the Market was at a low variance regime at the time of pricing the instrument.

3 Local USD Cupon rate, the risk-free interest at which USD can be invested.

4 BM&FBovespa stands for "Bolsa de Valores, Mercadorias e Futuros", Brazil's local Stock, Merchandise, and Futures Exchange, which serves as a benchmark for the local derivatives market.

5 Availabe at www.bmfbovespa.com.br under Boletim / Indicadores / Preços Referenciais / Preços Referenciais BM&F – Prêmios de Opções.

analysis using the actual strike (1.8744) and straight Black & Scholes formulae (with a single USD 1,000,000 option for each day of the Payoff Window) shows that this portfolio would have costed BRL 703,871 (USD 408,372). As expected, prices are very close, as according to BM&FBovespa (2010), as the Exchange European options are priced using the Black & Scholes model. These analysis did not take into account all the uncertainty involved at the time of pricing, as we have considered a hypothetical scenario when the instrument holder would have forecasted many aspects about the future. In a real scenario, trying to replicate the instrument with a portfolio of options would demand various tenors and strikes for the option. Such work is not in the scope of this paper.

5. Conclusion

This paper aimed to propose and model an instrument to offer protection against volatility in the BRL/USD exchange rate, the Volatility Triggered Range Forward, which is formed with long and short positions in calls and puts, both defined as options whose exercises necessarily occur whenever the currency pair volatility during a Variance Window exceeds a volatility threshold. The instrument replicates a forward option, whose strike is a volatility of the currency pair, and whose payoff may last for a number of periods previously defined.

Various derivatives are available to protect companies against swings in the FX markets. Vanilla options give them the upside of not withdrawing any cash in the event of unfavorable asset value moves. This is possible by paying a premium which, in many cases, may not be worth it in the manager’s view. Other alternatives pass through swaps and forward instruments, which do not require any down payment at inception. They do not guarantee, however, multiple payoffs nor a market level for the strike rate, as they are priced with “at the money” rates, which can significantly differ from spot rates at maturity. They cannot individually guarantee protection for a number of days, keeping the BRL/USD level steady for both parties of the deal. The VTRF incorporates forward characteristics, assuring a market rate level for the strike at maturity. This is achieved by a smaller premium when compared to a portfolio of similar vanilla options (which cannot replicate the instrument entirely).

The model is based on the SWARCH (Switching Markov ARCH) model, following the study of Santana and Bueno (2008), which concludes this is the best model to forecast the BRL/USD volatility (forecasting through the SWARCH is not applied in this paper). It uses the lattice method of pricing options, following the work of Yuen and Yang (2010), which incorporates the SWARCH characteristics of the series to this concept. The VTRF is priced in real scenarios and compared to portfolios of similar plain-vanilla options. The analysis suggests that the model is feasible and that the VTRF is a good alternative for those willing to remove the volatility exposure from their balance sheets. The model can, although, be enhanced. The lattice refinement (difference in the asset price between nodes) used does not seem to replicate the real world with accuracy and can be further studied. The volatility level proposed by Yuen and Yang (2010), for example, can be re-discussed for the BRL/USD reality. The risk-free interest rate can fluctuate between steps. Another area for further development would be the calculation of p_u, p_m, and p_d based on implied probability trees using BM&FBovespa plain-vanilla options (Arrow-Debreu approach). Risk management issues can also be explored in more details. Techniques to hedge the risks arising from a VTRF portfolio may be further discussed, in order to give the hedge trader better comfort to trade it with sales desks.

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