Identification of Rational Speculative Bubbles in IBOVESPA (after the Real Plan) using Markov Switching Regimes

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Abstract

The present article aims to verify the presence of rational speculative bubbles by identifying the regime switching of the return generating process in the Brazilian stock exchange market, BOVESPA, for the Real Plan period (July 1994 to March 2004). To achieve this goal, a Markov switching regime was used, and then the nonlinear structure of data was verified in relation to conditional mean and variance. As a result, the dynamics of the data generating process can be described as a function of two regimes (bull markets and bear markets). These cycles, however, were decomposed into other cycles, initial and final phases of the cycles of growth (bull) and decrease (bear). This decomposition proved more coherent with the concept of speculative bubble, in which there is a nonlinear relationship between stock prices and their fundamentals.

Keywords: Rational Speculative Bubble, Markov Switching Regimes, Nonlinearities, Market Efficiency

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O presente artigo procurou constatar a presença de bolhas especulativas racionais, a partir da identificação de mudança de regime do processo de geração de retornos no mercado brasileiro de ações na BOVESPA, para o período pós Plano Real (julho de 1994 a março de 2004). Para tentar lograr este fim, utilizou-se do modelo de regimes de conversão markovianos, que permite identificar a estrutura não linear dos dados seja em relação à média condicional, seja em relação à variância condicional. Como resultado a dinâmica do processo de geração dos retornos pode ser descrito como função de dois regimes ("bull markets" e "bear markets"). Estes ciclos, porém, puderam ser decompostos em outros ciclos, fases iniciais e finais do ciclo de crescimento ("bull") e de crescimento ("bear"). Esta decomposição mostrou-se mais coerente com o conceito de bolha especulativa, no qual há uma relação não linear entre o preço das ações e os seus fundamentos.

1 Introduction

The returns generating process, as pointed out by French (1980), has been the most widely investigated topic in finances and originated from the publication of Bachelier's thesis in 1900. Therefore, the origin of the Finance Theory is tangled up with this

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concern with market efficiency.

Tobin (1984) *apud* Barone (1990) describes the classification that establishes four types of efficiency: 1) efficiency in terms of **information**; 2) efficiency regarding the number of assets, that is, the market must be **complete**; 3) efficiency in terms of **operation** and; 4) efficiency in terms of **evaluation**, that is, the stock prices reflect or should reflect the present value of their future earnings (dividends). In this regard, the stock price should have an intrinsic value.

According to Jensen (1978), any market strategy that consistently produces economic gain, after discounting the risk, for a sufficiently long period, considering the transactions costs, is a piece of evidence against market efficiency. This concept is sufficiently general to incorporate the above-mentioned Tobin's taxonomy. Traditionally, the concern of researchers with efficiency can be translated as the hypothesis that the natural logarithm of stock prices behaves as a martingale difference in relation to filtration. This is the same as to say that the value expected from the excess rate of return is **on average equal to zero**, considering a measure of probability that discounts the risk premium, given a set of (historical, public or private) information.

Empirical evidence, especially from the 1960s, has overwhelmingly demonstrated an array of stylized facts, which gave rise to a vast literature on finances, such as: volatility clusters, non-normality of returns, negative asymmetry, excess kurtosis, stochastic volatility, autoregressivity of returns and volatility, market anomalies related to seasonality or to the operation of the market, market anomalies related to the size of the firm and its capital structure, mean-reverting process for returns and extreme values. Concomitantly with these findings, a series of theories (mainly economic ones) were posited on the nonlinearity of data, including fads, manias and panics and rational speculative bubbles.

The **aim** of the present study is to verify the presence of rational bubbles by detecting the regime switching of the returns generating process in the Brazilian stock market (BOVESPA), for the period following the Real Plan (July 1994 to March 2004). The use of the Markov switching model allows identifying the nonlinear structure of data either in relation to the conditional mean or to the conditional variance. As a result, the dynamics of the generating process may be a function of bull markets and bear markets. These markets may be decomposed into other cycles, such as initial and final stages of the bull and bear markets.

2 Bibliographic Review

2.1 Rational speculative bubble

The most direct empirical evidence is that which considers a persistent rise in asset prices for a sufficiently long period (rally) as a bubble, followed by a collapse in prices (crash). The concept of bubble can have different meanings. By considering the no-arbitrage and equilibrium arguments, the present value of an asset must equal the value expected from the flow of net benefits that this asset generates to its holders. However, in case of a stock, the observed value can be higher than the present value of its dividends. For some reason, the demand exceeds the supply of that good, causing its price to rise, for a certain period of time, supposing the nonexistence of a monetary phenomenon (inflation or hyperinflation). The origin and nature of this process that generates price movement will characterize the different types of bubble. In this study, we will deal with the rational speculative bubble, after regime switching takes place.

The original rational speculative bubble model was developed by Blanchard (1979) and Blanchard and Watson (1982). According to this model, the bubble appears when an asset price is an increasing and positive function of the expected future price movement. The assumption is that economic agents, under the conditions of forming their price expectations in a rational manner, do not commit errors in a systematic fashion and, therefore, the positive relationship between the current price and its future variation results in an equally positive relationship between its current price and its observed variation. Thus, the agents' expectations are self-fulfilling, causing the variation in price to drive the current price towards its expectation, regardless of its fundamentals. For a given period of time, economic agents follow this line of thought or belief and this causes prices to rise, regardless of the flow of dividends. Agents are aware of the possibility of a bubble burst, but the expected return makes it worthwhile taking on such risk. However, what is observed is that this deviation between the observed price and its intrinsic value can be so large that one may think of speculation. Hence the origin of the term rational speculative bubble.

Irrational decision-making models such as those proposed by Tversky and Kahneman (1981) may explain the speculative nature of markets. However, in the speculative bubble model, the rationality of agents is preserved and there is a representative investor. A broader concept of bubble encompasses the fads model proposed by Summers (1986), manias and panics proposed by Kindleberger (1989), speculative bubble by Blanchard and Watson (1982) and random speculative bubble by Weil (1987). In Summers' model (1986) there are two types of representative agents: agents that demand the asset in function of the expected future return and those who demand the asset in function of past returns. The latter aggregate noise to the market and can act according to "irrational" negotiation rules, reacting excessively to the latest news, with information asymmetry or necessitating a high cost to obtain them, acting in function of the estimated behavior of others. In Kindleberger's model (1989), collapse is more likely, the more distant the observed price and the intrinsic value of the asset, thus placing emphasis on its speculative nature. The model devised by Blanchard and Watson (1982) takes for granted that the bubble has a deterministic behavior, whereas that of Weil (1987) assumes its random nature. Both have a rational representative agent who admits the possibility of sale at a higher price, regardless of the flow of the fundamental asset value. **However, as shown by Van-Norden and Schaller (1996), all these models lead to regime switching**. Bubbles survive and collapse and, therefore, the generation of returns is associated with the presence of bubbles or with their collapse. Moreover, the probability of bubble collapse depends on bubble size, i.e., the larger its size, the larger the probability of collapse. Thus, the dynamics of bubbles naturally results in regime switching.

Nevertheless, the following aspect should be highlighted: regime switching can be a sign that bubbles are present. The behavior of stock prices, in case of the speculative bubble model, is related to the expected flow of dividends. In the fundamental model proposed by Van-Norden and Schaller (1996), stock price is related to macroeconomic fundamentals. Thus, regime switching may occur due to the change in macroeconomic fundamentals or not. In the latter case, the existence of a regime occurs independently of the presence of fundamentals that justify it, either because prices are being influenced by pieces of news that do not have an impact on the fundamentals, or because prices are related to their fundamentals in a nonlinear fashion. Also noteworthy is that the relationship between macroeconomic fundamentals and the expected dividends is not necessarily synchronized. Evidence of this is that decreases in stock prices do not necessarily occur when there is a drastic change in macroeconomic fundamentals. The review of the flow of dividends considers a longer-term perspective, whereas macroeconomic fundamentals consider government policies in a shorter period of time.

Initially, the tests for detection of bubbles aimed to spot any type of bubble, without specifying its nature, being therefore more general. To cite a few: Leroy and Porter (1981), Shiller (1981), Mankiw et al. (1985), Mattey and Meese (1986) and West (1987). Basically, these tests focused on defining limits to the variance by considering the observed asset prices and the present value of their dividends.

Later on, other authors tested for bubbles, considering their specificity and therefore providing more details about them. This is the case of Turner et al. (1989), who assessed the model in which the variance of excess return is a function of the state of the regime, using monthly data from 1946 to 1987, with the S&P500 composite index. Their results showed that the mean returns were inversely proportional to the level of risk of each state, indicating that moments of high volatility took agents by surprise.

Van-Norden and Schaller (1993) analyzed the predictability of regime switching in the Toronto Stock Exchange between 1956 and 1989. Their results confirmed the evidence that growth bursts that precede collapse result from the deviation from fundamentals, as suggested by the bubble model. Mcqueen and Thorley (1994) found evidence that the probability of a change in the continuously high and persistent stock prices of the New York Stock Exchange from 1927 to 1991, on a monthly basis, decreases in function of the length of this period (negative “hazard” function).

Van-Norden and Schaller (1996) used monthly data on price and dividends for the U.S. market in the period between 1926 and 1989 but did not find any evidence that the predictability of returns followed a nonlinear relationship. Additionally, the pieces of evidence confirmed that the higher stock prices during the growth period, the higher the probability of collapse, and that there exists a significant difference between returns in both regimes (rally and crash). By using monthly data for the U.S. stock market between 1926 and 1989, Van-Norden and Schaller (1997) proved that the fad and bubble models imply regime switching.

Maheu and Mccurdy (2000) found evidence of nonlinearity of the New York Stock Exchange monthly returns for the 1834-1995 period. The authors considered that the largest return occurred in periods of economic growth (bull market) and that the smallest return took place in a period of decrease (bear market). The period with the largest growth corresponded to lower conditional volatility and the period with smaller growth was that with higher conditional volatility.

Coe (2002) employed the Markov switching regime to investigate financial crises, especially the Great Depression in 1929. Evidence suggests that the crisis did not begin with the crash of the New York Stock Exchange, but with the bank crisis that followed it, and that the changes in regulatory milestones triggered the regime switching.

Brooks and Karasaris (2003) considered the Markov switching regime to have three stages: dormant bubble state, explosive bubble state and collapsing bubble state. Through monthly data, they analyzed the 1888-2001 period and found that abnormal volume is a significant predictor of bubble collapse.

In Brazil, Laurini and Portugal (2002) used the Markov switching regime to validate the hypothesis of market efficiency in relation to the nominal exchange rate (R$/US$). They analyzed the post-Real Plan until January 2002, using daily data, and validated the hypothesis of efficiency. However, the model identified periods with abnormal gains. Terra and Valadares (2003) also used the Markov switching model to verify the alignment or non-alignment of the real exchange rate in a sample of 85 countries. They found two regimes (calmness and crisis) for some countries and a higher persistence for those which showed a lower rate of appreciation. Valls-Pereira et al. (2004) applied the Markov switching regime to the stochastic volatility model, in order to analyze the level of persistence and the dynamics of the volatility process for indices (S&P500 and FTSE100) of the U.S. market on a daily and weekly basis. Disregarding the presence of regimes governing volatility resulted in higher persistence.

2.1.1 General rational speculative bubble model

The bubble model below follows the pattern described by Mcqueen and Thorley (1994). In efficient markets, the value expected from the future return should be the same as the observed value. This implies that, in a two-period model, the observed return should be the same as the present value of the dividend and of price variation in the period. Considering an infinite time horizon, respecting the transversality condition, the fundamental asset price (p^*) should be the present value of the cash flow generated by it, represented by the flow of dividends. Thus, the fundamental asset value is also a solution to the first equation.

$$E_t \left[R_{t+1} - r | \Omega_t \right] = 0 \tag{1}$$

$$R_{t+1} = \left[p_{t+1} - p_t + d_{t+1} \right] / p_t \tag{2}$$

$$p_t = E_t \left[p_{t+1} + d_{t+1} \right] / \left(1 + r_{t+1} \right) \tag{3}$$

$$p^* = \sum_{i=1}^{\infty} \frac{E_t\left[d_{t+1}\right]}{\prod_{j=1}^{i}\left(1 + r_{i+j}\right)} \tag{4}$$

The price, based on the construction of the model, may deviate from its fundamental value due to the size of the rational speculative bubble and this new price with bubble (p_t) also satisfies the equation of its fundamental value, if the nature of the bubble meets the arbitrage conditions. The expected bubble value should be the same as the expected return. Therefore, prices with or without bubble offer an expected rate of return adjusted by the risk it poses.

$$p_t = p^* + b_t \tag{5}$$

and

$$E_t\left[b_{t+1}\right] = \left(1 + r_{t+1}\right) b_t \tag{6}$$

However, the bubble value should not grow indefinitely. Thus, in this model, we suppose that the bubble can be reduced or eliminated with a probability $1 - \pi$, if there is a bubble collapsing regime or if the investor is remunerated for a value that compensates for the risk assumed by him with a probability π, if bubble survival takes place. If $a_0 = 0$, this model is reduced to that of Blanchard and Watson (1982).

$$\begin{aligned} b_{t+1} = & \frac{(1 + r_{t+1} b_t}{\pi} - \frac{1-\pi}{\pi} a_0 \text{with probability } \pi, \\ & \quad \text{in case of bull regime} \\ = & \, a_0 \text{ with probability } 1 - \pi, \\ & \quad \text{in case of bear} \end{aligned} \tag{7}$$

The unexpected variation in prices (ε_t) has two sources of uncertainty: the unexpected variation of fundamental price (δ_{t+1}) and the unexpected variation of the rational speculative bubble (η_{t+1}). To eliminate the possibility of arbitrage, the unexpected mean variation (innovation) should be equal to zero. However, innovation can be positive asymmetric in case of price rises or negative in case of bubble collapse, producing autocorrelation for returns. Excess kurtosis can be produced by mixing the distributions of probabilities of both regimes. The observations responsible for a smaller variance, compared to the total variance of observations, will produce more kurtosis and the observations responsible for a higher variance, compared to the total variance, will produce fatter tails.

$$\delta_{t+1} = p^*_{t+1} + d_{t+1} - (1 + r_{t+1})\, p^*_t \tag{8}$$

$$\eta_{t+1} = b_{t+1} - (1 + r_{t+1})\,.b_t \tag{9}$$

$$\varepsilon_{t+1} = \delta_{t+1} + \eta_{t+1} \tag{10}$$

$$\begin{aligned} \varepsilon_{t+1} &= \delta_{t+1} + \frac{1-\pi}{\pi}\left[(1 + r_{t+1})\,.b_t - a_0\right] \text{with probability } \pi \\ &= \delta_{t+1} + \left[(1 + r_{t+1})\,.b_t + a_0\right] \text{with probability } 1-\pi \end{aligned} \tag{11}$$

As corollary of the speculative bubble model we have regime switching. Nevertheless, regime switching may result from the presence of speculative bubble and also from the changes in fundamentals, mainly macroeconomic ones (interest rate, exchange rate, inflation rate, wage levels). Thus, the evidence of bubble from the regime switching in this study follows a Markov switching regime.

2.2 Markov switching regimes

Large fluctuations were observed when we analyzed the behavior of IBOVESPA in the study period. So, the assumption is that the nature of fluctuations results from the presence of rational speculative bubbles, which imply regime switching. The perception of agents is that there are more or less risky moments to invest in stock exchanges.

Ryden et al. (1998) proved that a series of stylized facts derives from a Markov switching regime model. Thus, the presence of these stylized facts implies evidence of bubble, in which each of the different phases follows different regimes.

The concept of nonlinearity refers to the change in regime or states, that is, certain properties of the time series, such as mean, variance and autocovariance function are remarkably different due to distinct regimes. Each of the regimes generates a series of observations that can be described by a linear process. However, the combination or sum of these processes generates a nonlinear dynamics. The process of transition from one state to another follows a Markov process. In this study, we adopt the model developed by Hamilton (1989), which is briefly described next.

Let a data generating process (returns) be such that it obeys the following equation:

$$R_t = \mu(S_t) + \sum_{i=1}^{p} \phi_i \left[R_{t-i} - \mu\left(S_{t-i}\right)\right] + \sigma(S_t)v_t$$
$$\text{se } S_t = \text{estado } j \in (1...M)\,,\ \text{where} \tag{12}$$

$R_t = 1n(P_t/P_{t-1})$; P is the number os lags or order of the regression process; $\mu(S_t)$ is the conditional mean, given the history of the process $\Omega_{t-1} = (S_{t-1}, S_{t-2}, ..., S_1, S_0, S_{-\tau+1})$ and $\tau =$ length in each regime, ν_t is the standard normal innovation ($\nu_t \sim NID(0,1)$) which is not dependent upon S_t and R_t.

For each observation of the return, its expected value, at a given moment, given the history of the (Ω_t) and the regime (S_t), assumes the following value:

$$E\left[R_t|\Omega_{t-1}, S_t\right] = \mu(S_t) + \sum_{i=1}^{p} \phi i \left[R_{t-i} - \mu\left(S_{t-i}\right)\right] \tag{13}$$

The difference between the observed value and its expected value is a martingale difference that follows a standard normal distribution with mean zero and whose variance-covariance matrix $\sum$ depends on regime S_t.

$$\mu_t = R_t - E\left[R_t|\Omega_{t-1}, S_t\right] \sim NID\left(0, \sum S_t\right) \tag{14}$$

However, the generating process above the returns is insufficient to elucidate the dynamics of the process, as the regime switching, due to the construction of the model, follows a Markov process. This process is characterized by a Markov chain that assumes discrete states (e.g.: bull or bear markets), as follows:

$$P = \begin{pmatrix} p_{11} & 1 - p_{22} \\ 1 - p_{11} & p_{22} \end{pmatrix}$$

where p_{ij} is the probability to go to state j, since this is state i.

The density probability function conditional on regime S_t and the history of the process Ω_{t-1} follows a normal distribution, given by:

$$f\left(R_t|S_t = j, \Omega_{t-1}, \theta\right) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left\{\frac{-\left(R_t - \phi_j' x_t\right)^2}{2\sigma^2}\right\} \qquad (15)$$

where $\mathbf{x}_t = (1, R_{t-1}, ..., R_{t-p})'$ and $\phi_j = (\phi_{0,j}, \phi_{1,j}, ..., \phi_{p,j})'$, $j = 1$ or 2 and $\theta = (\mu(S_t = 1), \mu(S_t = 2), p_{11}, p_{22}, \sigma^2)$.

The parameters of vector θ are estimated based on the information contained in Ω_{t-1}. The maximum likelihood function for the nth observation is given by:

$$\begin{aligned} L(\theta) = {} & 1n[f(R_t|\Omega_{t-1}, \theta)] = 1n[f(R_t|S_t = 1, \Omega_{t-1}, \theta)] \\ & + f(R_t|S_t = 2, \Omega_{t-1}, \theta)] \\ & = 1n[\sum_{j=1}^{2} f(R_t|S_t = j, \Omega_{t-1}, \theta).P(S_t = j|\Omega_{t-1}, \theta)] \end{aligned} \qquad (16)$$

The estimates of θ are obtained by maximizing the maximum likelihood function above using the EM algorithm of Dempster et al. (1977).

For the maximization process above, the conditional probability of being in regime $S_t = j$, given the history of the process Ω, $P(S_t = j|\Omega, \theta)$ encompasses three different types of inference, namely: a) the probability that considers the observations up to $t - 1$ (forecast), given by $P(S_t = j|\Omega_{t-1}, \theta)$; b) the probability that considers the observations up to t (filtering), given by $P(S_t = j|\Omega_t, \theta)$ and; c) the probability that considers all the information about the sample up to T (smoothed), given by: $P(S_t = j|\Omega_T, \theta)$.

The maximum likelihood estimates are:

$$\hat{p}_{ij} = \frac{\sum_{t=2}^{T} P\left(S_t = j, S_t = i|\Omega_T; \hat{\theta}\right)}{\sum_{t=2}^{T} P\left(S_{t-1} = i|\Omega_T; \hat{\theta}\right)} \tag{17}$$

that is, the transition probability is the number of states i followed by state j divided by the number of times state i occurred;

$$\hat{\sigma}^2 = n^{-1} \sum_{t=1}^{T} \sum_{j=1}^{M} \left(y_t - \phi_j' x_t\right)^2 .P\left(S_t = j|\Omega_T; \hat{\theta}\right) \tag{18}$$

$\phi_{j.}'$ is the coefficient vector of explanatory variables and $\mathbf{x}_t$ is the vector of explanatory variables represented by the lagged values of variable y and $\hat{\sigma}^2$ is the sum of residuals of T weighted least squares regressions, multiplied by n^{-1};

$$\hat{\sigma}_{'j} = \frac{\sum_{t=1}^{T} x_t(j).y_t(j)}{\sum_{t=1}^{T} x_t(j).x_t'(j)} \tag{19}$$

$$y_t(j) = y_{t.}\sqrt{P(S_t = j|\Omega_T; \hat{\theta}}$$

$$x_t(j) = x_t\sqrt{P(S_t = j|\Omega_T; \hat{\theta}}$$

$\hat{\phi}_{'j}$'s are obtained by ordinary least squares of the regression of yt against its lagged values. Chapter 22 in Hamilton (1994) describes the procedure above in detail.

3 Empirical Tests

3.1 Estimation

According to Krolzig (1998), the determination of the number of regimes, based on some test statistics, is not possible due to the nonexistence of a standard asymptotic distribution. This occurs, especially with regard to the likelihood ratio (LR), in function of nuisance parameters that are necessary for the estimation. Thus, the initial selection of the number of regimes was made using the theoretical framework of the speculative bubble model, which ranges from 2 to 3 regimes. The estimation results for the model with three regimes were omitted, due to the significance level of most parameters.

The procedure suggested by Granger (1992) *apud* Franses and Van-Dijk (2002) was used to estimate the model. This procedure goes from the specific model to the general one, in which 1) the level of autoregressivity of the corresponding linear model was observed, 2) the null hypothesis of linearity was tested in relation to the alternative model, 3) the alternative model was estimated and 4) the diagnostic test was eventually carried out.

The data refer to the monthly undeflated returns of IBOVESPA, from July 1994 to March 2004. The MSVAR software (Hans-Martin Krolzig) was used in the estimation. Two situations were taken into account: 1) the variable conditional mean, by supposing the same and constant conditional variance in each regime, MSM(M)$-$AR(p) restrictive models and 2) variable mean and conditional variance in each regime, MSM**H**(M)$-$AR(p) non-restrictive models, as shown by the following equations:

$$R_t = \mu(S_t) + \sum_{i=1}^{3} \phi_i [R_{t-i} - \mu(S_{t-i})] + \sigma(S_t) v_t$$
$$\text{if } S_t = \text{state} j \in (1\text{or}(2) MSM(2) AR(3)$$

$$R_t = \mu(S_t) + \sum_{i=1}^{4} \phi_i [R_{t-i} - \mu(S_{t-i})] + \sigma(S_t) v_t$$
$$\text{if } S_t = \text{state} j \in (1\text{or}(2) MSM(2) AR(4)$$

$$R_t = \mu(S_t) + \sum_{i=1}^{3} \phi_i [R_{t-i} - \mu(S_{t-i})] + \sigma(S_t) v_t$$
$$\text{if } S_t = \text{state} j \in (1, 2, 3, 4) MSM(4) AR(3)$$

$$R_t = \mu(S_t) + \sum_{i=1}^{4} \phi_i [R_{t-i} - \mu(S_{t-i})] + \sigma(S_t) v_t$$
$$\text{if } S_t = \text{state} j \in (1, 2, 3, 4) MSM(4) AR(4)$$

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$$R_t = \mu(S_t) + \sum_{i=1}^{4} \phi_i [R_{t-i} - \mu(S_{t-i})] + \sigma(S_t) v_t$$
$$\text{if } S_t = \text{state} j \in (1,2,3,4) MSMH(4)AR(4)$$

Table 1
Estimates for MSM(M)−Ar(p) models –Situation 1

	MSM(2)Ar(3) (8,1,2)	MSM(2)Ar(4) (9,1,2)	MSM(4)Ar(3) (20,3,12)	MSM(4)Ar(4) (21,3,12)
Mean μ_1 1 (phase 1)	-0.0744*	-0.0837*	-0.1661*	-0.1456*
Mean μ_2 2 (phase 2)	+0.0511*	+0.0532*	-0.0293	-0.0373*
Mean μ_3 3 (phase 3)			+0.0337*	+0.0351*
Mean μ_4 4 (phase 4)			+0.0934*	+0.0925*
σ	0.088712	0.085184	0.065437	0.061682
Trans Prob. 11	69.73	66.32	32.08	38.14
Trans Prob. 22	12.85	33.18	67.06	63.15
Trans Prob. 33			83.92	84.21
Trans Prob. 44			70.66	72.94
Uncond. Prob. R1	29.80	28.99	10.68	11.80
Uncond. Prob. R2	70.20	71.01	22.03	19.81
Uncond. Prob. R3			45.12	46.23
Uncond. Prob. R4			22.17	22.16
Lenght of R1	3.30	2.97	1.47	1.62
Lenght of R2	7.78	7.27	3.04	2.71
Lenght of do R3			6.22	6.33
Lenght of R4			3.41	3.70
Obs. R1	34.6	33.3	11.90	13.20
Obs. R2	79.4	79.7	25.20	22.60
Obs. R3			50.70	51.40
Obs. R4			26.20	25.80
LR_{msw} test	5.8736	5.7830	24.0539	24.6918
P-value	(0.0154)	(0.0162)	(0.0000)	(0.0000)
	(0.1179)+	(0.1227)+	(0.0006)+	(0.0004)+

Obs: * the parameters are significant at 5%.

*** (p,r,n) where p=no. of parameters; r=no. of restrictions and n=no. of nuisance parameters.

+ the procedure of Davies *apud* Krolzig (1998).

Source: the authors.

In the first situation, the null hypothesis of linear relationship between the variables (LR_{msw} test $\sim \chi 2$, table 1.) is rejected. The t statistic $LR_{msw} = L_{msw} - L_{ar}$ does not have a standard distribution and its critical values are obtained by simulation, according to Hansen (1992) and Garcia (1998). There is poor evidence in the case of two regimes, resulting in the conditional mean, and strong evidence in the case of four regimes, resulting in the conditional mean. In the case of two regimes, in the first regime the mean is negative and in the second regime the mean is positive. Therefore, the first regime can be seen as a period of decrease and the second one as a period of growth. In the case of four regimes, there are two phases with negative mean returns and/or zero (phases 1 and 2) and two phases with positive mean return (phases 3 and 4). We may observe a regime with a strongly negative mean and another one with a strongly positive mean, but with a smaller module.

By observing the unconditional probabilities and length, we may see that in the case of four regimes, persistence is higher in phase 3 (e.g.: MSM(4)Ar(4) model, $p_3 = 46.23\%$ and average length of 6.33 months). The sum of the lengths of the phases forms the cycle. The whole cycle lasted on average around 14 months. In the case of two regimes, the phase of price increases is more persistent (e.g.: MSM(2)Ar(4) model, $p_2 = 71.01\%$ and average length of 7.27 months). In this case, the whole cycle lasted about 11 months.

Table 2
Estimates for MSM**H**(M)-Ar(p) models – Situation 2

	MSM**H**(2)Ar(3)	MSM**H**(2)Ar(4)	MSM**H**(4)Ar(3)	MSM**H**(4)Ar(4)
	(9,2,2)	(10,2,2)	(23,6,12)	(24,6,12)
Mean μ_1 1 (fase 1)	-0.0253	-0.0379	-0.1785*	-0.2728*
Mean μ_2 2 (fase 2)	+0.0528*	+0.0571*	-0.0005	+0.0028*
Mean μ_3 3 (fase 3)			+0.0478*	+0.0557*
Mean μ_4 4 (fase 4)			+0.1316*	+0.1081*
σ_1	0.128480	0.127420	0.161100	0.150430
σ_2	0.064989	0.064301	0.092489	0.098499
σ_3			0.045528	0.043415
σ_4			0.052475	0.048820
Trans Prob. 11	85.38	73.50	~ 0	~ 0
Trans Prob. 22	85.70	77.84	93.99	94.56
Trans Prob. 33			95.47	94.40
Trans Prob. 44			~ 0	49.00
Uncond. Prob. R1	49.44	45.54	7.74	4.72
Uncond. Prob. R2	50.56	54.46	37.23	51.86
Uncond. Prob. R3			47.34	34.14
Uncond. Prob. R4			7.69	9.29
Length R1	6.84	3.77	1.00	1.00
Length R2	6.99	4.51	16.63	18.39
Length R3			22.09	17.86
Length R4			1.00	1.96
Obs. R1	58	51.8	9.3	4.9
Obs. R2	56	61.2	57.3	68.8
Obs. R3			38.1	29.4
Obs. R4			9.2	9.9
LR_{msw} test	15.6338	14.1172	42.8925	35.8845
P-value	(0.0004)	(0.0009)	(0.0000)	(0.0000)
	(0.0130)+	(0.0067)+	(0.0000)+	(0.0001)+

Obs: * the parameters are significant at 5%.

*** (p,r,n) where p=no. of parameters; r=no. of restrictions and n=no. of nuisance parameters.

+ the procedure of Davies *apud* Krolzig (1998).

Source: the authors.

In the second situation, the null hypothesis of linear relationship between the variables (LR$_{msw}$ test $\sim \chi 2$, table 2.) is rejected. In all models there is evidence of the existence of two and four regimes, generating conditional mean and variance. In the case of two regimes, in the first regime the mean is not significantly different from zero and in the second one the mean is positive. However, the variance of the regime with mean zero is twice as high as that of the regime with positive mean. Thus, the first regime can be seen as a period of decrease and the second one as a period of growth, and the risk of the first regime is much higher than the second one.

In the case of four regimes with different conditional variances, the sequence of phases is less evident than in the restricted model. The most acute phase of the crisis (phase 1) may alternate with the phase of largest growth (phase 4). This can produce a surprise effect to market agents. The most acute phase (phase 1) can also follow phase 2. By observing unconditional probabilities and length, one notes that in the case of four regimes persistence is higher in phase 2, only in the MSMH(4)Ar(4) model, with $p_2 = 51.86\%$ and average length of 18.39 months. In case of two regimes, persistence is higher in phase 2, in the MSMH(2)Ar(4) model, with $p_2 = 54.46\%$ and average length of 4.51 months. In this case, the total cycle lasted around 14 months and 8 months, supposing autoregressivity of order 3 and 4, respectively.

3.2 Diagnostic test

With regard to the number of regimes, the specification of models with four regimes proved more appropriate than that with two regimes, according to the information criteria (table 3.). The specification of models with four regimes, without restriction to the equality of variance proved the most appropriate according to the likelihood ratio test (table 4.), except regarding the MSM(4)Ar(4) model.

Table 3
Diagnostic TEsts of the Models

Criteria	MSM(2)Ar(3)	MSM(2)Ar(4)	MSM(4)Ar(3)	MSM(4)Ar(4)
AIC	-1.4644	-1.4649	-1.4134	-1.4199
HQ	-1.3865	-1.3768	-1.2186	-1.2142
SC	-1.2724	-1.2477	-0.9334	-0.9130
LogLik.	91.4736	91.7683	100.5637	101.2227
	MSMH(2)Ar(3)	MSMH(2)Ar(4)	MSMH(4)Ar(3)	MSMH(4)Ar(4)
AIC	-1.5325	-1.5210	-1.5260	-1.4658
HQ	-1.4449	-1.4230	-1.3020	-1.2308
SC	-1.3165	-1.2796	-0.9740	-0.8866
LogLik.	96.3536	95.9354	109.9830	106.8190

Source: the authors

According to Krolzig (1997), the selection of the models using statistical tests still requires a lot of improvement due to the nonlinearity of the model. In this regard, the tests for normality of errors should consider the nonlinear nature of the data and the tests for autocorrelation of residuals are only descriptive.

Table 4
LR teste

Unrestricted model	Restricted model	Statistical test	Critical value at 5%
msmh(4)ar(3)	msm(4)ar(3)	18.84	14.0671*
msmh(4)ar(4)	msm(4)ar(4)	11.19	14.0671
msmh(2)ar(3)	msm(2)ar(3)	9.76	7.815*
msmh(2)ar(4)	msm(2)ar(4)	8.33	7.815*

*rejected H_o: the unrestricted model has a better specification

Source: the authors

According to Granger and Terasvirta (1993) *apud* Clements and Krolzig (1998), the good performance of nonlinear models "within the sample" could only be obtained "out of the sample" if the nonlinearity pattern were the same. On top of that, the starting point for the prediction is essential in order to obtain a good result. **Therefore, nonlinear models do not provide good predictions "out of the sample", considering a certain regime, but they are a good predictor of regime switching**. Diebold and Nasson *apud* Clements and Smith (1999) cited a series of reasons for the weak performance of linear models, among which is the wrong selection of some nonlinear models.

Based on the ("smoothed") probabilities obtained, it is possible to classify the observations according to the probable regimes to which they belong, using $M^* = \arg\max \text{Prob}\ (S_t = M|Y_T)$, as shown in Figure 1 below:

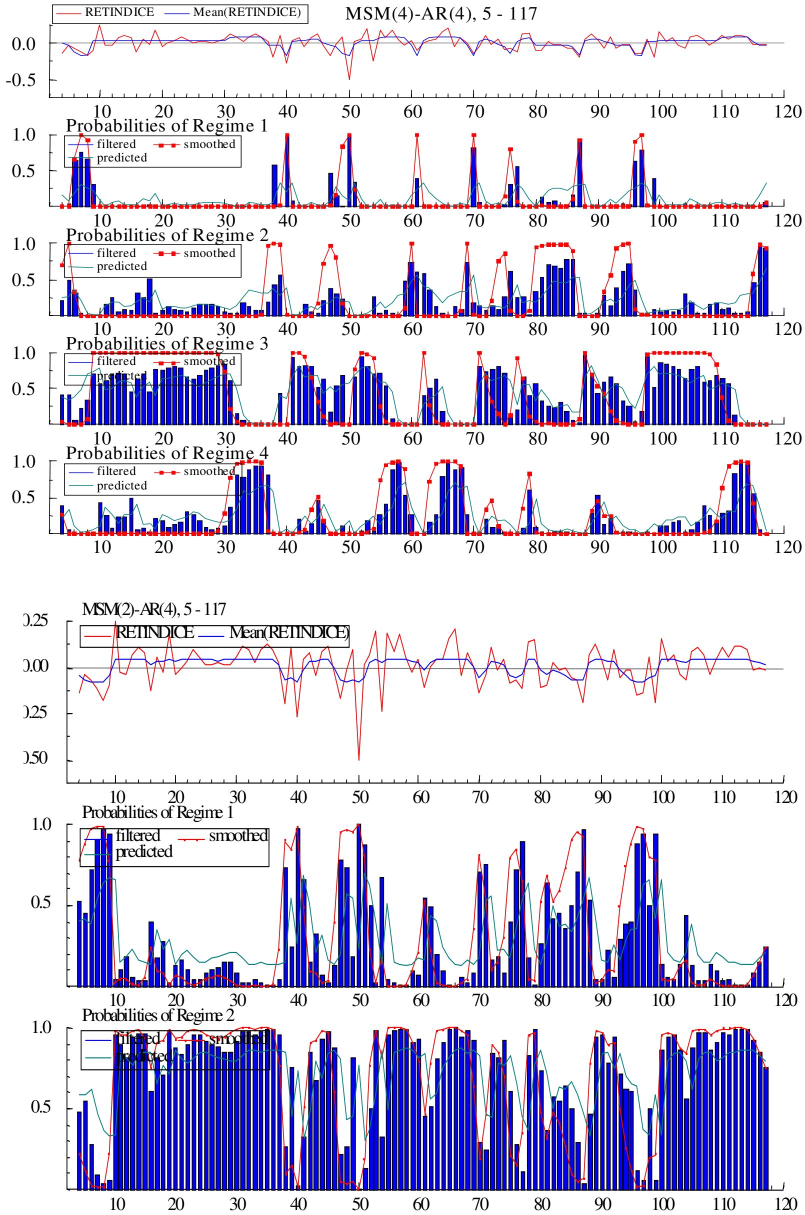
RETINDICE
Mean(RETINDICE)
MSM(4)-AR(4), 5 - 117
Probabilities of Regime 1
filtered
smoothed
predicted
Probabilities of Regime 2
Probabilities of Regime 3
Probabilities of Regime 4
MSM(2)-AR(4), 5 - 117

Fig. 1.

3.3 Conclusions

The models used are able to capture the nonlinear nature of the data, resulting in a better specification than linear models. The nonlinear models analyzed herein provide a better explanation about the series, regarding it as a combination of distributions, where each probability distribution is generated by the probable regime to which it refers. It is possible to better understand the generation of a series of stylized facts, such as excess kurtosis and fat tails through the identification of regime, creating the dynamics of the return generating process.

The verification of statistical nature provides evidence of rational speculative bubbles. Considering the presence of two regimes, the smaller conditional volatility was that of the period in which growth was more pronounced, whereas higher volatility occurred in the period with less growth. The periods of decrease in returns have a shorter length. However, the model with four regimes seems to be more consistent with the rational speculative bubble model due to the presence of a more acute crisis and higher return at the end of the cycle. Thus, phase 1 of the model with two regimes can be decomposed into two phases, which would correspond to phases 1 and 2 of the model with four regimes. In its turn, phase 2 of the model with two regimes could be decomposed into phases 3 and 4 of the model with four regimes. This decomposition obeys condition $M^* = \arg\max \text{Prob}\ (St = M|YT)$. The phases with the smallest and highest returns (phases 1 and 4) may be more appropriate for detachment or nonlinear relationship between prices and fundamentals, as a result of excessive pessimism or optimism of the agents, respectively. Thus, in the fourth regime, a larger number of naive investors may participate in the market. Especially in the case of speculative bubble, the return should be increased to compensate for the agents who speculate, which may occur in the fourth regime.

The exchange rate crises during this period strongly influenced the Brazilian stock market. These crises include the following: Mexican Crisis in December 30, 1994; Asian Crisis in October 24, 1997; Russian Crisis in August 4, 1998; and Brazilian Crisis in January 15, 1999. The Mexican and Asian crises gave rise to an acute phase of price decrease in the models with four regimes and with the same variance. The Russian crisis corresponded to the end of an acute phase and the excessive currency devaluation in Brazil originated an increase in prices (phase 4) in the models with four regimes and with the same variance.

4 Final Remarks

The existence of rational speculative bubbles implies regime switching. Detecting the presence of bubbles based on regime switching, without considering what happened to the dividends, is the evidence that bubbles exist, according to the speculative bubble model. However, regime switching may occur due to macroeconomic fundamentals, for instance, excess liquidity. In the present study, the flow of dividends was not assessed. Nevertheless, BOVESPA had to cope with a series of external and domestic crises. Moreover, Brazil went through some reforms, trade liberalization and changes in regulatory milestones, mainly from 1994 to 1998.

Regime switching, for example, as a result of some exogenous shock, may cause bubble collapse or predispose to it. However, this is not necessarily the harbinger of a collapse. The external shock may be either a sign of growth or collapse.

The use of Markov switching models is relatively recent in Brazil and promptly results in new studies. A suggestion for new studies is the incorporation of the GARCH effect into the Markov switching model, applied to the Brazilian stock market or the relationship of IBOVESPA returns with the flow of dividends in the same model. Another suggestion is the use of more complex switching models, such as the equilibrium correction model proposed by Krolzig and Toro (1999), considering the relationship between returns and negotiated volume.

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