Purchasing Power Parity: A Non-Linear Reversion Model for Brazil

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Abstract

This article applies the smooth transition autoregressive nonlinear model (STAR) proposed by Granger and Terasvirta (1993) to the Brazilian real exchange rate (RER), aiming to test the validity of the purchasing power parity (PPP) for Brazil. Initially a review of Brazilian and international literature is presented, describing the development of the econometric techniques that have been applied in the tests of the PPP. After that, the STAR model is presented and used for the PPP test for Brazilian data between 1959 and 2004. The results indicate that CPI-based RER reveals nonlinear behavior, being stationary when distant from the equilibrium and with an explosive tendency when close to parity. The WPI-based RER has shown linear stationarity, rejecting the null hypothesis of unit root.

Keywords: Exchange Rate, Purchasing Power Parity, Nonlinear Models
JEL Classification: F31, C51

Revista EconomiA *December 2004*

Este artigo aplica o modelo não linear auto regressivo com transição suavizada (STAR) proposto por Granger e Terasvirta (1993) à taxa de câmbio real brasileira, com o objetivo de testar a validade da paridade do poder de compra (PPP) para o Brasil. Inicialmente é realizada uma revisão da literatura nacional e internacional, apresentando-se a evolução das técnicas econométricas empregadas nos testes da PPP. Em seguida o modelo STAR é introduzido, sendo utilizado para o teste da PPP para dados brasileiros entre 1959 e 2004. Os resultados apontam que a taxa de câmbio real brasileira formada com base no IPC apresenta comportamento não linear, revelando-se estacionária quando distante do equilíbrio e com tendência explosiva quando próxima à paridade. A taxa de câmbio real apurada com base no IPA mostrou-se linearmente estacionária, rejeitando a hipótese nula de raiz unitária.

* The opinions expressed in this paper are those of the authors and not of the organizations to which the authors belong.

1 Introduction

According to the purchasing power parity (PPP) hypothesis, domestic price levels should be identical when expressed in the same currency. This occurs due to international goods arbitrage, under the assumptions of no-transactions costs and of barriers to international trade, considering perfect information and homogeneity of goods. However, the nonobservation of the hypotheses on which the theory is based results in remarkable PPP deviations, at least in the short run.

The real exchange rate (RER) is defined as a nominal exchange rate that is adjusted by relative prices. The empirical validation of PPP as a long-run relationship is defined by the RER stationarity, and variations in the RER represent PPP deviations. If the RER has a random walk or explosive behavior, the null hypothesis of PPP's non-validity cannot be rejected.

Controversial results have been described in the literature regarding the validity of PPP, i.e., the RER stationarity. The first econometric tests were performed in the 1970s by applying the ordinary least squares regression. Since the tests did not bring dynamics to the estimated equation, the first results often indicated the rejection of the hypothesis. The development of new econometric techniques in the last twenty years has allowed for remarkable improvement of stationarity tests for random variables, such as unit root tests and cointegration tests.

In line with the international literature, the empirical studies on the validity of PPP for Brazil reveal discrepant results with regard to the analyzed period, to the test applied and to the price indices used for the definition of the RER.

The conventional approach used in empirical tests for the RER stationarity assumes that this variable is formed through a linear autoregressive process. However, the presence of nonlinearities in the RER may carry considerable implications for the conventional tests for the validity of PPP (Michael et al. (1997) and Sarno and Taylor (2003)).

The nonlinearity of the RER autoregressive process results from the nonobservation of basic PPP hypotheses, such as the presence of transactions costs and barriers to international arbitrage. This creates a band of PPP deviations in which international trade costs exceed the benefits to be obtained from the arbitrage on the differences between domestic and foreign prices. Therefore, within this band where no international goods-arbitrage is expected to occur, the RER may have a random walk or explosive behavior. Nevertheless, in the presence of deviations greater than the band of inaction, international trade causes the reversion of the RER to its equilibrium at a speed that is proportional to the size of the deviation, thus characterizing the nonlinearity of the adjustment process.

The aim of the present study is to test the validity of PPP for Brazil in the 1959-2004 period by the application of the nonlinear smooth transition autoregressive (STAR) model proposed by Granger and Terasvirta (1993). The empirical tests were performed on the exchange rates estimated through consumer and wholesale price indices, so that the different methods described in the literature could be assessed.

In addition to the introduction, the paper is organized into three sections. Section 2 discusses the theoretical aspects of PPP, reviewing the available literature, showing the results obtained in national and international studies and the improvement of the techniques used to test the theory. Section 3 presents the model used in the current paper and the data used in the empirical tests. Afterwards, the econometric procedures are carried out according to the theory – selection of the autoregressive term, unit root tests, linearity tests, selection of the nonlinear transition function, specification of dummy variables and estimation of the STAR model. The last section summarizes and concludes.

2 Purchasing Power Parity: A Linear Approach

According to the PPP hypothesis, the domestic price levels are identical when expressed in the same currency. So we have:

$$P_t = S_t \times P_t^* \tag{1}$$

where P_t is the domestic price level, P_t^* is the foreign price level and S_t is the nominal exchange rate defined as the domestic price of the foreign currency, all variables expressed at date t. By expressing this relationship in natural logarithm, we have:

$$s_t = p_t - p_t^* \tag{2}$$

The real exchange rate (RER) can be defined as a measure of PPP deviations, and the reversion of the RER to the long-run equilibrium characterizes the validity of PPP as a long-run parity relationship:

$$q_t = s_t - p_t + p_t^* \tag{3}$$

The parity condition expressed by the PPP assumes perfect goods-arbitrage across countries. However, due to factors such as transactions costs, taxes, subsidies, nontariff barriers to foreign trade, existence of nontradable goods and services, imperfect competition between companies, government interventions in the exchange rate market and differences in the composition of consumption baskets and price indices across countries, the PPP would not be verified in the short run, being therefore regarded as a long-run parity relationship.

Empirical studies using long data series have supported the PPP hypothesis (Michael et al. (1997) and Taylor (1995)). However, analyses regarding the floating exchange rate period in industrialized countries after 1973 have yielded controversial results. By applying unit root tests, several authors could not reject the null hypothesis that the RER generating process contains a unit root (Meese and Rogoff (1988)).

Several studies using long series and data panels reveal similar results regarding the half-life of PPP deviations: from 3 to 5 years (Rogoff (1996)). Considering that real shocks (e.g.: changes in technology and preferences) cannot account for most of the short-run volatility of the real exchange rate and that nominal shocks only produce an effect during the period in which nominal prices and wages are stick, a puzzle would be given by the high level of real exchange rate persistence, that is, by the long necessary time for this rate to return to its long-run equilibrium level.

Even though few authors agree that PPP is continuously observed in the real world, many instinctively believe in some PPP variant as a long-run “anchor” for the RER (Rogoff (1996)); therefore, it is used in several macroeconomic theories, at least in the long run.

The empirical evidence of PPP is extremely extensive and the tests have improved concomitantly with econometric methods. The tests used in the linear approach to PPP are divided into five stages: ordinary least squares regressions; unit root test of the real exchange rate; cointegration tests; studies using long data series and data panels.

2.1 Ordinary least squares regression

Absolute PPP establishes that the nominal exchange rate is identical to the division between the relevant price levels of two countries. Relative PPP determines that changes in the exchange rate are the same as the changes in domestic relative prices. Let us consider the following equation:

$$s_t = \alpha + \beta p_t + \beta^* p_t^* + u_t \tag{4}$$

The test of hypothesis $\beta = 1$ and $\beta^* = -1$ is interpreted as the absolute PPP test, whereas the same restriction with first-difference variables represents the relative PPP, that is, the rate of currency depreciation is the same as the inflation difference. Quite often, a distinction is made between the tests, in which β and β^* are identical but have different signs (symmetry condition), and the tests in which they are equal to one and to minus one, respectively (condition of proportionality).

The first PPP tests, performed until the late 1970s, estimated β and β^*. Such tests did not bring dynamics into the estimated equation, and did not make a distinction between short and long run, usually indicating rejection of the PPP.

The first estimates did not investigate the stationarity of the residuals of the estimated equation. If the exchange rate and relative prices are nonstationary and are not cointegrated, then the estimated equation represents a spurious regression and the conventional ordinary least squares method is not valid (Granger and Newbold (1974)). Nevertheless, if the error is stationary, then there is a long-run linear relationship between the exchange rate and relative prices, but conventional statistical inference is not valid due to the bias in the estimated standard deviation.

2.2 Unit root test on the real exchange rate

At this stage, the stationarity of the real exchange rate q_t was assessed as described in equation [3], which imply the validation of PPP in the long run. Otherwise, this rate would contain a unit root and would not tend to revert to a long-run equilibrium level.

From the mid-1980s, studies have employed the augmented Dickey-Fuller (ADF) test (Dickey and Fuller (1979)) to verify whether the real exchange rate contains a unit root:

$$\triangle q_t = \alpha + \rho q_{t-1} + \sum_{j=1}^{p-1} \phi_j \triangle q_{t-j} + \varepsilon_t \tag{5}$$

where $\triangle$ denotes the first-difference operator and ε_t stands for a white noise process.

Testing the null hypothesis $\rho = 0$ means testing whether q_t contains a unit root, which implies the lack of a long-run real exchange rate equilibrium. The alternative hypothesis, in which PPP prevails, requires $\rho < 0$.

The tests performed with the major currencies in the floating exchange rate period (after 1973) have suggested permanent PPP deviations, indicating that the real exchange rate contains a unit root and does not revert to the long-run equilibrium (Enders (1988), Mark (1990) and Taylor (1988)). A thorough review of the tests performed during this period can be obtained in Froot and Rogoff (1994).

The unit root tests applied to Brazilian data show controversial results. Rossi (1991) tests the PPP hypothesis using monthly data between 1980 and 1988 and wholesale price indices (WPI).

According to the author "*It is possible that the analyzed period is not sufficiently long for a more definitive test, since PPP deviations, in particular, could take longer than one decade to be eliminated, especially when there is government intervention in the exchange rate determination, as the case analyzed herein, due to problems with foreign debt.*" In fact, the author could not reject the unit root null hypothesis of the real exchange rate series constructed with the official and black market exchange rates, and with the basket of currencies (weighted by the total number of exports and by the export of manufactured products).

Pastore et al. (1998) applied the unit root tests to Brazilian data between 1959 and 1996. They concluded that the CPI-based real exchange rate (FIPE – Institute of Economic Research) contains a unit root. Likewise, by using the national CPI (IBGE – Brazilian Institute of Geography and Statistics) in the period between 1979 and 1996, the null hypothesis that the series contains a unit root could not be rejected. However, the authors found evidence of stationarity in the WPI-based real exchange rate.

Kannebley-Jr. (2003)[1] conducted unit root tests with monthly data between 1968 and 1994, using real exchange rate series constructed with the CPI (FIPE) and WPI (FGV – Getulio Vargas Foundation). The test results for the relative PPP indicate that this version is not rejected in the Brazilian case, regardless of the price indices used and of the periods analyzed. The results for absolute PPP are controversial. Whereas the WPI-based real exchange rate has a stationary behavior during the 1968-1978 period, the CPI-based real exchange rate contains a unit root. When the whole period (1968-1994) is analyzed with a structural break in the level of the series, the WPI-based exchange rate has a stationary behavior. The unit root null hypothesis cannot be

[1] Kannebley-Jr. (2003) performs a thorough review of PPP tests for Brazil.

rejected at a 10% significance level for the CPI-based exchange rate.

2.3 Cointegration

Cointegration was originally developed by Engle and Granger (1987). According to the theory, two nonstationary series, integrated of the same order, are cointegrated if there is a linear combination between them that is stationary [2]. In this case, the nonstationarity of a series is exactly compensated for the nonstationarity of the other one, and a long-run relationship is established between both variables. In the exchange rate analysis, if s_t and $\pi_t = (p_t - p_t^*)$ are stationary after being differentiated d times, that is, they are integrated of order d or $I(d)$, then the linear combination $z_t = s_t + k\pi_t$ will also be $I(d)$ if the real exchange rate presents a random walk process.

In the PPP context, s_t and π_t are expected to be $I(1)$ and z_t is expected to be $I(0)$, that is, it is expected to be mean-reverting. In this case, one does not reject the long-run relationship between both variables with a common trend. However, if the hypothesis of non-cointegration cannot be rejected, the estimated regression is a spurious regression, devoid of economic meaning. Under the null hypothesis (of non-cointegration) the error has to be nonstationary.

The main difference between the use of cointegration for the PPP test and for the nonstationarity test of the real exchange rate lies

[2] Stationary term is understood as stationary covariance: a time series has stationary covariance if it has a constant mean, finite variance and if the covariance between two observations is only a function of the time elapsed between the observations.

in the fact that in the cointegration the conditions of symmetry and proportionality ($\kappa = -1$ in equation $z_t = s_t + \kappa\pi_t$) are not imposed[3]. In practice, the cointegration represents the unit root test for the residual (z_t) of the regression between both variables s_t and π_t. However, the critical values tabulated by Fuller (1976) cannot be used to test the nonstationarity of cointegration residuals. The appropriate critical values, computed by Engle and Granger (1987) through the Monte Carlo simulation, have been employed to test the stationarity of cointegration residuals.

The first cointegration tests revealed absence of mean reversion in the real exchange rate for the floating exchange rate period after 1973 (Taylor (1988) and Mark (1990)). Nevertheless, more recent works have demonstrated the rejection of the null hypothesis of non-cointegration across the currencies of the major industrialized economies (Kim (1990) and Cheung and Lai (1994)).

The data used in the studies are crucial for the determination of results. With the application of the wholesale price index (WPI), the null hypothesis of non-cointegration is more easily rejected than in those studies that use the consumer price index (CPI) or the GDP deflator. This can be easily explained by the lower relative participation of nontradables in the WPI, which brings it closer to the ideal index for PPP analysis than does the CPI or GDP deflator.

The cointegration tests used to check the PPP for Brazilian data have yielded inconclusive results. Rossi (1991) obtained evidence of relative PPP utilizing the WPI-based real exchange rate. However, absolute PPP could not be confirmed in his monthly data analysis between 1980 and 1988.

[3] Relaxation occurs due to the barriers to international goods-arbitrage and to the differences between the price indices used and those that are theoretically correct for the calculation of PPP.

Holland and Pereira (1999) performed cointegration tests to verify the validity of PPP for Brazil by assessing whether the real exchange rate has a mean-reverting behavior such that deviations of the exchange rate from the long-run equilibrium are transient. The authors employ monthly observations corresponding to the 1974-1997 period, based on FIPE's CPI, U.S. CPI and WPI and Brazilian WPI. Tests were conducted with the samples, which were split into two periods: 1974-1985 and 1986-1997.

Ambiguous results were obtained due to the price index used and to the period covered by the tests. According to the authors: "*The PPP model cannot be rejected even under such restrictions. In fact, exchange rate fluctuation tends to maintain the real exchange rate, while periods of rampant inflation weaken this conclusion.*"

Marçal et al. (2003) conducted tests with quarterly data between 1980 and 1994 to verify the validity of PPP for Brazilian data. The tests considered the CPI-based (FIPE) and WPI-based (FGV) real exchange rates. By using cointegration techniques, the authors did not find any evidence that the real exchange rate obtained through wholesale prices is stationary. However, in contrast to the literature, there is flimsy evidence of the stationarity of the CPI-based real exchange rate. In their conclusions, the authors provide evidence that PPP deviations are related to the difference between domestic and foreign interest rates.

By using cointegration techniques, Pastore et al. (1998) obtained similar results to those derived from unit root tests. Based on monthly data between 1959 and 1996, they assessed the mean reversion of the WPI-based real exchange rate, but the null hypothesis of non-cointegration could not be rejected when the CPI was used.

2.4 Studies using long data series

The tests applied in the 1980s to assess real exchange rate stationarity (unit root and cointegration tests) had low power to reject the null hypothesis of non-reversion of this variable to the floating exchange rate period after 1973. This occurred because the real exchange rate reverted to the mean during long time periods. Therefore, the analysis of a single exchange rate for a period of approximately 15 years would not provide enough information for the detection of the slow reversion of the real exchange rate.

Several authors used long series (above 80 years) and obtained results that favored the mean reversion of the real exchange rate with a half-life between 3 and 5 years (Lothian and Taylor (1996) and Cheung and Lai (1994)).

For Brazilian data, the null hypothesis of real exchange rate nonstationarity could not be rejected. Zini-Jr. and Cati (1993) tested the absolute PPP for Brazil using annual data between 1855 and 1990, rejecting the validity of PPP. According to Marçal et al. (2003) the data used by Zini-Jr. and Cati (1993) may not be appropriate to test PPP, since data on the Brazilian implicit GDP deflator (with a large number of nontradables) and foreign price indices from two different countries (English and U.S. wholesale price indices) were used.

2.5 Data panel studies

Another way to overcome the low power of unit root and cointegration tests to reject the false null hypothesis of real exchange rate nonstationarity for short data series is to increase the number of exchange rates to be analyzed.

By using the data panel, several authors have obtained results in favor of PPP, even when only the floating exchange rate after 1973 was considered (Flood and Taylor (1996) and Wu (1996)).

According to Sarno and Taylor (2003), the main problem with the data panel for the unit root test of the real exchange rate is that the null hypothesis often considers that all series are generated by unit root testing, increasing the probability of rejection of the null hypothesis if only one of the series under consideration is stationary.

The linear PPP approach has yielded controversial results, regardless of the currencies analyzed, of the periods assessed and of the statistical methods used. Despite extensive research into this issue, the behavior of the RER and the elucidation of PPP deviations still are some of the major areas of investigation in macroeconomics. According to Enders and Dibooglu (2001), the vast literature about PPP shows the importance of the matter and the ambiguity of the conclusions.

3 Purchasing Power Parity: A Non-linear Approach for Brazil between 1959 and 2004

In conventional PPP tests, the null hypothesis considers that the RER generating process contains a unit root, assuming a linear autoregressive process, where adjustment occurs continually and at a constant speed, regardless of the size of the PPP deviation. The augmented Dickey-Fuller (ADF) unit root test is based on an autoregressive process (AR(p)), represented by equation [5]. The null hypothesis represents a unit root process $H_0 : \rho = 0$, whereas the alternative hypothesis $H_1 : \rho < 0$ defines the stationarity of the process. Considering a nonlinear data generating process, the linear approach used in the ADF test has low power to reject the false null hypothesis (Taylor et al. (2001)), not meaning, however, that the nonlinear process is not stable.

The presence of transactions costs and other barriers to international goods-arbitrage may result in nonlinearity of the adjustment process, with important developments regarding the conventional stationarity tests of PPP deviations (Michael et al. (1997) and Sarno and Taylor (2003)). Therefore, the presence of real exchange rate nonlinearity can shed some light on the rejection of the PPP hypothesis in several studies. The nonlinear approach considers the presence of market frictions that restrict the possibility of arbitrage, causing the real exchange rate to adjust towards the long-run equilibrium through a nonlinear process. Transactions costs and barriers to international goods-arbitrage form a band of inactivity within which the price difference across countries does not result in arbitrage. Only price differences that exceed the transactions costs, outside the band of inactivity, allow for arbitrage.

Considering that RER is an aggregate process, formed by several agents with heterogeneous preferences and which therefore do not act simultaneously, and also considering that price indices comprise the prices of different goods, each of them with different international arbitrage costs, the changes in the RER should present nonlinear characteristics with smoothed variations.

The described characteristics suggest that the RER would revert to its equilibrium value with an intensity that is proportional to the size of the deviation from the equilibrium, since large deviations would result in the arbitrage of a larger amount of goods proportionally to small deviations, moving the RER more quickly towards parity. However, when the RER is close to its equilibrium level, offering few opportunities for arbitrage, the

RER could have unit root (random walk behavior) or even an explosive behavior.

3.1 The model

The RER generating process can be characterized by the nonlinear smooth transition autoregressive (STAR) model proposed by Granger and Terasvirta (1993). In this model, the speed of reversion to the equilibrium level varies according to the deviation from parity:

$$q_t = \alpha + \sum_{j=1}^{p} \beta_j q_{t-j} + \left[\alpha^* + \sum_{j=1}^{p} \beta_j^* q_{t-j} \right] \times F\left[\gamma; q_{t-d} - \mu\right] + \varepsilon_t \tag{6}$$

where q_t is stationary, $F[\cdot]$ is a transition function defined between zero and one, $F[\cdot] : \Re \rightarrow [0,1]$, ε_t is an i.i.d. process with mean zero and finite variance. The main property of the model is the "smoothed transition," contrary to the reversion at a constant speed observed in the linear approach.

Transition function $F[\cdot]$ determines the degree of reversion to equilibrium, being governed by parameter γ, which effectively determines the speed of reversion, and by parameter μ, which represents the equilibrium level of q_t. Granger and Terasvirta (1993) and Terasvirta (1994) suggested two transition functions: LSTAR and ESTAR.

The *Logistic* STAR (LSTAR) function is characterized by asymmetric adjustment:

$$F\left[\gamma; q_{t-d} - \mu\right] = \left[1 + \exp\left\{-\gamma\left[q_{t-d} - \mu\right]\right\}\right]^{-1} \tag{7}$$

where parameter γ is positive and measures the speed of reversion of q_t to its long-run equilibrium level associated with PPP (μ), and q_{t-d} is the endogenous transition variable that represents the time necessary for the RER to start its reversion process in response to a shock.

The LSTAR model assumes that the process has an asymmetric behavior in function of the difference between the transition variable and the equilibrium level of q_t. If $q_{t-d} \to -\infty$ we have $F(\cdot) = 0$ and if $q_{t-d} \to \infty$, $F(\cdot) = 1$, with $F(\cdot) = 0,5$ when $q_{t-d} = \mu$. The smaller the parameter γ, the smoother the transition. If $\gamma = 0$, function $F[\cdot]$ becomes constant and the model converts into a linear model. On the other hand, if $\gamma \to \infty$ there is a very quick transition in function of $q_{t-d} - \mu$, with $F[\cdot]$ varying quickly between zero and one.

The exponential STAR (ESTAR) function allows for a symmetric adjustment of the real exchange rate for deviations greater or less than the equilibrium level, that is, it has a symmetric dynamics in function of the difference $q_{t-d} - \mu$:

$$F\left[\gamma; q_{t-d} - \mu\right] = 1 - \exp\left\{-\gamma\left[q_{t-d} - \mu\right]^2\right\} \tag{8}$$

where, just as in the LSTAR function, the parameter γ is positive and measures the speed of reversion of q_t to its equilibrium level μ, and q_{t-d} is a transition variable. When the real exchange rate is in equilibrium ($q_{t-d} = \mu$), we have $F[\cdot] = 0$ and the model results in a linear $AR(p)$ model:

$$q_t = \alpha + \sum_{j=1}^{p} \beta_j q_{t-j} + \varepsilon_t \tag{9}$$

Conversely, if $q_{t-d} \to \pm\infty$ we obtain $F[\cdot] = 1$, and the model converts into another $AR(p)$ model:

$$q_t = \alpha + \alpha^* + \sum_{j=1}^{p} \left[\beta_j + \beta_j^*\right] q_{t-j} + \varepsilon_t \tag{10}$$

In order to analyze some characteristics of the current model, we can reparameterize the STAR equation:

$$\begin{aligned} \triangle q_t = {} & \alpha + \rho q_{t-1} + \sum_{j=1}^{p-1} \phi_j \triangle q_{t-j} \\ & + \left[\alpha^* + \rho^* q_{t-1} + \sum_{j=1}^{p-1} \phi_j^* \triangle q_{t-j}\right] \times F\left[\gamma; q_{t-d} - \mu\right] + \varepsilon_t \end{aligned} \tag{11}$$

where $\triangle q_{t-j} = q_{t-j} - q_{t-j-1}$. Thus, the main parameters are ρ and ρ^*. As previously discussed, the larger the PPP deviation, the stronger the movement towards equilibrium. Therefore, small deviations of q_t from the equilibrium may mean a random walk behavior (or even an explosive behavior), that is, we can have $\rho \geq 0$. However, for large deviations from the real exchange rate, there is an equilibrium reverting process, so we must have $\rho^* < 0$ and $(\rho + \rho^*) < 0$ so that the process is totally stationary.

According to Granger and Terasvirta (1993) and Terasvirta (1994), the autoregression order (p) should be chosen by the inspection of the partial autocorrelation function (PACF). Granger and Terasvirta (1993) and Terasvirta (1994) also suggest a sequence of tests F based on the equation below used to verify the nonlinearity of the data [4], for definition of order (d) of the transition

[4] The null hypothesis of linearity $\gamma = 0$ cannot be directly tested since parameters α^*, β_j^* and μ are not defined under this hypothesis. Consequently, those authors recommend the expansion of Taylor transition function around $\gamma = 0$.

variable and for the selection of the transition function to be used:

$$q_t = \beta_{00} + \sum_{j=1}^{p} \left[\beta_{0j} q_{t-j} + \beta_{1j} q_{t-j} q_{t-d} + \beta_{2j} q_{t-j} q_{t-d}^2 + \beta_{3j} q_{t-j} q_{t-d}^3\right] + \varepsilon_t \quad (12)$$

Regarding the parameter (d) as fixed, the linearity test of the model consists in estimating the equation above through ordinary least squares and testing the null hypothesis:

$$H_{0L} : \beta_{1j} = \beta_{2j} = \beta_{3j} = 0 \quad (13)$$

The null hypothesis assumes that the linear autoregressive (AR) model is the correct specification of the series being tested, against the alternative hypothesis of nonlinearity of data. The linearity test should be repeated with different values for the parameter (d), and the one that minimizes the probability associated with the linearity test should be chosen [5] .

The next stage in the construction of the model is to select the smoothed transition function (LSTAR or ESTAR) that is appropriate for series modeling. To do that, the following tests of hypothesis should be carried out:

$$H_{03} : \beta_{3j} = 0 \quad (14)$$

$$H_{02} : \beta_{2j} = 0/\beta_{3j} = 0 \quad (15)$$

$$H_{01} : \beta_{1j} = 0/\beta_{2j} = \beta_{3j} = 0 \quad (16)$$

The rejection of H_{03} can be interpreted as a rejection of the ESTAR model. The non-rejection of H_{02} represents evidence in favor of the LSTAR model. The acceptance of H_{03} and H_{02} with

[5] Economically, low values are expected for parameter d, and there are no logical reasons for long periods before the real exchange rate is adjusted in response to a shock (Taylor et al. (2001)).

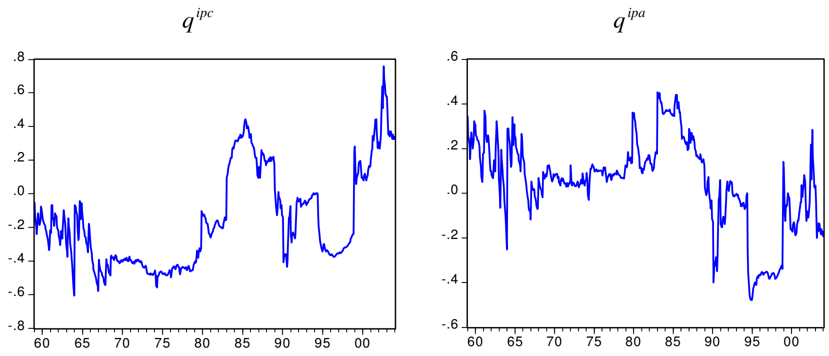

Fig. 1. Real Exchange Rates

rejection H_{01}, indicates the LSTAR model. The non-rejection of H_{01} after the rejection of H_{02} corroborates that the ESTAR model is the most appropriate.

3.2 Data

Monthly observations of the Brazilian and U.S. consumer price index (CPI) [6] and of the wholesale price index (WPI) were made from 1959M01 to 2004M02. The real exchange rates (in natural logarithms) based on the CPI (q^{ipc}) and on the WPI (q^{ipa}) were determined through the nominal exchange rate (defined as the price in national currency of the U.S. dollar at the end of each month). Both were normalized such that $q^{ipc}(1994M06) = q^{ipa}(1994M06) = 0$, and are plotted in Figure 1. All data were extracted from the IPEA database (*www.ipeadata.gov.br*).

[6] The Brazilian Consumer Price Index consists of the FIPE's CPI until 1979M12 and of IBGE's broad CPI from 1980M01 onwards.

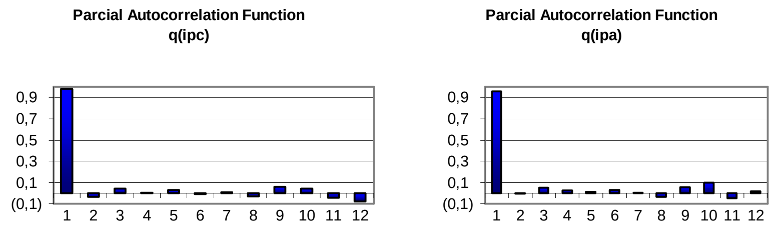

Fig. 2.

3.3 Data analysis

The evaluation of time series was implemented by the analysis of the partial autocorrelation function of both real exchange rates, as shown in Figure 2. The inspection of these functions reveals that, for both analyzed series, only the first partial autocorrelation coefficient is significantly different from zero.

Another method for the evaluation of the autoregressive term involves the Akaike-Schwartz criteria, shown in Table 1. These tests show similar results to those obtained through the inspection of the partial autocorrelation function. This way, we have $p = 1$ for both real exchange rates.

Table 1
Selection Criteria for the Autoregressive Term

Criteria	q^{ipc}	q^{ipa}
Akaike		
$p = 1$	-2,940971	-2,891205
$p = 2$	-2,938055	-2,887604
$p = 3$	-2,939267	-2,888976
Schwartz		
$p = 1$	-2,925121	-2,875355
$p = 2$	-2,914281	-2,863830
$p = 3$	-2,907568	-2,857277

Once the autoregressive term has been chosen, ADF unit root tests were applied to each series in the current study (Table 2). The four price indices used (Brazilian CPI, Brazilian WPI, U.S. CPI and U.S. WPI) and the exchange rate (BRL/USD) contain unit root in the level. The U.S. CPI contains unit root also when evaluated in first difference, being an integrated order 2 series $I(2)$. The Brazilian CPI, the WPIs and the exchange rate have stationarity in first difference at a 1% significance level.

Real exchange rates q^{ipc} and q^{ipa} have discrepant results. In line with a large number of studies on the real exchange rate, including those conducted by Pastore et al. (1998), Kannebley-Jr. (2003) and Marçal et al. (2003)[7] for Brazil, the null hypothesis of could not be rejected at conventional significance levels, showing a stationary behavior only when evaluated in first dif-

[7] Marçal et al. (2003) found flimsy evidence in favor of the real exchange rate stationarity obtained from consumer price indices.

ference. This result would at first represent a violation of PPP for Brazil, since its deviations would be nonstationary. However, the presence of nonlinearities in the process of real exchange rate adjustment can explain why conventional unit root tests (including ADF) are unable to reject the false null hypothesis of nonstationarity (Taylor et al. (2001)).

Table 2
ADF Unit Root Test

	X	$\triangle X$	$\triangle^2 X$
Brazilian CPI	0,099073	-4,363471	-15,92748
U.S. CPI	-1,248741	-2,284465	-10,11485
Brazilian WPI	-0,378752	-4,975268	-24,39869
IPA EUA	-0,869558	-9,051558	-12,59229
BRL/US$	1,167834	-15,24519	-12,01141
q^{ipc}	-1,968982	-22,81845	-13,81990
q^{ipa}	-3,430975	-23,93641	-13,49458

Notes: Statistics t of the ADF test for the null hypothesis of unit root. X denotes the log level of the series, except for the real exchange rates. $\triangle$ is the first-difference operator. All tests include intercept. The critical values for rejection of the null hypothesis are -3.44 at 1%, -2.87 at 5% and -2.57 at 10% of significance (McKinnon (1991)).

Consistently with the PPP hypothesis, q^{ipa} shows a stationary behavior in the level. Similar results were obtained by Pastore et al. (1998), Holland and Pereira (1999) and Kannebley-Jr. (2003) for Brazilian data. In fact, the previously defined linearity test was applied to q^{ipa}, and the linearity of the series could not be rejected. As the nominal exchange rate is primarily adjusted through tradable goods and, therefore, liable to international arbitrage (represented by the WPI) and, later on, the effects are transferred to the nontradable goods (with a large participation in the CPI), the real exchange rate stationarity defined according to the wholesale prices is in line with the PPP hypothesis.

The linearity tests of q^{ipc} and q^{ipa} were performed as previously described and could not reject the null hypothesis of linearity of q^{ipa}. However, q^{ipc} was strongly nonlinear. Table 3 shows the results of these tests for $1 \leq d \leq 12$, indicating the selection of $d = 1$ as a parameter that minimizes the probability of null hypothesis of linearity of q^{ipc}. This way, we have q_{t-1}^{ipc}.

The sequence of hypothesis tests defined in Terasvirta (1994) was applied to the CPI-based real exchange rate (Table 4), and indicated the ESTAR model as the most appropriate for the modeling of q^{ipc}. In line with several studies, the nonlinear adjustment process of the real exchange rate deviations has a symmetric behavior for positive or negative deviations from the equilibrium, and it is economically difficult to justify different speeds of adjustment for the real exchange rate in function of its position in relation to the equilibrium level (Taylor et al. (2001) and Michael et al. (1997)).

Table 3. Linearity Test [12] and H_{0L} [13]

d	1	2	3	4	5	6	7	8	9	10	11	12
q^{ipc}	0,003	0,050	0,045	0,070	0,028	0,005	0,008	0,546	0,534	0,598	0,697	0,718
q^{ipa}	0,626	0,350	0,359	0,490	0,327	0,401	0,569	0,636	0,857	0,833	0,592	0,497

Notes: : Probabilities of test F of the null hypothesis of linearity (Granger and Terasvirta (1993) and Terasvirta (1994)).

The properties of the ESTAR model are appealing to the modeling of the RER since they allow for the smoothed reversion to equilibrium and symmetric adjustments in relation to the deviations above or below parity. Once the autoregressive term has

Table 4
Tests of Hypothesis for the definition of the Transition Function [14]–[16]

Teste	Probabilidade
H_{03}	0,3455
H_{02}	0,0006
H_{01}	0,3003

Notes: Probabilities of the test F (Granger and Terasvirta (1993) and Terasvirta (1994)).

been selected, the unit root tests of the analyzed series have been performed, the nonlinearity of the CPI-based RER has been verified, the parameter d that indicates the time for the implementation of the adjustment has been checked and the transition function to be employed has been defined, the ESTAR model, described in equations [6] and [8], was estimated for the series q^{ipc} using nonlinear least squares based on several initial values, in order to obtain a global optimum:

$$\hat{q}_t^{ipc} = \underset{\substack{(-0{,}7512) \\ [0{,}4529]}}{-0,0065} + \underset{\substack{(6{,}4604) \\ [0{,}0000]}}{1,3333} q_{t-1} + \{\underset{\substack{(0{,}8222) \\ [0{,}4113]}}{0,0072} - \underset{\substack{(-1{,}6968) \\ [0{,}0903]}}{0,3492} q_{t-1}\}$$
$$\times \{1 - \exp\{\underset{\substack{(1{,}5294) \\ [0{,}1268]}}{-84,6057}\} \times \{q_{t-1} + \underset{\substack{(-0{,}2163) \\ [0{,}8289]}}{0,0071}\}^2\}\} + \hat{\varepsilon}_t$$

R-squared	0.963455	Mean dependent var	-0.158007
Adjusted R-squared	0.963114	S.D. dependent var	0.287785
S.E. of regression	0.055272	Akaike info criterion	-2.942109
Sum squared resid	1.637450	Schwarz criterion	-2.894560
Log likelihood	803.3114	Durbin-Watson stat	1.920094

Notes: Statistic t in parenthesis and significance level in square brackets.

The modeled time series (q^{ipc}) is strongly influenced by government interventions such as economic plans, either via the exchange rate (e.g.: changes in the exchange rate regime), or via the influence over the price level (e.g.: price freeze). Thus, dummy variables were defined for the different economic plans implemented in the 1980s and 1990s, as shown in Table 5:

Table 5
Dummy Variables

Variable	Month	Economic Plan
D8603	March 1986	Cruzado Paln
D8902	February 1989	Summer Plan
D9003	March 1990	Collor I Plan
D9102	February 1991	Collor II Plan
D9407	July 1994	Real Plan
D9901	January 1999	End of the fixed exchange rate regime

The estimation results of the model after the introduction of the dummy variables are shown below:

$$
\begin{aligned}
\hat{q}_t^{ipc} = & \underset{\substack{(-0,6765)\\ [0,4990]}}{-0,0042} - \underset{\substack{(-6,6648)\\ [0,0000]}}{0,0461} \times D8603 - \underset{\substack{(-28,4478)\\ [0,0000]}}{0,1548} \times D8902 \\
& - \underset{\substack{(-40,3726)\\ [0,0000]}}{0,2606} \times D9003 - \underset{\substack{(-30,6647)\\ [0,0000]}}{0,1577} \times D9102 - \underset{\substack{(-20,2202)\\ [0,0000]}}{0,1212} \\
& \times D9407 + \underset{\substack{(171,6051)\\ [0,0000]}}{0,4878} \times D9901 \\
& + \underset{\substack{(8,7006)\\ [0,0000]}}{1,2851} q_{t-1} + \{\underset{\substack{(0,6902)\\ [0,4903]}}{0,0042} - \underset{\substack{(-2,0591)\\ [0,0400]}}{0,2996} q_{t-1}\} \\
& \times \{1 - \exp\{\underset{\substack{(1,8073)\\ [0,0713]}}{-75,5251} \times \{q_{t-1} - \underset{\substack{(0,3150)\\ [0,7529]}}{0,0105}\}^2\}\} + \hat{\varepsilon}_t
\end{aligned}
$$

R-squared	0.971680	Mean dependent var	-0.158007
Adjusted R-squared	0.971092	S.D. dependent var	0.287785
S.E. of regression	0.048930	Akaike info criterion	-3.174941
Sum squared resid	1.268922	Schwarz criterion	-3.079843
Log likelihood	872.4090	Durbin-Watson stat	1.959443

Notes: Statistic t in parenthesis and significance level in square brackets.

Based on the estimated values, the following hypothesis was tested in order to provide greater parsimony to the model:

$$H_0 : \alpha = \alpha^* = 0$$

In line with previously obtained results (Michael et al. (1997)), the null hypothesis could not be rejected at the significance levels that are normally used.

Considering this result, the ESTAR model [6] and [8] was estimated again after the introduction of the previously defined dummy variables, represented by:

$$\begin{aligned}
\hat{q}_t^{ipc} =& -\underset{\substack{(-14{,}7593)\\ [0{,}0000]}}{0,0461} \times D8603 - \underset{\substack{(-32{,}6201)\\ [0{,}0000]}}{0,1535} \times D8902 - \underset{\substack{(-39{,}5031)\\ [0{,}0000]}}{0,2602} \\
& \times D9003 - \underset{\substack{(-30{,}6264)\\ [0{,}0000]}}{0,1578} \times D9102 - \underset{\substack{(-31{,}88074)\\ [0{,}0000]}}{0,1253} \times D9407 \\
& + \underset{\substack{(224{,}9073)\\ [0{,}0000]}}{0,4878} \times D9901 + \underset{\substack{(9{,}1120)\\ [0{,}0000]}}{1,3210} q_{t-1} - \underset{\substack{(-2{,}3330)\\ [0{,}0200]}}{0,3354} q_{t-1} \\
& \times \{1 - \exp\{-\underset{\substack{(1{,}9995)\\ [0{,}0461]}}{82,7172} \times \{q_{t-1} - \underset{\substack{(0{,}1389)\\ [0{,}8896]}}{0,0026}\}^2\}\} + \hat{\varepsilon}_t
\end{aligned}$$

The estimated parameter $\beta_1 = 1.3210$ indicates that the process

R-squared	0.971671	Mean dependent var	-0.158007
Adjusted R-squared	0.971192	S.D. dependent var	0.287785
S.E. of regression	0.048846	Akaike info criterion	-3.182012
Sum squared resid	1.269314	Schwarz criterion	-3.102763
Log likelihood	872.3252	Durbin-Watson stat	1.961072

Notes: Statistic t in parenthesis and significance level in square brackets.

has an explosive behavior when the real exchange rate is in its long-run equilibrium level, that is, when $q_{t-1} - \mu = 0$. However, the process is globally stable, since $\beta_1 + \beta_1^* < 1$.

Figure 3 shows the behavior of the transition function $F[\cdot]$ in function of the transition variable $q_{t-d} = q_{t-1}$, based on the estimated parameter γ:

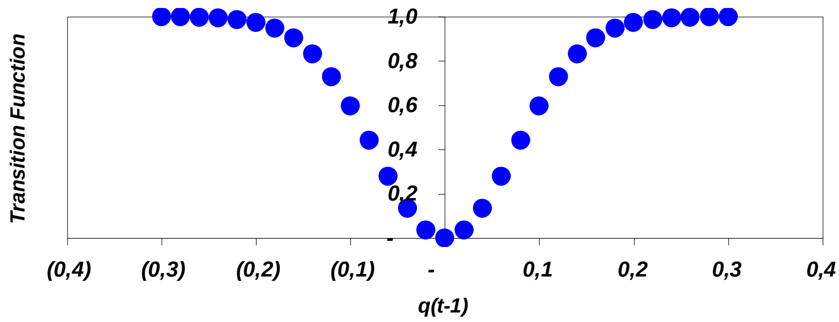

Fig. 3. Values assumed by the transition function in function of the values assumed by q_{t-1}

Note: $F[\cdot] = 1 - e^{-\gamma(q_{t-d}-\mu)^2} = 1 - e^{-82,7172(q_{t-1}-0,0026)^2}$.

One may observe that a deviation of q_{t-1} greater than approximately 0.2 causes the transition function to assume value 1, converting the ESTAR model into a linear autoregressive model.

Figure 4 shows the values assumed by the transition function in the analyzed period:

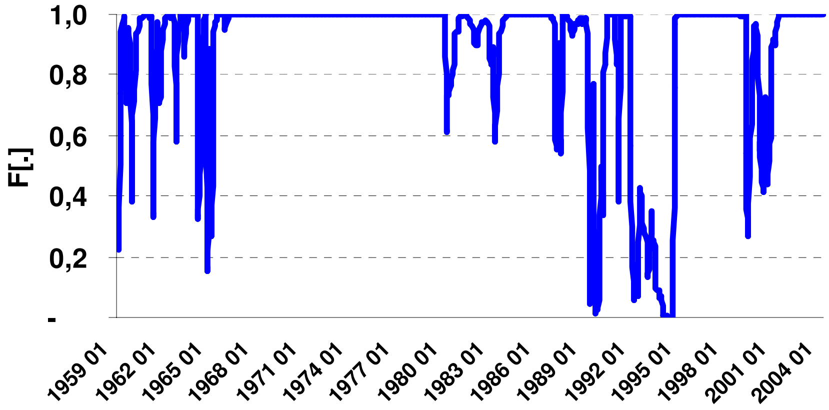

Fig. 4. Values assumed by the transition function in the analyzed period

Notes: $F[\cdot] = 0$ indicates an exchange rate in equilibrium, with an explosive behavior. $F[\cdot] = 1$ indicates an exchange rate distant from equilibrium, with a tendency towards reversion.

The maintenance of this function in its maximum value during most of the period is significant, indicating that the real exchange rate is usually far from its equilibrium level, with a tendency to mean reversion.

The real exchange rate series based on the consumer prices used in the present study (q^{ipc}) was subdivided into two samples (1959:01 to 1979:12 and 1980:01 to 2004:02), in order to observe their properties in the respective periods.

The null hypothesis of unit root was rejected for the first sample (1959:01 to 1979:12). After verifying the stationarity of the series, the linearity test was applied, showing strong rejection of the null hypothesis of linearity of the exchange rate in this period. The application of the appropriate tests of hypothesis indicated the Logistic STAR (LSTAR) transition function for the modeling of the series, with the following estimation [8].

$$\hat{q}_t = \underset{\substack{(-4{,}0284)\\ [0{,}0001]}}{-0,0703} + \underset{\substack{(22{,}1658)\\ [0{,}0000]}}{0,8351} q_{t-1} - \underset{\substack{(-2{,}2320)\\ [0{,}0265]}}{0,1400} q_{t-1}$$
$$\times \{1 + \exp\{\underset{\substack{(1{,}0135)\\ [0{,}3118]}}{-58,7030} \times \{q_{t-1} + \underset{\substack{(14{,}3662)\\ [0{,}0000]}}{0,3544}\}\}\}^{-1} + \hat{\varepsilon}_t$$

R-squared	0.828131	Mean dependent var	-0.360584
Adjusted R-squared	0.825348	S.D. dependent var	0.129260
S.E. of regression	0.054020	Akaike info criterion	-2.979300
Sum squared resid	0.720773	Schwarz criterion	-2.909272
Log likelihood	380.3918	Durbin-Watson stat	1.857684

Notes: Statistic t in parenthesis and significance level in brackets.

Figure 5 shows the values assumed by $F[\cdot]$ in function of q_{t-1}:

[8] $p = 1$ were selected by the inspection of the partial autocorrelation function and $d = 1$ by the linearity test.

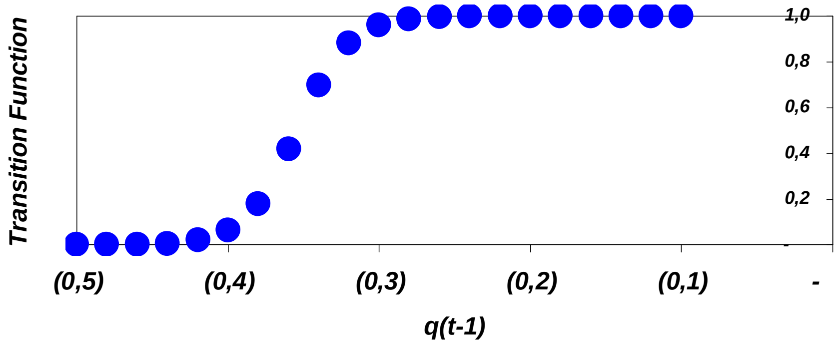

Fig. 5. Values assumed by the transition function in function of the values assumed by q_{t-1}

Note: $F[\cdot] = [1 + e^{-58,703(q_{t-1}+0,3544)}]^{-1}$.

It is possible to observe that the process has a stationary behavior, regardless of the distance between q_{t-1} and μ. If $q_{t-1} > -0,3$ we have $F\left[\cdot\right] = 1$, with the conversion of the LSTAR model into a linear stationary autoregressive model represented by:

$$\hat{q}_t = -0,0703 + 0,6951 q_{t-1} + \hat{\varepsilon}_t$$

Likewise, when $q_{t-1} < -0,4$ we have $F\left[\cdot\right] = 0$, and another linear stationary model is obtained:

$$\hat{q}_t = -0,0703 + 0,8351 q_{t-1} + \hat{\varepsilon}_t$$

The results obtained confirm the stationarity detected in the unit root test on the CPI-based real exchange rate in the first period.

The analysis of the second sample (1980:01 to 2004:02) shows different results from the first period. According to the ADF test, the series contains a unit root, and is integrated of order 1, or $I(1)$.

The inspection of the partial autocorrelation function indicates that only the first autoregressive term is significant, pointing to the selection of $p = 1$. Based on this parameter, the linearity tests were performed with $1 \leq d \leq 12$, indicating the selection of $d = 1$. Despite the non-rejection of the null hypothesis of linearity of the series at the usual significance levels [9], we attempted to estimate the ESTAR and LSTAR models for the second sample. Convergence was obtained only by the ESTAR model, estimated as shown below [10]:

$$\begin{aligned}\hat{q}_t = &\underset{\substack{(-1{,}3053)\\ [0{,}1928]}}{-0,0064} + \underset{\substack{(38{,}3459)\\ [0{,}0000]}}{1,0348} q_{t-1} - \underset{\substack{(-2{,}2331)\\ [0{,}0263]}}{0,0924} q_{t-1}\\ &\times \{1 - \exp\{\underset{\substack{(0{,}5556)\\ [0{,}5789]}}{-11,2334} \times \{q_{t-1} - \underset{\substack{(2{,}1327)\\ [0{,}0338]}}{0,2661}\}^2\}\} + \hat{\varepsilon}_t\end{aligned}$$

R-squared	0.953174	Mean dependent var	0.018026
Adjusted R-squared	0.958601	S.D. dependent var	0.271381
S.E. of regression	0.055217	Akaike info criterion	-2.937986
Sum squared resid	0.868954	Schwarz criterion	-2.874712
Log likelihood	431.0080	Durbin-Watson stat	1.830608

Notes: Statistic t in parenthesis and significance level in square brackets.

The estimated parameters reveal that the CPI-based real exchange rate series, analyzed between 1980 and 2004, has an explosive behavior ($abeta = 1.0348$) when the real exchange rate

[9] Significant level of test F of the null hypothesis of linearity = 0.1429.

[10] There was no convergence of the ESTAR model when the dummy variables corresponding to the economic plans were used.

is close to its equilibrium level, converting into a stationary autoregressive linear model ($\beta + \beta^* = 0,9424$) in the presence of large PPP deviations.

According to the results obtained for the two samples, we may observe that the equilibrium values are considerably different in both periods. In the sample comprising the period between 1959 and 1979, the equilibrium real exchange rate corresponded to -0.3544, whereas in the 1980-2004 period it was 0.2661. When the whole series was evaluated (1959-2004), the value for the long-run equilibrium is around zero. Figure 6 shows the graphs for the CPI-based real exchange rate regarding the two analyzed periods and the whole series. The horizontal lines represent the respective equilibrium values:

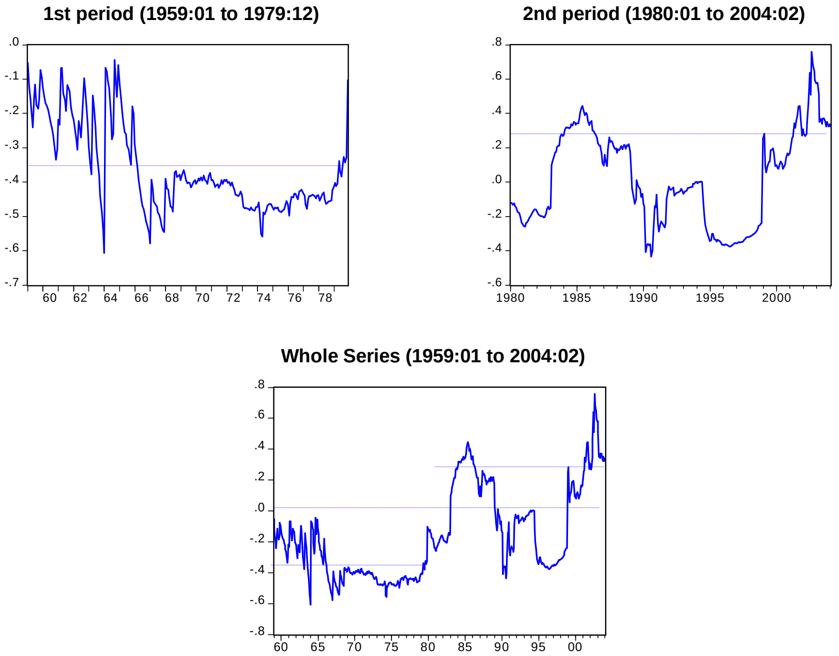

Fig. 6. Graphs for q^{ipc} regarding the analyzed periods and the respective equilibrium rates

4 Conclusion

PPP has been one of the major themes discussed in economic studies in the last few decades due to its importance to several macroeconomic models and to the controversial conclusions. Several studies do not reject the null hypothesis of unit root of the real exchange rate generating process, invalidating the PPP as a long-run relationship with parity.

However, transactions costs on international trade may cause remarkable nonlinearities in the reversion to the real exchange rate equilibrium. Under small PPP deviations, the real exchange rate may have a random walk or explosive behavior. Nevertheless, large deviations would render the process stationary, in such a way that the larger the deviations of the real exchange rate from the PPP, the stronger the tendency towards moving to an equilibrium.

The modeling of the real exchange rate through the STAR model proposed by Granger and Terasvirta (1993) has characteristics that are compatible with the nonlinear behavior of PPP deviations. This study modeled the Brazilian real exchange rate between 1959 and 2004 through the STAR model, providing evidence of nonlinearities in parity reversion. Linearity was rejected for the CPI-based real exchange rate, but was not rejected when the WPI was used as benchmark price index. The null hypothesis of unit root of the WPI-based RER was rejected, indicating stationarity of this variable.

The application of the tests defined by Terasvirta (1994) indicated the exponential smoothed transition autoregressive (ESTAR) model as the most appropriate for the Brazilian nonlinear modeling of the CPI-based real exchange rate. The inclusion of dummy variables proved appropriate, due to the different economic plans implemented in the Brazilian economy during the study period.

Despite the high persistence of PPP deviations, the estimated parameters were significant, confirming that the Brazilian real exchange rate has an explosive behavior whenever close to parity and a stationary tendency whenever far from the long-run equilibrium.

Besides the study on the whole series (1959 to 2004), analyses were carried out about the CPI-based real exchange rate in two different periods (1959/1979 and 1980/2004). The series for the first period was strongly nonlinear, being modeled through the LSTAR function. The estimated parameters showed stationarity of the series, which is in line with the results obtained from the ADF unit root test.

The series for the second period contained unit root and the hypothesis of linearity was not rejected at the usual significance levels. However, the nonlinear STAR model was estimated, showing that the real exchange rate in this period has a similar behavior to that of the whole series, with stationarity in the presence of large deviations and explosive behavior whenever close to an equilibrium.

The equilibrium exchange rates revealed significant differences in relation to the study period. Between 1959 and 1979, this value corresponded to -0.3544 and between 1980 and 2004, to 0.2661. For the whole series, the long-run equilibrium exchange rate was approximately zero.

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