Corruption and Tropical Deforestation in the Amazon Forest: A Differential Game Model

1. Introduction

Deforestation is an environmental problem especially for countries with large areas of tropical forests as is the case, among others, of Thailand, Malaysia, Indonesia, Congo, Ghana and Brazil. Studies have analyzed the various facets of the process of deforestation, and usually they highlight the importance of economic decisions involved, as one of the major cause, see for instance Pellegrini and Gerlagh (2006a) (2006b).

In China, according to Liu (2005), the Government provides educational programs through funds to support environmental programs, and the main purpose is to alert people about the adverse results from the loss of forests. A study by Li et al. (2007) indicates that the exploitation of rubber has been the main cause of deforestation recorded in the region of Xishuangbanna, China.

In the study by Ichikawa (2007), for Malaysia, a systematic loss of large forests was identified as mainly the result of cultivation of tradable goods, specially palm oil. In fact many empirical research at national or international level, found that variables as the price of meat, timber, beans, have a great impact in the level of deforestation see for example Young (1997) and Margulis (2003).

Recently, jointly with the variables cited above, the inexistence of solids institutions, have been considered as major variable responsible of deforestation (Global Monitoring Report, 2008, Greenpeace-Brazil, 2010). Weak institutions always encourage corruption behavior and it is included in the so-called indirect causes of deforestation.

Empirical evidences have found robust results, showing that corruption have a great impact on deforestation; see for instance Pellegrini and Gerlagh (2006a). Beyond this short introduction, the paper is structured in following way: in the first subsection of the next section it is presented some evidence of the deforestation in Amazon forest; in the latter subsection it is analyzed some of the international evidence on the effect of corruption in deforestation. In the second section we make a short survey about the literature in differential games. The third section presents a basic model with two different markets: illegal and legal market. In the fourth section we present a second model where it is included the corruption in the model. Final remarks are given in the fifth section.

1.1. Deforestation in Amazon Forest

The Amazon forest cover an area of nine Brazilian states, namely Acre, the state of Amazonas, Rondônia, Roraima, Pará, Tocantins, Amapá, Mato Grosso and part of the Maranhão. Due its extensive area and biodiversity, its considered one of the great human wealth, and its great loss in the last two decade has promote an increasing worldwide (including at national level) concern about its future (Muller et, al. 2008).

The main government policy used by the government has been the coercive delimitation of the area that can deforest and the application of penalties for those directly engaged in illegal deforestation. The government organization for the ministry of environment, IBAMA, has been the main instrument for such policies. These policies makes part the government's plan for the prevention and control of the deforestation in the Amazon forest, which have as one of the target point the immediately reduction of the deforestation. However, despite of the plan's application, every year the forest has been losing its size, and direct variables as international price of meat, soy price, etc, are pointed to be one of mains causes of deforestation (Margulus, 2003; Keohane e Olmstead, 2008).

The increasing demands for commercial products have increased the occupation of virgin areas, which due the great government's investment in social capital, induces more deforestation in the Amazon forest (Caldas, et al., 2001). Recently satellite images have shown that the state of Mato Grosso is the leader state in deforestation among the Brazilian states – in 2008 about 1,120 km^2 has been deforested in Amazon, and 70% of this deforested areas occurred in Mato Grosso (GEENPEACE, 2008).

There is a lot of empirical paper analyzing the causes of deforestation, however they mainly were concerned with those called direct causes of deforestation. The existence of corruption undermine all the government's effort to control or stop deforestation. The result of the use of coercive policies are not so trivial, the applicability of such policies in the field is an crucial part of all the policy, therefore government policies may turn innocuous under the assumption of existence of corruption (Global Monitoring Report, 2008).

According with Amacher (2006) and Viana (1988) 80% of all product collected are illegally harvested. Hence, in this environment the corruption appears to be an important issue that needs to be investigated.

1.2. Corruption and the forest sector

The link between the corruption and the forest sector is not a new issue, researchers have claimed that the governments shall incorporate such phenomenon in their policies, for example see: Damania (2002); Pellegrini and Gerlagh (2006); Amacher (2006), Koyunen e Yilmaz (2009); Transparency International (2002), (2007). For instance, Pellegrini e gerlach (2006) analyzed the impact of the entrance of new countries of the Europe union in the environment policies. Using cross-section estimation, they found that the corruption is the main responsible for the effectiveness of the environment policies in these countries. Koyunen e Yilmaz (2009) analyzed through the cross-section data, the impact of corruption in the deforestation level in 100 countries. They used many corruption indexes in their estimation, e.g., corruption perception index (CPI), International country risk, etc. They found a strong correlation between corruption and deforestation.

Despite the knowledge of the impact of corruption in the deforestation phenomenon, the way that they are correlated is little known (Tranparency International, 2007). Pellegrini (2006b), through the cross-sections methodologies, have analyzed the effects of corruption, democracy on deforestation. He found that the democracy have non statistical significance in the model, in the other hand, corruption was proved to be an important variable to explain difference in deforestation rate around the world.

Welsch (2004) analyzed the impact of corruption and GDP in the level of pollution. He found a positive and monotonic relationship between corruption and pollution and a negative relationship between GDP and pollution.

As cited above, there is an international consensus of the impact of corruption on the level of deforestation. However, for the case of Brazil, little attention has been given by the Brazilian researchers. There is a lot of empirical works that focus their attention in the so called direct caused of deforestation, however, the existence of weak institutions in several region in Brazil and the salary scheme used by the government may increase the potential existence of corruption related with the forest sector. It is a fact that many cases of corruption has been showed in the Brazilian media in the last years. Cases of collusion between the IBAMA's official and enterprise/ landowners have been showed to co-exist during long periods of time.

2. Differential Games and Deforestation

Since the work of Hoteling (1931), the economic analysis entered in a new area – the economics of natural resources. This increasing study of the natural resources issues began since the first oil crisis in 70's. Since there, a lot of theoretical work has been done incorporating many different types of resources and environment see for instance Solow (1974), Dasgupta (1980); Kemp (1980), Quyen (1988); Stavins and Jaffe (1990), Stiglitz and Dasgupta (1981).

These works usually uses optimal control theory and dynamic programing to analyze the effects of human activity on natural resources. Others works, namely, Amacher (2006 a, 2006 b), Rose-ackerman (1975), Hamilton (2006), and Mendes et al. (2008); change the focus of the analysis and highlights that the direct owner of the property are not the only agents responsible for deforestation, the existence of corrupted officials usually incentives more deforestation.

The present works, follows the same framework of the earlier papers, that is, the present paper aims to use the optimization by a "*representative*" agent to show the effect of his activity in the deforestation. However, in the present paper we use differential game to model the impact of corruption in the rate of deforestation, hence our work presents two important points in the deforestation issue, namely we used differential game and we analyzed the *open-loop stackelberg equilibrium*, and finally we introduced a new variable in the analysis, the corruption[1].

The use of the differential game to model deforestation in tropical forest is not a new one, Fredj et al., (2006), analyzed, through an application of differential game, the impact of the subsidies from rich countries (north) to the poor countries (the south), they followed some assumptions from earlier work, namely, Ehui et al. (1990) and Van Soest & Lensink (2000).

The use of differential games is not without a sense, given that the structure of relationship between those engaged in collusion behavior (landowner and government's official), and supposing that there is a state variable involved, the differential game is perfect tool for this case[2].

3. The Basic Model – Model 1

As argued earlier the institutional issues have a big importance in the level of deforestation in several countries. Our first model wants to analyze the effect of the existence of a market of illegal selling of timber or related products. In many countries, e.g., Brazil, one of the many ways to guarantee the provenience of the timber is the certification of the wood. In Brazil this certification is made by the IBAMA. However, as related case of corruption shows, many cases of corruption has been found in this accreditation process. Basing in these anecdotal evidences, the basic model developed here wants to highlight the effects of the "black" market in the incentive of the landowner to clean up his land.

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1 We define the *stackelberg open-loop equilibrium* when the strategies of the players are simple function of time.

2 The differential game was first introduced by Isaacs (1954). These games supposes the existence of constrains on the strategies used by players, these constrains are in fact, first order differential equation for the state variable or the so called "*kinematic equations*". The equilibrium definition used in the differential games depends of the structure of information under which the game is played (Basar and Olster, 1982). Under the case of perfect state information we may have*: closed-loop no memory, feedback, and open-loop*. For our case, we used the notion of stackelberg equilibrium. The name stackelberg game comes from the "traditional" view of stackelberg games, that is, there is a leader and follower in the game and the equilibrium solution is obtained by using backward induction. For a simple survey about stackelberg equilibrium please see Sethi et al., (2007) or for introduction to differential games see Feichtinger and Jorgehnen (1983).

In the present model there is two market, namely: a "legal" market for selling the forest product (related with deforestation), in this market the accreditation works perfectly. However, the landowner may choose to target his production to the "black" market or "illegal" market, this market the landowner can sell his timber without a certification. These two markets may have different characteristics: different demands, selling related costs, price, etc. Given that there is no audition by the IBAMA officials in this first case we are not including the potential corruption behavior), the landowner can choose spontaneously how much quantity to target to both market[3]. Therefore, in our model the landowner may choose how much to log but also how to split this amount between the legal and the illegal market.

Assuming a finite horizon optimization problem, the landowner may choose at each point of time, the quantity of his product to maximize his net revenue[4]. We define his revenue, R(t), as:

$$R(t) = p(t) * Q(t)$$

Where p(t) represents the price of the product and Q(t) the quantity. In fact we can define Q(t) as[5]:

$$Q(t) = Q^L(t) + Q^{IL}(t)$$

Therefore $Q^{IL}(t) = \alpha Q(t)$ and $Q^L(t) = (1-\alpha)Q(t)$, so by choosing the $q^j(t),\ j = L, IL$, he is choosing automatically the proportion and the quantity of timber to be harvested. Following the international literature and using the inverse demand, the landowner´s revenue can be written as:

$$R(t) = p^L(t).Q^L(t) + p^{IL}(t).Q^{IL}(t)$$

or

$$R(t) = \left(\overline{p^L} - \lambda q^L(t)\right).q^L(t) + \left(\overline{p^{IL}} - \theta q^{IL}(t)\right).q^{IL}(t) \qquad (1)$$

Where $\overline{p^j},\ j = IL, L$ represent the highest price to be paid when the quantity is zero, i.e., $\lim_{q^j \to 0} p^j = \overline{p^j},\ j = IL, L$. θ, λ represents positive parameters. Note that, increasing θ or λ means that lower will be market price, or otherwise it will be higher.

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[3] This assumption will be relaxed later.

[4] For the sake of simplicity, we suppose that the capital cost or labor cost in the production is zero.

[5] As discussed in Ehui et al. (1990), Van Soest and Lensink (2000), and Fredj (2006), the timber can be extracted in general form or selective logging's methods; however for sake of simplicity we ignore these differences and suppose that there is non-selective logging. Another note that we must bear in mind is that, following Fredj (2006), $Q^j,\ j = I, IL$, is the rate of deforestation, therefore, if we suppose that there n valuables stems per unit of land, the quantity of wood/timber produced is equal to *n* times $q^j,\ j = I, IL$, therefore if we normalized *n* to unity, we have that: $q^j(t) = Q^j(t),\ j = I, IL$, that is, the quantity of timber is the same of the rate of deforestation.

The forest size develops according with following dynamic equation (we omit from now on the time argument for the sake of simplicity of writing):

$$\dot{X} = -\left(q^{L} + q^{IL}\right) + rX \qquad (2)$$

Where X represents the stock of forest owned by the landowner, and *r* represent the rate of natural growth of forest. Thus the net growth of the forest at each point in time will depend of the weight of each element – more logging will decrease the stock of forest and more natural rate of growth, *r*, will increase the stock of forest. Note that we used a linear case of growth equation, this is different of the logistic function usually used in these studies – in the present paper the forest would increase indefinitely over time, that is, without the loggings from the landowner, the forest would increase - there was no "natural" steady state in the future for the stock of forest. This assumption is not so far from the reality, because we are talking about rain forest in the tropics, hence the natural causes for logging as fire, etc. don't exist. Hence, in the present case it is the human activity rather than the natural causes that is key element in constraining the growth of the natural resource (see Long et al., (2001) for the case of linearity of the kinematic function in the case of fish biomass).

Given the controls variable and the kinematic equation, we define that the landowner wants the maximize the streams of revenue over the an finite horizon, $t \in [0, T]$, therefore his optimization problem is[6]:

$$Z = \underset{\{q^{L}, q^{IL}\}}{MAX} \int_{0}^{T} R(t)dt + \phi X(T) \qquad (3)$$

Where $\phi F(T)$ represents the "scrap" value function for the final period. Following Fredj (2006) We suppose that the scrap value is linear in its arguments, even though it is usually non-linear function[7].

The landowner optimization depends of the following constrains:

$$\dot{X} = -\left(q^{L} + q^{IL}\right) + rX$$

And the transversality condition (see Long et al.,(1992) about this condition):

$$\frac{\partial}{\partial X}(\phi X(T)) = \phi = \pi(T) \qquad (4\text{ a})$$

And the boundary constrain

$$X(0) = x_0 \qquad (4\text{ b})$$

Where $\pi(T)$ is the co-state variable at terminal date.

The optimal solution for the landowner is found through the application of Pontryagin's maximum principle. In such way we organize the following Hamiltonian:

$$H(t, q^{j}, \pi, X) = \overline{p^{L}}.q^{L} - \lambda\left(q^{L}\right)^{2} + \overline{p^{IL}}.q^{IL} - \theta\left(q^{IL}\right)^{2} + \pi\left[-\left(q^{L} + q^{IL}\right) + rX\right] \qquad (5)$$

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[6] We do not incorporate the discount factor because we suppose that the *T* is not so large. Therefore the loss in terms of analysis is minimum. However, further explanation can be found in Freyd et al. (2006).

[7] About these issues see Freyd et al. (2006).

Solving the Hamiltonian and assuming an interior, will provide the optimal path for the controls variables and the state variable.

Remark 1: Even though we didn't use an logistic function for the state variable, the second order differentiation guarantees that the controls found are optimal, given that the first principal minor is negative, i.e., $\frac{\partial^2 H(.)}{\partial^2 q^j} < 0,\ j = IL$ [8].

Results1: The stock of forest will decline in $t \in [0,T]$, and this decline will depend of the amount of the logging and the natural growth rate. Given that the logging rate will depend of the parameters of the inverse demand, these will indirectly determinate the stock of forest in the final period (**please see figures - 2, 3 for numerical illustration**).
Proof: supposing an interior solution, the first order condition (FOC) for this problem is:

$$\overline{p}^{IL} - 2\theta q^{IL} - \pi = 0 \tag{6}$$

$$\overline{p}^{L} - 2\lambda q^{L} - \pi = 0 \tag{7}$$

$$\dot{\pi} = -r\pi \tag{8}$$

$$\dot{X} = -\left(q^{IL} + q^{L}\right) + rX \tag{9}$$

From the co-state differential equation, and using the condition (4 a), we find that:

$$\pi(t) = \phi e^{r(T-t)} \tag{10}$$

For the first condition in q^{IL}, q^{L}, we know that the quantity of timber logged depends of the co-state variable or the shadow price of unity of forest. Substituting the co-state equation in the quantity we find that

$$q^{IL} = \frac{\overline{p}^{IL} - \phi e^{r(T-t)}}{2\theta} \tag{11}$$

$$q^{L} = \frac{\overline{p}^{L} - \phi e^{r(T-t)}}{2\lambda} \tag{12}$$

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[8] In the present case, following Leonard and Long, (1992), to analyze the sufficient condition for a maximum, we must see if the Hamiltonian is concave. Hence, is straightforward to show that the Hessian associate with the Hamiltonian is negative definite, and it is given by

$$H = \begin{bmatrix} -2\theta & 0 \\ 0 & -2\lambda \end{bmatrix} = H_2$$

Hence, given that $\theta > 0, \lambda > 0$ by definition, we have that the determinant of the second principal minor is positive, i.e., $|H_2| > 0$. Therefore, the optimal solution found is a maximum.

Substituting these equations in the kinematic equation, and supposing that $\lambda = \theta$, we can obtain the time dependent equation for the stock of forest given by

$$X(t) = \frac{4\lambda r x_0 e^{rt} + 2\left(\overline{p}^L + \overline{p}^L\right)\left(1 - e^{rt}\right) + 2\phi\left(e^{r(T+t)} - e^{r(T-t)}\right)}{4\lambda r}$$

As we can see at $t = T$ we have

$$X(T) = \frac{4\lambda r x_0 e^{rT} + 2\left(\overline{p}^L + \overline{p}^L\right)\left(1 - e^{rT}\right) + 2\phi\left(e^{2rT} - 1\right)}{4\lambda r}$$

Easily, by using static comparative, we see, by using linear approximation, that the stock of forest in the final period will depend, ceteris paribus, of price parameters and the natural rate growth the forest.

$$\Delta X(T) = \Delta r \frac{\partial X(T)}{\partial r} + \Delta p^j \frac{\partial X(T)}{\partial p^j},\ j = IL, L$$

Using calculus we can see that (this conditions is always satisfied)

$$\frac{\partial X(T)}{\partial r} = \frac{16(\lambda r)^2 T x_0 e^{rT} + 8e^{2rT}\phi\lambda(Tr-1) + 8e^{rT}\lambda\left(p^{IL} + p^L\right)(Tr-1) + 8\lambda\left(\phi - \left(p^{IL} + p^L\right)\right)}{16(\lambda r)^2} > 0$$

And

$$\frac{\partial X(T)}{\partial p^*} = \frac{\left(1 - e^{rT}\right)}{2\lambda r} < 0$$ [9]

Where $p^* = p^{IL} + p^L$.

Hence, everything constant, the net effect in the final stock of forest will depend of the weight of each effect. That is we have:

$$\begin{cases} \dfrac{\partial X(T)}{\partial r} > \dfrac{\partial X(T)}{\partial p^j} \rightarrow \Delta X(T) > 0,\ j = IL, L \\ \dfrac{\partial X(T)}{\partial r} < \dfrac{\partial X(T)}{\partial p^j} \rightarrow \Delta X(T) < 0,\ j = IL, L \end{cases}$$

Where we suppose $\Delta r > 0, \Delta p^* > 0$.

This concludes the proof. ■

[9] We have defined the inverse demand as $p^{IL}(t) = \overline{p^{IL}} - \theta Q(t)$ for the illegal market and $p^L(t) = \overline{p^L} - \lambda Q(t)$ for the legal market, therefore, the price will be affect jointly by $\overline{p^j}$ or the parameters λ and θ. Hence, to see the effects of price in the stock forest we can do it by supposing the change in one of this parameters. For the present case we are supposing the change in the intercept of the inverse demand function.

As we can see in the figures 2 and 3, the increasing of the natural rate of growth will increase the amount in the forest at final stage (**Figure-2**). This result is intuitive given that, through the kinematic equation we can see that the natural rate have a positive impact in the growth rate for the forest. For se same reason, as expected theoretically, the increased price at market (**figure-3**) will increase the logging level, and therefore, decreasing the amount of forest in the final period. As we can see in both figures, for short horizon the stock of forest is increasing and after some point it will turn decreasing. The reason for this is the different weight of the impact of the natural rate of growth and the quantity logged over the period. As we can see, for shorter period the impact of the natural rate of growth is higher that the quantity of timber logged, and this explain why the amount of forest is growing up. However for larger period the quantity logged will overcome the weight of the natural growth´s impact and therefore the size of the forest will decrease.

Therefore the quantity logged, in both market, in $t \in [0,T]$ will depend of the inverse demand parameters and natural growth rate of the forest. In the present case the total logged quantity will be (considering that $\lambda = \theta$)

$$Q(t) = \frac{\left(\overline{p}^{L} + \overline{p}^{L}\right) - 2\phi e^{r(T-t)}}{2\lambda} \tag{13}$$

According with optimal path of the quantity for both market we see (**please see figure-1 for numerical simulation – Appendix C**) that, the logged quantity will increase over time, for $t \in [0,T]$. Given the constant rate of growth, we find that, in the finite horizon specified, the stock of forest will decrease over the time.

Result 2: The amount of "*environment*" benefits received from forest, for all $t \in [0,T]$, is higher when the lower is the price.[10] **(see figure-3**).

Proof:

To proceed to proof this result we must bear in our mind that, the benefits from the environment depends of the stock of the forest (for instance, see Green Peace. 2008; Amacher et al., 2006), hence it is normal to think that during all $t \in [0,T]$, the magnitude of the forest's benefits will fluctuate according with the stock of forest at each time and not only in $t = T$. The explanation is straightforward – the forest stock can be high for any $t \in [0,T)$ and rapidly became low in the neighborhood of $t = T$. In this case, if we analyze just the final stock we could underestimate the real benefits of the forest over the time. Hence, to not engage in such problem of underestimation or overestimation of the real "*environment*" benefits of the forest, the best manner deal with this problem is to calculate the total forest's areas over the period.

Let's suppose that the "*environment*" benefits from the forest are a linear function of the stock of forest, therefore we can write the benefit function as $BEN(t) = a + bX(t)$. Supposing that $a = 0, b = 1$, we have that $BEN(t) = X(t)$. Hence, to proof the statement made in result 2, we must derived the respective area of the forest for the all period, i.e., $t \in [0,T]$.

[10] This proof suppose that $\lambda = \theta$.

First step: As demonstrate earlier, substituting the optimal for the controls in the kinematic equation, and integrating we can find the optimal path for the state variable (after some algebraic manipulation), is given by (**more details in the Appendix A**)

$$X(t)=\frac{4\lambda r x_0 e^{rt}+2\left(\overline{p}^{L}+\overline{p}^{L}\right)\left(1-e^{rt}\right)+2\phi\left(e^{r(T+t)}-e^{r(T-t)}\right)}{4\lambda r} \tag{14}$$

Considering that $\lambda=\theta$.

Second step[11]**:**

Let's define the following two areas in the Cartesian plan $\left[X\times[0,T]\right]$

$$d^{1}=\int_{0}^{T}X(t,\overline{p^{IL}}^{1},\overline{p^{L}}^{1})dt \qquad d^{2}=\int_{0}^{T}X(t,\overline{p^{IL}}^{2},\overline{p^{L}}^{2})dt \tag{15}$$

Where $\overline{p^{IL}}^{1}>\overline{p^{IL}}^{2}$ and $\overline{p^{L}}^{1}>\overline{p^{L}}^{2}$. That is, we are supposing that the price would change to higher level given that the intercept will change from $\left(\overline{p^{IL}}^{2},\overline{p^{L}}^{2}\right)to\left(\overline{p^{IL}}^{1},\overline{p^{L}}^{1}\right)$.

Therefore, is straightforward to show that

$$\int_{0}^{T}X(t,\overline{p^{IL}},\overline{p^{L}})dt=\frac{4x_0\lambda r(e^{rT}-1)+\left(2rT-2e^{rT}+2\right)\left(\overline{p}^{IL}+\overline{p}^{L}\right)+2\phi\left(e^{2rT}-2e^{rT}\right)}{4\lambda r^{2}} \tag{16}$$

Thus, given that $2rT+2<2e^{rT}$ (defined for all $t\in\left[0,T\right]$)

We have that

$$d^{2}-d^{1}=\int_{0}^{T}X(t,\overline{p^{IL}}^{1},\overline{p^{L}}^{1})dt-\int_{0}^{T}X(t,\overline{p^{IL}}^{2},\overline{p^{L}}^{2})dt \tag{17}$$

$$d^{2}-d^{1}=\left(2rT-2e^{rT}+2\right)\left(\overline{p^{IL}}^{2}+\overline{p^{L}}^{2}-\overline{p^{IL}}^{1}-\overline{p^{L}}^{1}\right) \tag{18}$$

Thus,

$$d^{2}-d^{1}>0 \tag{19}$$

That is, an increase in the price in both markets will decrease the stock of forest for all specified time horizon. Therefore, given that the benefits at any $t\in\left[0,T\right]$ are a linear function of the forest's stock for any $t\in\left[0,T\right]$, we conclude that lower lever of price means higher stock of forest at any $t\in\left[0,T\right]$,and consequently higher level of "environment" benefits at any $t\in\left[0,T\right]$. This concludes the proof.

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[11] In this analysis we are supposing that the price in both markets will change, but this is not necessary to show the aimed results. That is, we would have the same results if we supposed that just the price of one market has changed while the price the other market remained constant.

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Remark 2: The market price, in both market, have a negative impact on the level of forest stock, since

$$\left(\overline{p}^{L}+\overline{p}^{L}\right)\left(1-e^{rt}\right)<\phi\left(e^{r(T+t)}-e^{r(T-t)}\right)$$

This is always satisfied, given that: $1-e^{rt}<0, \forall t\in[0,T]$

Result 3: The split of the logged quantity of timber between the "legal" and "illegal" market will depend of the difference of price in these markets.

Proof: this can be obtained easily by computing the differences between the obtained logging function, that is $q(t)^{IL}-q(t)^{L}$, we find that

$$q(t)^{IL}-q(t)^{L}=\frac{\left(\overline{p}*-\Omega\right)(\lambda-\theta)}{2\lambda\theta}\underset{>}{<}0 \qquad (20)$$

Where $p*=\overline{p}^{IL}\, or\, \overline{p}^{L}$, $\Omega=\phi e^{rt}$.

The results will depend of the signal of the numerator, and therefore of the signal of $(\lambda-\theta)$ .suppose that $\left(\overline{p}*-\Omega\right)>0$ (this is satisfied for small $t\in[0,T]$ or large $\overline{p}*$ and small ϕ hence the signal of $q(t)^{IL}-q(t)^{L}$ will the signal of $(\lambda-\theta)$, thus we have the following cases

$$\begin{cases} q(t)^{IL}-q(t)^{L}>0 & if\,\lambda>\theta \\ q(t)^{IL}-q(t)^{L}<0 & if\,\lambda<\theta \end{cases} \qquad (21)$$

Hence, the proof is completed. ■

This results shows that if it is more profitable for the landowner to shift from the market with accreditation "legal" market to the "illegal" market, due the higher price paid, he will do so. If $\lambda>\theta$ that means that the price in the legal market, given that $\overline{p}^{IL}=\overline{p}^{L}$, is lower than in the illegal market, therefore the landowner will shift the production from the legal market to the illegal market. In this simple model we introduce the effect of "illegal" timber market in the strategies used by the landowner to maximize his profit. However, we know that in fact there may be auditing for these individuals and that, usually this auditing is made by internal government's institution responsible for environment issues (in the case of Brazil this auditing are made by IBAMA).

The existence of auditing by the government and the profitability of the logging activity, mainly due the existence of the "illegal" market, may contribute to the existence of "active" corruption. Is such framework, it is important to see how corruption may influence the optimal path of deforestation over time. In the following section we used a modified model of the earlier model to analyze this new issue, the corruption.

4. The effects of corruption on Deforestation – Model 2

To model this new scenario, we suppose, first of all, that there are three elements in the game, two active, the landowner and the government official, and one passive: the government itself. The passive one is responsible for defining the roles of the game, in the present case the government is responsible for the salary scheme, penalty function, and for the surveillance of the land, however there is no auditing in the field of the official´s work. Hence, the government's official gains, is only the his constant salary in each $t\in[0,T]$. Beyond the government official, in the active side, there is the landowner, which objective is to find the optimal way for the maximized profit. To model this section we suppose that the differential game (there is just one state variable represented by a single kinematic equation) between the landowner and the official follows the following structure: first, in the last stage the official choose his control variable as way to maximize his net gain, given the strategies used by the landowner. Given the strategies used by the official the landowner will incorporate this optimal path in his problem and choose his best strategy, i.e., amount of timber to be cut down.

The information structure of the differential game used to solve this model is open-loop equilibrium in a stackelberg fashion. We suppose that the leader in this game would be the landowner and the follower would be the government´s official. It is plausible to suppose this structure given that in this kind of illegal behavior we expect that the landowner will interact with the official to see the amount of bribe to give, therefore he is the leader.

4.1. The Government Role

As discussed the government is responsible for the penalty function that the landowner will be fronted with for illegal deforestation.

We assumed that the penalty function is given by

$$F(t) = f(t)Q(t)e(t) \tag{22}$$

Where $f(t)$ represents the penalty rate used for the government (the control variable for the government) for each $t \in [0,T]$, if we suppose that the government is an passive agent in this relationship, therefore we must suppose that in fact (in this case we follow the Staher (1996))[12] the penalty rate is constant over time: $e(t)$ represents the effort that comes from the official. Note that we suppose a linear function of the penalties and the efforts, i.e., when effort is zero the penalty to be paid by the landowner will zero.

Remark 3: we suppose that there is a direct link between the effort applied by the official and the amount of penalties because, if he doesn't apply effort means that the landowner is not audited and therefore no illicit act is found. Thus, for any possible collusion between the landowner and the official the latter must find any illicit act, and this can only be done according with the effort applied.

$$F(t) = \overline{f}Q(t)e(t) \tag{23}$$

We suppose that the penalty rate is constant over time, i.e.,., $f(t) = f_0$: at initial period the government announces his penalty policy and this policy will remain until the final period[13]. For simplicity we suppose that the penalty is a linear function of the quantity of timber.

Remark 4: Usually in many countries there a legal limit volume for the state variable. In Brazil, for instance, each landowner can deforest at 20% percent of his land. The clearing of this 20% of the land will not be charged any penalty. For Mathematic simplicity, in the present model we are supposing that these legal limits are actually physically archived by the landowner, but he has the free will to choose to illegally deforest or not. In this case the control variable for the landowner will be just the quantity for the illegal market, i.e., $Q(t) = q^{IL}(t)$.

3.1 The Government´s official Role

[12] He used the same approach, in a different case.

[13] In a good sense this is the situation in most part of the countries, for example in Brazil each new government usually begins with announcement of the environment policies (and also others policies) for the all legal period (five years until the next elections).

For sake of simplicity we supposed that the official from the government wants to maximize his utility over finite period. His utility is disjointedly given by the consumption and the cost of his work (effort)

$$U(c(t),e(t)) = c(t) - v(e(t))$$

Where we define the consumption at each $t \in [0,T]$, as $c(t) = w_0 + B(t)$, therefore his objective is to maximize the following function value

$$V^1 = \int_0^T \left[\left(w_0 + B(t) \right) - v(e) \right] dt \qquad (24)$$

Where w_0 indicates the salary paid for $t \in [0,T]$, and it is constant (this assumption is true for most part of world – public officials received a constant salary per period). However, in case of illegal deforestation he can be bribed by the landowner and receive a non-negative amount, $B(t)$. We define the bribe paid as function of the penalty function adopted by the government.

Remark 5: we could make the bribe paid as function of the profit, however we are supposing that the official doesn't know the amount of the landowner profit, but he knows the amount of fine that would be paid by the landowner given the quantity of wood arrested/logged.

Given remark 5, the bribe function is defined as

$$B(t) = b(t)F(t)$$

Where $b(t)$ represents in an economic sense, the bargaining power of the official. For sake of simplicity we suppose that the bargaining power is constant over the specified period, hence $b(t) = b_0$. The bribe function can be stresses in the following way

$$B(t) = b_0 f_0 Qe \qquad (25)$$

To make feasible the game played between the landowner and the official, we defined $b_0 \in [0,1)$, that is, the bargaining power need to be less than one unity. If we had $b_0 = 1$, that would mean that the landowner would pay all the penalties to the official, therefore is this case is better for him to not engage in any collusion with the official, because he wouldn't have nothing to gain. In this case it would be rationally better for him to pay directly the penalties to the government. Given this explanation, we suppose that when $b_0 = 1$, we are presence of the " the optimum state", in the sense that there is no corruption.

4.2. Determination of the Stackelberg equilibrium[14]

To derive the equilibrium in this model, we must bear in mind that the open-loop equilibrium will in fact results in optimal controls time dependent. Hence the use of optimal controls is critical. First we derive the strategy for the official and after the landowner´s strategy. Given these strategies the model can be solved. Given the strategy used by the leader (the landowner), and the path of his co-state variable, we can derive jointly the path of the effort chosen by the official, and the stock of forest.

[14]The use of open-loop structure for the game demands some explanation: usually as stressed by Sethi et al. (2007) the open-loop equilibrium is time inconsistent given the interest of the leader to change his strategies at any time $t \in (0,T]$. However, we suppose that the leader can credibly pre-commit to his strategies at the beginning of the game. This approach has been applied for many works in the literature (Sethi et al., 2007).

The official objective is to choose the level of effort to maximize his gain over a finite period of time[15].

$$\max_{\{e\}} \int_0^T \left[\left(w_0 + f_0 b_0 e Q \right) - v(e) \right] dt$$

We suppose a quadratic form for the cost function, that is $v'(.) > 0, v''(.) > 0$

To obtain the optimal path for his control we applied the maximum principle. Hence for the official we have the following reduced Hamiltonian[16]

$$H(.) = w_0 + f_0 b_0 Q e - e^2 \tag{26}$$

From the Hamiltonian we know that the optimal path for the official control will be

$$e = \frac{b_0 f_0 Q}{2} \tag{27}$$

Result 4: The effort applied by the government official will depend, essentially, of the amount of bribe. And therefore of the quantity of timber, his bargaining power, and the penalty rate applied by the government.

This result is intuitive, given that the bribe that he (the official) will receive will depend of the amount of penalties, and the later will depend of the amount logged, the agent will apply more effort to discover the illicit act. The same analogy can be applied to the bargaining power and the rate of penalties.

As a leader the landowner will incorporate this result in his optimization problem. The landowner objective is to choose the quantity of timber to maximize his profit, given the kinematic equation showed in the earlier section

$$Z = \underset{\{Q(t)\}}{MAX} \int_0^T \left[R(t) - b_0 F(t) \right] dt + \varphi X(T)$$

s.a. $\dot{X} = -Q + rX$ (28)

$$\mu(t) = \mu_0 e^{-t} \tag{29}$$

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[15] Note that if we had no bribe for the official, the maximum principle would mean that

$$2e = 0$$

Therefore, the equilibrium would be trivial, and the optimal effort chosen by the official would be zero for any $t \in [0, T]$.

[16] We refer this Hamiltonian as a reduced from, because we didn't included the co-state variable for the official. In fact, given that the control variable, from perspective of the official, is influenced by the official controls, this simplicity will not change the mains results in this section. If we had added the co-state equation in his Hamiltonian we would have

$$H(.) = w_0 + f_0 b_0 Q e - e^2 + \mu(-Q + rX)$$

Applying the maximum principle we would find that the path way of the co-state variable-time dependent is

$$\mu(t) = \mu_0 e^{-rt}$$

where μ_0 the initial value of the co-state variable.

$$\pi(T)=\frac{\partial\left[\varphi X(T)\right]}{\partial X}=\varphi \tag{30}$$

Where $\varphi X(T)$ represents our scrap value function.

The short Hamiltonian for the landowner will be (**see Appendix B for more information about this Hamiltonian**)

$$H(t,Q,\pi,X)=\overline{p}Q-\theta Q^2-\frac{f_0^2b_0^2Q^2}{2}+\pi\left[-Q+rX\right]$$

Assuming an interior, and applying the maximum principles we find that

$$\overline{p}-2\theta Q-f_0^2b_0^2Q-\pi=0 \tag{31}$$

$$H_\pi=\dot{X}\Rightarrow\dot{X}=-Q+rX \tag{32}$$

$$\pi=-H_X\Rightarrow\dot{\pi}+\pi r=0 \tag{33}$$

$$\mu(t)=\mu_0e^{-rt} \tag{34}$$

From the stated above condition we find that

$$Q=\frac{\overline{p}-\pi}{2\theta+f_0^2b_0^2} \tag{35}$$

The results show that the quantity of timber logged will depend positively of the market price, negatively with the shadow price of each unity of forest, and negatively of the bargaining power of the official and the penalty rate.

Result 5: The open-loop stackelberg equilibrium in this game, is given the pair of strategies time-depend

$$Q(t)=\frac{\overline{p}-\varphi e^{r(T-t)}}{2\theta+f_0^2b_0^2} \tag{36}$$

$$e=\frac{b_0f_0\left[\overline{p}-\varphi e^{r(T-t)}\right]}{4\theta+2f_0^2b_0^2} \tag{37}$$

And the optimal path for the co-state variable is

$$\pi(t)=\varphi e^{r(T-t)}$$

$$\mu(t)=\mu_0e^{-t}$$

Proof: using condition (33) and integrating it, we find $\pi(t)$, and substituting it in the condition (31) we find $Q(t)$. Finally, we can obtain $e(t)$ by substituting the last one in in equation (27). ■

As we can found the earlier model, the deforestation rate is negatively influenced by the, the scrap value. The deforestation rate is positively influenced by the price in the timber market. However, in this new environment (the potential existence of corruption) the quantity of timber will be affected by the penalty rate and the official bargaining power. Increasing the penalty rate or the bargaining power of the official will decrease the quantity logged. These results are intuitive, given that the official have a real appraisal of the illicit act, increasing the bargaining power will increase the landowner cost, hence lowering the profit. The same analogy can be made in terms of the penalty rate – increasing the penalty rate will increase the potential cost of timber activity and therefore lowering the landowner's profit.

The interesting thing about the result is the effect of the penalty rate and the bargaining power in the optimal effort path by the official. The penalty rate and the bargaining power affects the effort by two channel, the first one is the through the impact directly on the penalties and the share in the penalties. The second channel is due the effects of these parameters in the quantity of timber – the quantity of timber is decreased when we increase these variables. Therefore increasing these variables have a double impact, however, since $b_0 < 1, f_0 < 1$ we have an increase in the effort when these parameters are increased. However if $f_0 > 1$ the increase of the penalty rate decrease the effort, because in this case the negative effect of this penalty rate in the quantity of timber is bigger than the direct, positive effect, of this rate on the level of effort chosen. In our numerical simulations we supposed that $f_0 > 1$.

Results 6: The presence of corruption, i.e., $b_0 < 1$, will decrease the stock of forest in the final period.

Proof:

From the results of the co-state variable from the landowner, the optimal path for the quantity of timber, we can derive the forest stock equation

$$X(t) = \frac{2r\Delta x_0 e^{rt} + 2\overline{p}\left(1 - e^{rt}\right) + \varphi\left(e^{r(T+t)} - e^{r(T-t)}\right)}{2r\Delta} \tag{38}$$

Where $\Delta = 2\theta + f_0^2 b_0^2$

In the final period, $t = T$, we find that

$$X(T) = \frac{2r\Delta x_0 e^{rT} + 2\overline{p}\left(1 - e^{rT}\right) + \varphi\left(e^{2rT} - 1\right)}{2r\Delta} \tag{39}$$

Hence, after some algebraic manipulation, we have that

$$X(T, b_0 < 1) - X(T, b_0 = 1) = \frac{\chi\left(f_0^2 - f_0^2 b_0^2\right)}{\Gamma} < 0 \tag{40}$$

Where $\Gamma = 2r\Delta\left(2\theta + f_0^2\right)$ and $\chi = 2\overline{p}\left(1 - e^{rT}\right) + \varphi\left(e^{2rT} - 1\right)$. ■

Clearly the amount of the forest in the final period will depend of the length of the period, that is of the signal of χ. As we increase the period, the amount of forest, for small value of b_0, will decrease over the period (**see figure-3 for numerical simulation).** Hence, the proof is concluded. As we can see the decrease of the of the bargaining power of the official will increase the quantity of timber and this will lead to higher level of deforestation, i.e., the stock of forest will tend to zero in a shorter period of time.

If the bargaining power of official is important for the final stock of forest, it will interesting to check which variables can influence the bargaining power of the official. It is normal to suppose that one of the variables that may influence the official's bargaining power is the salary received by the agent. Intuitively we can expect that lower the salary the lower is the bargaining power. The explanation is very intuitive, when the official has a low salary, and he finds an illicit, the landowner can easily convince him to receive any small amount of bribe given is salary situation – in this case he has more to lose if he doesn't accept the bribe. However, if we suppose that the salary if high he has little to lose by not accepting the bribe.

Result 7: In the presence of potential corruption, the static comparative shows that the salary scheme played by the government will determinate positively the bargaining power of the official, and indirectly (negatively) the stock of forest in final period.

Proof:

The present case is a subcase of the previous one, therefore, the proof follows the previous one, we investigate the forest stock in two different salary scheme, namely w_1 and w_2 where $w_1 > w_2$, hence $b_1(w_1) > b_2(w_2)$ because we are supposing a linear bargaining function, i.e., $b_i = \sigma(w), and \sigma_w > 0, \sigma_{ww} = 0$.

$$X(T,b_1) - X(T,b_2) = \frac{\Upsilon\left(f_0^2 b_2^2 - f_0^2 b_1^2\right)}{\Psi} > 0 \qquad (41)$$

Where $\Psi = 2r\Delta_1\Delta_2$ and $\Upsilon = 2\overline{p}\left(1-e^{rT}\right)+\varphi\left(e^{2rT}-1\right)$, $\Delta_1 = 2\theta + f_0^2 b_1^2$ $\Delta_1 = 2\theta + f_0^2 b_2^2$.

since $2\overline{p}\left(1-e^{rT}\right)+\varphi\left(e^{2rT}-1\right) < 0$. Hence, the proof is completed. ■

Remark 6: It is important to analyze that the signal of the difference above depends of the signal of $2\overline{p}\left(1-e^{rT}\right)+\varphi\left(e^{2rT}-1\right)$, as we can see this function is not always negative however it will negative before t=2, hence for any time $T \in \left(2, T^{\infty}\right)$ this expression is true. Note that this assumption is satisfied given that, as we supposed in the beginning, these policies are applied, by the government, in ate least one election, therefore for our model $T \geq 4$.

The results above shows that the strategy of the government to use independent salary scheme, i.e., salary independent of output or effort, can increase the possibility of bribes, and lower salary will increase the rate of deforestation. Our results are similar with corruption literature, in the sense that the salary scheme is a good policy that can be used to avoid the problem of corruption, and in this particularly case to fight illegal deforestation.

5. Concluding Remarks

The existence of countries with huge forest areas and with high level of corruption seems to be a perfect environment to promote non-sustainable deforestation. Empirical evidence in a national level shows that countries with higher level of corruption, on average, have higher rate of deforestation. In Brazil, attention are concentrated in the so called direct causes of deforestation, however, the Amazon forest includes the poorest regions or states in Brazil – North and Northeast. On average, is this states the institutions are weaker and this situation helps the existence of potential corruption. In recent years, through the media, it has been an increasing delivering of cases of corruption where officials from IBAMA were involved.

Despite the international evidence (and the national cases of corruption) of the effect of weak institutions in the rate of deforestation, researches in Brazil didn't give the adequate importance to the matter. In this sense, this paper is the first attempt to show how the corruption may have a positive effect in the rate of deforestation and therefore decreasing the stock of forest.

The use of differential game is not new, however, to our knowledge, this is the first work that tries to model it (the effect of corruption) using the methodology of differential games. Even though, there a lot of empirical (qualitative and quantitative) work that analyzed the effect of corruption in the deforestation, to our knowledge, this work is the first work that models the effect of corruption in the deforestation.

The model developed here is based in some critical assumptions used as a matter of mathematic simplicity. First we supposed a control problem defined in a finite period of time. We suppose that the time range is relatively small, and in such case there is no need to use discounted values. In second place, we supposed that kinematic equation is linear, therefore without any quantity logged by the landowner the forest would growth without limits over the years. For the case of forest stock this approach seems to be real given that in tropic forest, the possibility for natural degradation of the natural forest is very limited.

The model that we used is based on the "representative agent" fashion; however the results are robust if we suppose that may exist a huge number of similar agents. Our models focus mainly in the case of Brazil, however given that many others countries, with huge tropical forest, shares the same institutions problems, the findings in this paper can be generalized for such countries.

The results from our model are consistent with the main empirical evidence, that corruption increases the rate of deforestation over given period of time. The model shows that the salary scheme can be an important tool, for the government, to control the potential corruption behavior. When the government's official receive a lower salary and this salary is constant over time, that is independent of his effort, his bargaining power in the collusion game is lower and therefore, he is more likely to receive an small amount of bribe, because in this case his potential loss given the situation that he rejects the bribe is higher. Note that, our models considers that landowner knows that he will be fined, however he knows that he can bribe the official. Hence, our results shows that the existence surveillance cannot have a guaranteed results, given that if there is corruption, the bribe paid will lower than the fine, and this bribe will lower as lower is the salary paid by the government.

Our model suggest that the existence of "bad" salary scheme controlled by the government, and high international price for timber will constitute the worst possible scenario – in this case corruption is more likely to happen and the deforestation rate will be the in the highest level. These findings help to understand why and how the salary scheme played by the governments and the international price of timber, jointly promotes higher level of deforestation around the world. However, through these findings, we must advocates that the policies adopted by the governments must internally solve the agency problem that creates a potential environment for corruption behavior. Finally, our model suggests that any policy adopted by the governments must include two main approaches, first dealing with the "direct" causes of deforestation and in second place dealing with the indirect cause of deforestation.

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6. Appendix.

A) Deriving the stock forest equation

To derive the equation, first of all we investigate the kinematic equation, that is given by

$$\dot{X} = -\left(\bar{q}^{L} + \bar{q}^{IL}\right) + rX$$

Where $\bar{q}^{IL} = \dfrac{\bar{p}^{IL} - \phi e^{r(T-t)}}{2\theta}$ and $\bar{q}^{L} = \dfrac{\bar{p}^{L} - \phi e^{r(T-t)}}{2\lambda}$

Note that $\phi e^{r(T-t)}$ represents the solution for the co-state variable in the equation (8) we may see that

$$\dot{\pi} + r\pi = 0$$

Hence, the solution this first order differential equation is given by

$$\pi(t) = e^{-\int r dt}\left[A\right] \tag{42}$$

Where *A* is a constant. Using the condition (4 a) we can easily find the solution expressed in equation (10).
Using (11) and (12), and substituting in the kinematic equation (9) we obtain

$$\dot{X} - rX = -\left(\frac{\overline{p}^{IL} - \phi e^{r(T-t)}}{2\theta} + \frac{\overline{p}^{L} - \phi e^{r(T-t)}}{2\lambda}\right) \tag{43}$$

Hence, the solution for the stock of forest is given by

$$X(t) = e^{\int r dt}\left[B - \int\left(\frac{\overline{p}^{IL} - \phi e^{r(T-t)}}{2\theta} + \frac{\overline{p}^{L} - \phi e^{r(T-t)}}{2\lambda}\right)e^{-\int r dt} dt\right] \tag{44}$$

Using the method of integral by substitution and the condition (4 b), and supposing that $\lambda = \theta$, after some algebraic manipulation we obtain the equation (14).
Note that to obtain the equation (38) we use the same methodologies, given that they are similar there is no need to present it here.

B) Deriving the conditions (31)-(34)

In the text we presented a short version of the Hamiltonian when we were analyzing the optimal problem for the landowner. However, the open-loop stackelberg equilibrium requires that we introduce, in the leader problem, a co-state variable for the co-state variable of the follower, thus, in this case, the landowner Hamiltonian should be

$$H(t,Q,\pi,X) = \overline{p}Q - \theta Q^2 - \frac{f_0^2 b_0^2 Q^2}{2} + \pi\left[-Q + rX\right] + \gamma\left[-\mu r\right] \tag{45}$$

Where γ represent the landowner's co-state variable related with the follower's co-state variable.
Given this expanded Hamiltonian, the maximum principle are

$$\overline{p} - 2\theta Q - f_0^2 b_0^2 Q - \pi = 0 \tag{46}$$

$$H_\pi = \dot{X} \Rightarrow \dot{X} = -Q + rX \tag{47}$$

$$\pi = -H_X \Rightarrow \dot{\pi} + \pi r = 0 \tag{48}$$

$$H_\gamma = \dot{\mu} \Rightarrow \dot{\mu} = -r\mu \tag{49}$$

$$\gamma = -H_\mu \Rightarrow \dot{\gamma} - r\gamma = 0 \tag{50}$$

Note that for the present model, since there no direct effects of the follower's optimal response in to the forest kinematic equation, the inclusion of the condition (49) and (50) have no effects on the optimal path for the quantity of timber or the landowner forest's co-state variable. Hence, we omitted in the text.

C) Numerical Simulations

The numerical simulation follows, for some parameters, the same values found in Freyd et al. (2006) and Van Soest and Lensink (2000). The parameters used are

$$\varphi = 17000 \quad T \geq 5 \quad \bar{p} = 50000 \quad \theta = 20 \quad \lambda = 20 \quad \bar{p}^{IL} + \bar{p}^{IL} = 95000$$

$$r = 0.2 \quad f_0 = 2 \quad X(0) = 2000 \quad \phi = 9000 \quad \bar{p}^{IL} = 50000$$

Figure-1: Total Quantity of timber logged in both market.

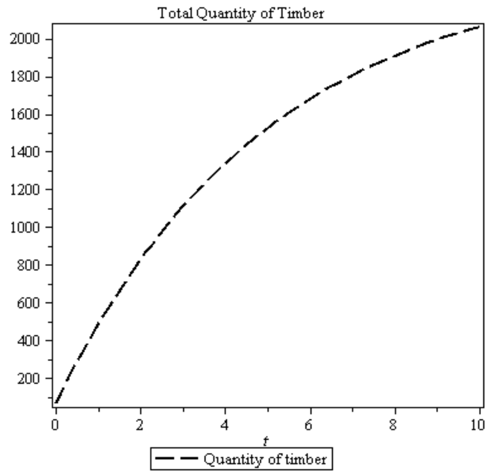

Figure-2: Stock of Forest over period: the effect of different rate of natural growth.

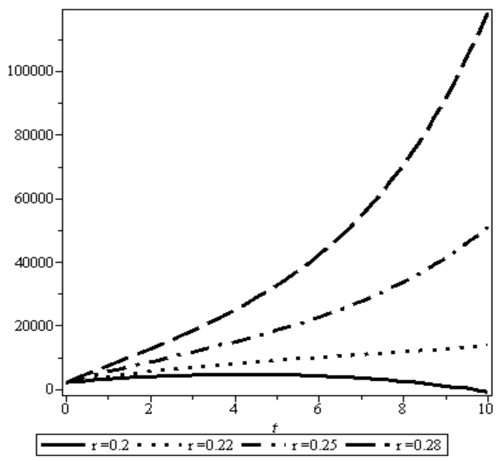

Figure-3: Stock of Forest over period: the effect of higher price in the both market.

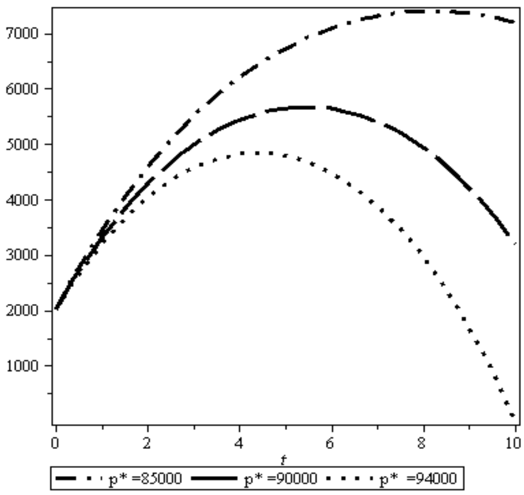

Figure-4: Stock of Forest over period: the effect of different salary scheme