Group Characteristics Evolution Arising from Asymmetric Information

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Abstract

In asymmetric information problems, agents with less information (principals or contractors) usually take as given the preferences of agents with more information (agents or contractees). Moreover, the distribution of characteristics of contractees is supposed to be invariant. In this article we consider a mixed framework of asymmetric information (adverse selection followed by moral hazard) where those two assumptions are excluded. Specifically, the contractor only knows the current distribution of characteristics and the contractees may change them after signing the contract, if this improves their welfare. Thus, we find that the asymmetric information problem leads to a group effect (changes of characteristics). This feedback defines a sequence of temporary equilibria. We provide conditions for the convergence of that sequence to a stationary long run equilibrium. We also prove that both temporary equilibrium and long-run equilibrium coincide with the equilibrium in classical models of adverse selection and the moral hazard problem vanishes in the long-run.

Keywords: Asymmetric Information, Mixed Models, Group Effect, Characteristics Evolution

JEL Classification: D82, D83

Resumo

Em problemas de informação assimétrica agentes com menos informação (principais ou contratadores) usualmente tomam como dadas as preferências dos agentes com mais informação (agentes ou contratados). Além disso, a distribuição de características dos contratados é suposta constante. Neste artigo consideramos uma estrutura mista de informação assimétrica (seleção adversa seguida de prejuízo moral) onde essas duas suposições são excluídas. Especificamente, o contratador somente conhece a distribuição corrente de características e os contratados podem mudá-las após a assinatura do contrato, se isto melhorar o seu bem-estar. Dessa maneira, encontramos que o problema de informação assimétrica conduz a um efeito grupo (mudanças de características). Esta interação define uma seqüência de equilíbrios temporários. Proporcionamos condições para obtermos convergência dessa seqüência a um equilíbrio de longo prazo estacionário. Também provamos que ambos equilíbrios, o temporário e o de longo prazo coincidem com o equilíbrio nos modelos clássicos de seleção adversa e que o problema de prejuízo moral desaparece no longo prazo.

1. Introduction

Changes in the characteristic profiles of populations are usually assigned to externalities or to some peer-group effects. For instance, Glaeser et al. (1996) explains that the increase in the crime rate is due to social interactions. de Bartolome Ch (1990) supposes that expenditures in education, individual ability and in particular, the proportion of more able agents in the society affect the individual educational achievement. As a consequence, this produces heterogeneous and dissimilar communities. Guryan (2004) analyzes the reduction in dropout rates of black students when they are included in white students groups. Calvo-Armengol and Jackson (2004) studies the effect in the employment rate of peer-group associations through network information. As we can see, changes in the profile distribution of characteristics result from peer-group effects acting as an externality in the utility function of the agents

However, some empirical works point out to the fact that relevant information included in some characteristics of the population and which affect economic variables may also induce changes in individual characteristics. In the credit market we have evidence of changes in individual bankruptcy decisions in response to the aggregate bankruptcy of the location. Using the Panel Study of Income Dynamics (PSID) dataset for the period 1984-95, Fay et al. (2002) run probit regressions to analyze the decision of a household filed for bankruptcy in one year. They found that household bankruptcy decisions are influenced by average bankruptcy filing rates in the localities where they live. We can also find that sort of characteristics changes in the labor market. Sillamaa (1999) showed that the worker effort depends on the wage taxation. Assuming that the wage depends on the marginal productivity of the (effective) number of hours that a group of workers operates then, those findings would indicate that the worker decision about the level of effort depends on the net wage he receives and therefore that is affected by the group level of effort. Finally, in the insurance market, the excessive risk premium of an insurance policy due to a high risk profile of the group may induce an insured to change her risk profile. As a consequence, the risk profile of the group will deteriorate and this will increase the risk premium for the next period.

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The situations listed above have the following common features:

1) The market does not have the full information of each individual.
2) Due to that lack of information, individuals are ranked according to the group (mean) characteristic, which may be different from their particular characteristics, and they receive the same treatment as the "mean" or "representative" member of the group.
3) The imputation of a common characteristic to each individual generates a second effect after signing the contract, each individual may change her initial characteristic if it improves her welfare. In this way we obtain a group effect even in the absence of interaction between individuals of the same group.

Thus, the group effect resulting from the imputation of the same characteristic to each individual does not rely on any direct externality from the group on the particular individual.

In that vein, this work proposes a framework that allows us to analyze changes in the population distribution of characteristics or traits as resulting from asymmetric information problems. Specifically, we consider a contractual system (one representative contractor and many contractees) where the less informed part (contractor or principal) only knows the distribution of characteristics of the more informed part (contractees or agents). Using that information, the contractor decides the supply of contracts and then the equilibrium price of the contracts is found by equalizing the supply with the demand of contracts. Once the contracts are signed, the contractees may change their traits if they feel better by doing it. Since the contractor does not have information about the contractees' preferences, he cannot foresee that change and therefore he can only use the available information, which is the distribution of characteristics; this is the central key of our model. As a consequence we will have a group effect (changes in the distribution of traits), which does not result from any externality nor peer-group effect. In this sense this framework competes with others in the literature explaining a group effect arising just from asymmetric information (applied to the group of contractees). After concluding the period of the contract, a new distribution of characteristics profiles is available for the contractor to offer a new contract menu and thus the cycle restarts. That sequence of contract revisions may converge and its properties are also analyzed.

As mentioned above, the main aspect of our analysis is that we are arguing that the employers (or the lenders, or the insurance companies) do not know the reaction function of their employees (or borrowers, or insured). As a result, they are not able to maximize their payoff in the traditional way. [1] To maximize their payoffs the employers will use the distribution of group characteristics as the best available information about the employees, in other words, they will use the available *a priori* information about the characteristics of the group.

[1] In the traditional literature, the contractor has knowledge of the contractee's reaction, meaning that he may implement contracts that are compatible incentive and which satisfy the participation constraint.

As we can see, we are dealing with a mixed model of asymmetric information, where in the first stage the contractor is solving an adverse selection problem and afterward he faces a moral hazard problem that is solved by revising the contracts. The difference is that we consider the adverse selection variable (type or characteristic) as being the same or highly correlated with the moral hazard variable (effort). Making that identification we may obtain an evolution of the group characteristics. Examples of mixed models can be found in literature (Laffont and Martimort 2002), however to the best of our knowledge none of them deals with the same variable for both problems, and as we can verify in the examples described below, it is not difficult to find real situations where that happens.

A final question that we can respond is if the sequence of revised contracts converges and if that “long-run contract” has some important property. We prove, under technical conditions, that the long-run contract is stationary (namely, the contractees do not change their characteristics), it coincides with the classical adverse selection equilibrium and the moral hazard problem is not longer present. In this way we are providing an alternative approach to these kind of models, where the contractors does not use the utility functions of the contractees when designing optimal mechanisms.

The paper is divided into five sections. In Section 2 we present the general framework which shows how a group effect may emerge from asymmetric information. This produces a sequence of temporary equilibrium contracts, which may converge to a long-run equilibrium. Existence and stability of the long-run contract are proved. In Section 3 we provide three examples in order to illustrate the framework given in the Section 2. The examples were selected to show the diversity of equilibria that our structure may support. In Section 4 we present the theorem relating the stationary equilibrium of our framework with the equilibrium in classical asymmetric information models. The conclusions are given in Section 5 and all the proofs are provided in the appendix.

2. General Framework

In this section we are going to define what a *contractual system* is and how a group effect may emerge from a mixed model of asymmetric information. We begin with a population of agents (called *contractees*) which wants to sign a contract with a group of principals (for the sake of simplicity we will consider a representative principal, called *contractor*). The contractees may have different characteristics or traits relevant to the contract and each contractee knows her own characteristic, which is not observed by the contractor.

The idea is to build a framework which captures the changes in the distribution of characteristics of contractees due to the asymmetric information. To this end, it will be supposed that the contractor has *a priori* beliefs about the traits of contractees, but he is unable to foresee the changes in the distribution of types, because he does not know the preferences of the contractees, this is the key element of our framework. In this sense, our model requires less information than the classical models of asymmetric information, where the contractor knows the utility function and the reservation utility of the contractee. The unique information that our contractor has is the statistical distribution of characteristics, usually provided by market research.

Specifically, let $T \subset R^n$ be the set of characteristics of individuals in a group. The proportion of contractees with characteristics lower than $t \in T$ is given by $F(t)$ and this is the unique information that the contractor knows about the characteristics.

There are two types of variables in the model. The *control variables*, represented by $x \in X$ which are endogenous in the individual decision problem and the *state variables* represented by $y \in Y$, which are endogenous in all the system but are taken as given by each individual. Both X and Y are Euclidian spaces. Usually x represents quantities (hours of work, amount of loans or quantity of insurance) and y prices (wages, interest rates or insurance premium).

Each contractee has a payoff function that he maximizes in order to obtain his optimal decision. The contractee with characteristic t has a payoff function $U(t, ., .) : X \times Y \rightarrow$.[2] The representative contractor has a payoff function given by $B : X \times Y \times \triangle(T) \rightarrow R$, where $\triangle(T)$ is the set of probability distributions defined on T. Usually this function has the form of an expected profit value of the contract, *i.e.* $B(x, y, F) = \int_T b(x, y, t)dF(t)$ where $b(x, y, t)$ (called the *kernel* of B) is the profit of establishing the contract with individual of type t.

Then the contractual system is defined by $\mathbf{S} = \{T, X, Y, F, U, B\}$. Notice that we are dealing with heterogeneous agents and therefore the evolution of the distribution of characteristics is relevant information for the system.

2.1. *Short and long run equilibria in a Contractual System*

Now we are going to define the concepts of contractual system equilibria in the short and long run. Furthermore, we will see how the short run equilibrium may generate a group effect. Consider the contractual system $\mathbf{S} = \{T, X, Y, F, U, B\}$.

It will be supposed that changes in individual characteristics are not instantaneous. That assumption holds when we consider habits formation on those characteristics, cultural endowments or any other reason that might produce rigidity in those changes. Since the adjustment of the equilibrium price of a contract may take some short interval of time, it is innocuous to suppose that characteristics remain constant in that interval, while the equilibrium is reached. For those reasons we state the following definition for a short-run equilibrium in a contractual system.

Definition 2.1: A short-run equilibrium for the contractual system $\mathbf{S}$ is a vector $(y^*, x^*_S, (x^*_{D,t})_{t \in T})$ such that:

[2] To keep the analysis simple, we are supposing that all individuals with the same characteristic have the same payoff function.

1) For all $x \in X, B(x, y^*, F) \leq B(x^*_S, y^*, F)$;
2) For each $t \in T, U(t, x, y^*) \leq U(t, x^*_{D,t}, y^*), \forall x \in X$;
3) $x^*_S = \int_T x^*_{D,t} dF(t)$.

As usual, conditions 1 and 2 of Definition 2.1 represent the optimality conditions which define the supply and the individual demand of contracts and condition 3 is the market clear condition. From this definition, conttactees and contractors are "price-takers", since they are taking the variable $y \in Y$ as given when making their decisions.

Let us notice that the supply and the demand of contracts depend on the state variable y and in addition the former depends on the initial distribution of characteristics (F). Finally, the equilibrium value of the endogenous variable y^* depends on the same distribution. The following theorem states conditions for continuity of those functions. [3]

Theorem 1. *Suppose that the sets T, X and Y are compact and convex subsets of Euclidean spaces and the functions B and U are continuous in all their arguments and strictly concave in x.*

i) *The functions: $x^S : Y \times \triangle(T) \rightarrow X$ and $x^D : T \times Y \rightarrow X$ defined by:*

$$x^S(y, F) = \operatorname*{argmax}_{x \in X} B(x, y, F) \tag{1}$$

and

$$x^D(t, y) = \operatorname*{argmax}_{x \in X} U(t, x, y) \tag{2}$$

are continuous. As a consequence, the excess demand function defined by: $Z(y, F) = \int_T x^D(t, y) dF(t) - x^S(y, F)$ is continuous.

ii) *If in addition, for each $F \in \triangle(T)$ there exists a unique $y^* = y(F)$ such that $Z(y^*, F) = 0$ and $\frac{\partial Z}{\partial y}(y, F)$ is a continuous function and non-singular at (y^*, F), then the function $y : \triangle(T) \rightarrow Y$ defined by $Z(y(F), F) = 0$ for all $F \in \triangle(T)$, is a continuous function.*

The general hypotheses of Theorem 1 are quite standard in this kind of models. The additional hypotheses in part ii) are more restrictive although necessary for performing the construction of a long-run equilibrium. In subsection 3.2 we exhibit an example that shows the necessity of assuming those hypotheses.

The proof of part i) of Theorem 1 is not difficult and it is given in the Appendix. Part ii) is straightforward using the Implicit Function Theorem for topological spaces (see Schwartz 1967, Chapter 3, Section 8, Theorem 25).

The key aspect of this model is the possibility of agents changing their individual characteristics after signing the contract, in order to attain greater payoff values. Those delays in changes are reasonable if we suppose that the characteristics remain unaltered while the equilibrium is being reached by the forces of the

[3] For simplicity, we will consider the case of single valued supply, demand and equilibrium correspondences.

market. Therefore, we have the following definition:

Definition 2.2: Given the contractual system **S**, the short-run equilibrium price $y^* = y(F)$ and the contracts demand $x^D(t,y)$ defined by (1), the characteristics change function is defined by:

$$t^{new}(t,F) = \underset{t' \in T}{\operatorname{argmax}}\, U\left(t', x^D(t,y^*), y^*\right)$$

That is, $t^{new}(t,F)$ is the new characteristic of the contractee t.

At this point, it is important to comment on the maximization problems of the contractor and the contractee. The apparently naïve nature of our contractor is easily explained if we do not assume that the agent is *super-rational.* It means that he would not be able to foresee the changes in the distribution of types. That is the case if the contractor does not know the preferences of the contractees and the only available information is the market research information about the distribution of characteristics. Regarding the optimization problem which defines the new characteristic of the contractee, as mentioned above, we may consider some level of rigidity in the changes of characteristics (motivated by the tradition, habits formation or cultural aspects, which are elements present in the variables that we will illustrate in the models of the next section). Furthermore, the decision of changing the characteristic may only be taken after knowing the equilibrium value of the state variable (price of the contracts). Thus, in a first stage (before signing the contract) the contractee only decides her contracts demand. After signing the contracts, in a second stage, she may change her type/characteristic, if it improves her own welfare.

Using the Theorem of the Maximum we obtain the following result:

Theorem 2. *Under hypotheses of Theorem 1, the function* $t^{new} : T \times \triangle(T) \to T$ *is continuous.*

Therefore, the characteristics change function is continuous (and therefore measurable), so the new distribution of characteristics can be defined by:

$$\Phi(F)(t) = Prob\left[t^{new}(s,F) \le t\right] = Prob\left[s \in \left(t^{new}(.,F)\right)^{-1}\left([-\infty,t]\right)\right] \qquad (3)$$

Definition 2.3:

1) The group effect in the contractual system **S** is defined by $\Phi(F)$.
2) A stationary equilibrium is a (short-run) equilibrium where $\Phi(F) = F$.

Therefore the group effect is the change in the population profile of characteristics. Note that this change does not result from any externality of the group on the individual trait or peer-group interaction. It is a consequence of the asymmetric information in the contract and the equal treatment received by the contractees with different characteristics.

2.2. *The dynamics of the contracts revisions*

Since the contracts are periodically revised (due to changes in the population profile of characteristics), an evolutionary dynamics of the distribution of characteristics is defined by the sequence of revisions of those contracts. If that dynamics converges we will have the long run equilibrium of the system. To define this concept we are going to use the following notation. Let $F_0 = F$ and F_n the probability distribution associated to $\Phi^{(n)}(F)$ (recall that $\Phi : \triangle(T) \to \triangle(T)$).

Definition 2.4: A long-run equilibrium of the contractual system **S** is a vector $(y^*, x_S^*, (x_{D,t}^*)_{t\in T})$ satisfying 1, 2 and 3 of Definition 2.1, where the distribution of characteristics is a F^* such that $F_n \to F^*$ in the weak topology.

If the distribution of characteristics of the long run equilibrium is a Dirac measure then the asymmetric information will vanish, otherwise the problem will prevail. The following theorem relates the long run equilibrium with the stationary equilibrium of a contractual system.

Theorem 3. *Under hypotheses of Theorem 1, any long run equilibrium is a stationary equilibrium.*

In general, the sequence $(F_n)_{n\geq 0}$ may not converge. It may have cycles or accumulation points, depending on the initial condition $F_0 \in \triangle(T)$. It is important to analyze the behavior of this sequence in order to know if group effect will be persistent. The relevance of the stationary equilibrium will be analyzed in Section 4.

The next theorem gives sufficient conditions for existence of a long run equilibrium and therefore of a stationary equilibrium.

Theorem 4. *In addition to hypotheses of Theorem 1, suppose that T is a compact interval in R. Long run equilibrium exists if at least one of the following conditions is satisfied:*

a) *For all $t \in T$ and $F \in \triangle(T), t^{new}(t, F) \geq t$, (or $t^{new}(t, F) \leq t$).*
b) *For all $F \in \triangle(T), t^{new}(t, F) = t^{new}(t)$, and the function $t^{new}(t)$ has a global attractor.*

3. Some examples

In this section, three examples will be provided in order to illustrate the framework developed in Section 2. They are based on classical models given in the asymmetric information literature. It is not our intention, however, to explain the phenomena analyzed in those articles. Rather, we use these examples to calculate the long run and stationary equilibria and finally to illustrate the diversity of equilibria that may arise in this structure.

3.1. *The Labor Market Model*

In an experiment performed by Sillamaa (1999) the response of the worker effort to wage taxation is analyzed. It is an indication that the worker decision about the level of effort depends on the net wage he receives. Using that fact we elaborate the following example. A firm (contractor) is hiring workers (contractees) whose skills (and therefore their productivities) are not observable by him. Suppose that the level of effort and skill are highly correlated;[4] it will allow us to consider the variables "skill" in the adverse selection problem and "effort" in the moral hazard problem as being the same. This kind of identification, which seems reasonable in some situations, has not been included in the standard mixed models of the asymmetric information literature.

The effort/productivity of a worker affects her individual labor supply in the following way: if the worker with productivity $t \in T = [a, 1](a > 0)$ offers $x \geq 0$ units of labor then the effective labor supply will be tx. Since the firm does observe neither the skills nor the individual production, it takes into account the prior distribution of productivities of workers defined by the distribution F in order to decide the labor demand. As above, $F(t)$ is the proportion of workers with productivity lower than t.

If $f : R \to R$ is the individual production function, which associates the effective units of labor offered by a worker to the number of units to be produced by her, the problem of the risk neutral firm is:

$$\max_{x \geq 0} \int_T (f(tx) - yx)\, dF(t)$$

where $y > 0$ is the real wage of the worker. Notice that the objective function of the firm is the summation of the profits given by the contract with each worker. The first order condition (sufficient for convex technologies) is:

$$y = \int_T f'(tx) t dF(t)$$

From this equation we can find the labor demand $x^D(y, F)$.

Regarding the labor supply, the worker with personalized skill t (her private information) has utility function $u(t, c, l,)$, which depends on the private consumption c and leisure l. The worker's problem is to decide on these three variables, which is done in two stages:

First Stage: Given the personalized skill t, she chooses private consumption and individual labor supply in order to:

[4] This is the case if the wealth effect strongly dominates the substitution effect. In that situation, a highly skilled worker will prefer to apply more effort (rather than increase her leisure) in order to obtain more gains.

$$\begin{aligned} &\max_{x \geq 0} u(t, c, l) \\ \text{subject to } & c = yx \\ &(x/t) + l = 1 \end{aligned}$$

In this model, the worker is contracted (and she receives a wage) to work x units of time. However, depending on her skill, the working time may be lengthened, making lower the effective time of leisure.

The first order condition of the problem above is: $yu_c(t, c, l) - (1/t)u_l(t, c, l) = 0$, where c and l are defined by the problem constraints. From this we obtain $x^S(t, y)$, the individual labor supply of worker t.

Therefore the equilibrium wage $y^* = y^e(F)$ is given by the following equation:

$$\int_T x^S(t, y^*)dF(t) = x^D(y^*, F)$$

As asserted in Section 2, the worker does not decide the level of effort in this first stage because she may have a habit formation which prevents it. Thus, any changes in the level of effort only may take place after signing the contract. The firm is only able to evaluate the current distribution of skills in order to maximize its profits and to define the supply of labor contracts. The market then decides the equilibrium wage. Once the wage is defined and the contract is signed, the worker may decide whether she continues with the initial level of effort or changes it, depending on her total payoff. That sort of behavior (the choice of the level effort depending on the net wage) was already analyzed by Sillama (1999).

Second Stage: Given the first stage equilibrium wage $y^* = y^e(F)$ and the labor supply contracted $x^* = x^D(t, y^e(F))$, the t-worker redefines her level of effort in order to:

$$\max_{t' \in T} u\left(t', y^* x^*, 1 - (x^*/t')\right)$$

The solution of problem above $t^* = t^{new}(t, F)$ will provide the new distribution of productivities $t^{new}(\tilde{t}, F)$ (where $\tilde{t}$ is the random variable associated to the distribution F) and the group effect results.

To illustrate the model above, let us consider the following functional forms: $f(x) = (1/\alpha)x^\alpha; \alpha \in (0, 1), u(t, c, l) = \ln(c) + \beta \ln(l) - \gamma \ln(t); \beta > 0, \gamma > 0$. With this, we will obtain the following:

$$\begin{aligned} &x^D(y, F) = \left(\frac{E[\tilde{t}_0^\alpha]}{y}\right)^{1/(1-\alpha)}; x^S(t, y) = \frac{t}{1+\beta}; y^e(F) = \frac{(1+\beta)^{1-\alpha} E[\tilde{t}_0^\alpha]}{(E[\tilde{t}_0])^{1-\alpha}}; \\ &t^{new}(t, F) = (1 + \beta/\gamma 1 + \beta)\, t \end{aligned}$$

Recall that if the right side of the equation that defines, $t^{new}(t, F)$ is greater than 1 then $t^{new}(t, F) = 1$ and if it is lower than a then $t^{new}(t, F) = a$. Notice that for each $F \in \triangle(T)$, there exists a unique $y^e(F)$, which is continuous in F. We can also find the long run equilibrium of the system. We can use the Theorem 4a to conclude that:

1) If $\gamma < 1$ then for all $F_0 \in \triangle(T), F_n \to \delta_1$, (the Dirac measure concentrated in "1");
2) If $\gamma > 1$ then for all $F_0 \in \triangle(T), F_n \to \delta_a$ and
3) If $\gamma = 1$ then for all $F_0 \in \triangle(T), F_n = F_0$.

Then for the cases 1) and 2) the asymmetric information problem and the group effect vanish in the long run. On the other hand, for the case 3) any initial distribution is invariant and therefore, a long-run equilibrium; so the asymmetric information problem may prevail.

3.2. *The Loan Market Model*

An empirical study developed by Fay et al. (2002) found evidence that households are more likely to file for bankruptcy if they live in districts which have higher aggregate bankruptcy filing rates. It suggests that bankruptcy may have a contagion effect in a group of borrowers. The following example illustrates how the loan market itself may trigger that contagion. Consider a loan market with one representative lender (contractor) and many borrowers (contractees). Each borrower has a personalized default rate on loans. [5] In a mixed model of asymmetric information we deal with the personalized rate of default as being the adverse selection variable whereas the effort to repay the debt would be the moral hazard variable. However we may suppose that those variables are perfectly correlated if the uncertainty is completely relegated to the effort to repay the debt.

Thus, let t be the default rate (*i.e.* $1-t$ is the payment rate or the effort rate to repay the debt) of one individual and $F(t)$ the proportion of individuals with default rate lower than t. The support of F is the interval $T = [0, 1]$. If y is the interest rate on loans, then the return of one unit of loan given to individual t is $(1-t)(1+y)$. Each lender has an expected utility function $U^L(c_0, c_1) = u_L(c_0) + u_L(c_1)$, where c_0 and c_1 are the consumption plans for periods 0 and 1 respectively. For simplicity we will suppose that lender's initial endowment is $w_0 = 1$ and $w_1 = 0$.

The lender problem is:

$$\max u_L(c_0) + \int_T u_L(c_t)dF(t)$$
$$\text{subject to } c_0 + x = 1$$
$$c_t = (1-t)(1+y)x$$
$$\Rightarrow \max_{m\geq 0} u_L(1-x) + \int_T u_L\left((1-t)(1+y)x\right)dF(t)$$

[5] The determinants of personalized default are analyzed by Barron and Staten (1998) and Gropp et al. (1997).

As in the former model, the objective function is the summation of the utilities attained from the contract signed with each borrower. The first order condition for an interior solution is:

$$u_L'(1-x^*) = \int_T (1-t)(1+y)u_L'\left((1-t)(1+y)x^*\right) dF(t)$$

the solution of this problem provides the loans supply $x^S(y,F)$.

There exists a great number of borrowers without initial endowment in the first period and with one unit of initial endowment in the second period ($w_0 = 0, w_1 = 1$). In Dubey et al. (2005) the payoff function of a borrower is:

$$V\left(c_0, c_1, \varphi, D\right) = u(c_0, c_1) - \lambda\left[pA\varphi - D\right]$$

where c_0 and c_1 are the consumption plans in each period, pA is the nominal return of one unit of the financial asset, φ is the total sales of assets and D is the delivery on the total debt $pA\varphi$. The parameter λ is the utility penalty on the non-paid debt. We can rewrite that function in the following form:

$$u(c_0, c_1) - \lambda\left[1 - \frac{D}{pA\varphi}\right] pA\varphi = u(c_0, c_1) - \lambda t pA\varphi$$

where $t = 1 - \frac{D}{pA\varphi}$ is the (personalized) rate of default. Therefore, we consider the payoff function of individual t as being $V(t, c_0, c_1, x) = u_B(c_0) + u_B(c_1) - \lambda t(1+y)x$.

The optimal decision of a borrower will be made in two stages:

First Stage: Given the contractual interest rate on loans y, the borrower t solves the following problem:

$$\begin{aligned} &\max_{x \geq 0} V\left(t, c_0, c_1, x\right) \\ &\text{subject to } c_0 = x \\ &c_1 + (1-t)(1+y)x = 1 \end{aligned}$$

or equivalently:

$$\max_{x \geq 0} u_B(x) + u_B\left(1 - (1-t)(1+y)x\right) - \lambda t(1+y)x$$

The solution of that problem is the loans demand of agent $t : x^D(t,y)$.

With this information, the equilibrium interest rate of loans $y^* = y^e(F)$ satisfies:

$$\int_T x^D(t, y^*) dF(t) = x^S(y^*, F)$$

The dependence of the equilibrium interest rate on the default distribution was detected by Gropp et al. (1997). They found evidence that households living in states with higher bankruptcy exemptions (where the bankruptcy filing is more likely) pay higher interest rates on loans than households which live in states with lower bankruptcy exemptions.

If we suppose that the personalized rate of default is affected by the habits in defaulting, the borrower will take it as fixed when deciding the loans demand. However, after signing the contract, she may change that habit if it improves her welfare. Those individual decisions may vary the default profile of the group and increase the default rate in the market. That kind of deterioration in the risk composition of borrowers was proposed by Gross and Souletes (2002) as an explanation for the increase in the default rate of the market.

Second Stage: At this stage, agent t will revise her decision of default rate. It means that given the equilibrium interest rate $y^* = y^e(F)$ and her former decision of demand for loans $x^* = x^D(t, y(F))$ she will find a new default rate t^* that solves:

$$\max_{t' \geq 0} u_B(x^*) + u_B\left(1 - (1-t')(1+y^*)x^*\right) - \lambda t'(1+y^*)x^*$$

Notice that the decision of changing the personalized default rate depends on the evaluation of the net cost for defaulting. This is another explanation given by Gross and Souletes (2002) for the increase in the default rate of the market.

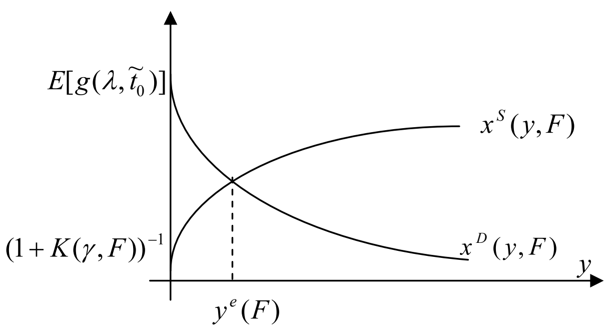
Solving the maximization above, we obtain the new default rate of borrower $t : t^* = t^{new}(t, F)$ and the new distribution of default rates is given by the random variable $t^{new}(\tilde{t}, F)$. This is the group effect in this model.

Using the following specifications for the model we explicitly obtain the equilibrium. Let $u_L(c) = c^{1-\gamma}/(1-\gamma); \gamma < 1$ and $V(t, c_0, c_1, x) = \ln(c_0) + \ln(c_1) - \lambda t(1+y)x$ be the utility functions of the contractor and contractees respectively. Then we will obtain:

$$x^S(y, F) = \frac{1}{1 + (1+y)^{1-1/\gamma} E[(1-\tilde{t}_0)^{1-\gamma}]^{-1/\gamma}}; x^D(t, y) = \frac{g(\lambda, t)}{1+y}$$

where: $g(\lambda, t) = \frac{\lambda t + 2(1-t) - \sqrt{\lambda^2 t^2 + 4(1-t)^2}}{2\lambda t(1-t)}$. Before finding the characteristics change function t^n, we show the graphic of the demand and supply of loans in Figure 1. Let $K(\gamma, F) = (E[(1-t_0)^{1-\gamma}])^{-1/\gamma}$.

Fig. 1. Demand and supply of in the Loan Market model



It is easy to see that a sufficient condition for existence of equilibrium is $(1 + K(\gamma, F))^{-1} \leq E[g(\lambda, \tilde{t}_0)]$. This condition is violated, for example, for the following case: $\lambda = 3; \gamma = 0.9$ and $F = \delta_{0.5}$. Therefore the additional hypothesis of Theorem 1 ii) must be assumed since equilibrium could not exist. On the other hand, it is not difficult to prove that if $\lambda \in (0, 2]$ then $g(\lambda, t) \geq 1/2, \forall t \in [0, 1]$ and therefore $E[g(\lambda, \tilde{t}_0)] \geq 1/2 \geq (1 + K(\gamma, F))^{-1}$. This implies the existence and uniqueness of equilibrium. Thus, if $y^e(F)$ exists then it must satisfy:

$$\frac{1}{1 + (1+y)^{1-1/\gamma}(E[(1-\tilde{t}_0)^{1-\gamma}]^{-1-\gamma}} = \frac{E[g(\lambda, \tilde{t}_0)]}{1+y}$$

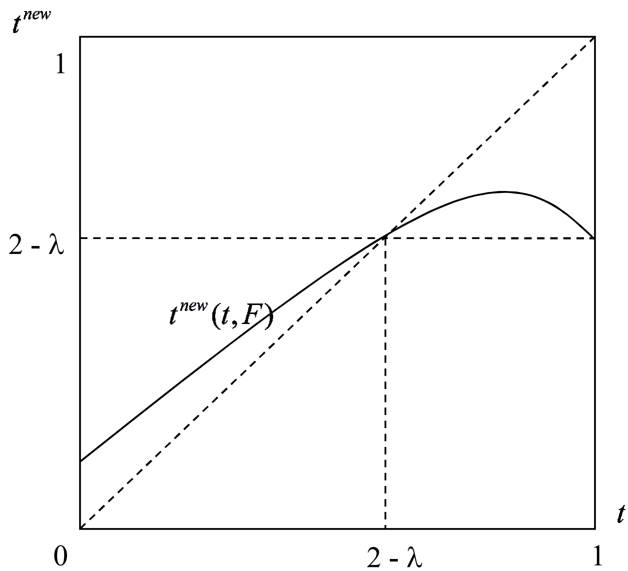
with that $y^e(F)$, the characteristics change function:

$$t^{new}(t, F) = 1 - \frac{(1-\lambda^{-1})}{g(\lambda, t)}$$

Notice that if the right side of the equality above is greater than 1 then we must choose $t^{new}(t, F) = 1$ and if it is negative we must choose $t^{new}(t, F) = 0$. Figure 2 shows the function t^{new} for a value of $\lambda \in (1, 2)$. It can be proven that for all $\lambda \in (1, 2)$ the function t^{new} has a unique interior fixed point equal to $\bar{t} = 2 - \lambda$ which is a global attractor for the dynamics generated by the iterations of t^{new}.

Due to the existence of a global attractor of the transition function t^{new}, Theorem 4b allows us to conclude that for any $F_0 \in \triangle(T), F_n \to \delta_{\bar{t}}$. Therefore the asymmetric information problem, as well as the group effect, vanishes in the long-run.

Fig. 2. The characteristics chanbe function in the Loan Market model



3.3. *The Insurance Market Model*

This last example provides an evolutionary dynamic for the insurance market, where the contracts are revised in the form described in Section 2. Ania et al. (2002) defined another evolutionary dynamics followed by bounded rational insurance firms; they offer menus of insurance contracts which are periodically revised: profitable competitors' contracts are imitated and loss-making contracts are withdrawn. In the words of Wilson (1977, p. 205), it lacks "an explicitly dynamic model, which describes how firms adjust their policies over time". In this section we provide a model of dynamic correction of contracts due to changes in the risk profile of the insured population.

There is an insurance company offering to pay part of the losses suffered by individuals in case of hazard damages. Each individual is characterized by his personal probability of damage $t \in T = [0, 1]$, which depends on the effort to avoid the lose. In mixed models of insurance markets, it is usual to consider the personal probability of damage as the adverse selection variable, whereas the effort to avoid the loss is the moral hazard variable. In our model here we are going to suppose that both variables are perfectly negative correlated; thus greater effort to avoid the loss corresponds to lower probability of damage. The heterogeneity of abilities to avoid the risk of hazard damage was already used in Landsberger and Meiluson (1996). In that work the probabilities of suffering losses are different.

Thus, we will suppose that the effort is $e = 1 - t$. The company takes as given $y \in Y = [0, 1]$, the insurance premium per dollar of coverage. The distribution of personalized rates of damages (and therefore of level of efforts) of the population is given by F, thus $F(t)$ represents the proportion of individuals with probability of hazard damage lower than t. The insurance company is risk neutral and will choose the insurance supply x in order to maximize the sum of the profits earned from each individual:

$$\max_{x \geq 0} \int_T (yx - tx) dF(t)$$

The solution of the problem above is a totally elastic insurance supply:

$$\int_T \{y^* - t\}\, dF(t) = 0 \Rightarrow y^* = E[\tilde{t}_0]$$

In this case, we can conclude in advance that $y^e(F) = y^*$.

Let us find the aggregate demand for insurance. Suppose that the individual t may lose her initial wealth $w_0 = 1$ with probability t and she has an expected utility function, which also depends on her effort level in avoiding the damage. Specifically $u(w, e)$ is the utility of the individual with final wealth w and effort level e. Then the individual's problem will be solved in two stages:

First Stage: Given the risk premium per dollar $y \in Y$ and the individual risk profile $t \in T$, the individual has to find the optimal level of insurance coverage in order to:

$$\max_{x\in[0,1]} tu\,(x - yx, 1 - t) + (1 - t)u\,(1 - yx, 1 - t)$$

The solution of the problem above provides the individual demand for insurance $x^D(t, y)$. Since the equilibrium insurance premium is y^* given above, the aggregate demand results:

$$\int_T x^D(t, y^*)dF(t)$$

Second Stage: Given the premium per dollar of coverage y^* and the individual demand for insurance $x^* = x^D(t, y^*)$, the individual will find the new level of effort $e = 1 - t^*$ in order to:

$$\max_{t'\in T} t'u\,(w - L - y^*x^* + x^*, 1 - t') + (1 - y^*)u\,(w - y^*x^*, 1 - t')$$

the solution of this problem provides the new risk profile that the individual will adopt: $t^* = t^{new}(t, F)$. Finally, the new distribution of characteristics will be given by the distribution of $t^{new}(\tilde{t}, F)$.

Using the following specification for the utility functions of contractees $u(w, e) = \ln(w) + \beta\ln(1 - e)$ we obtain:

$$x^D(t, y) = \begin{cases} t/y & \text{if it is not greater than 1} \\ 1 & \text{otherwise} \end{cases}$$

The aggregate demand for insurance results $\int_T x^D(t, y)dF(t) = 1 - (\int_T F(t)dt)y^{-1}$. With this, the total demand (and supply) for insurance in equilibrium is:

$$x^S = 1 - \frac{\int_0^{y^*} F(t)dt}{y^*}$$

Solving the second stage problem of the contractee we obtain:

$$t^{new}(t, F) = \begin{cases} 1; & \text{if } t \geq \hat{t} \\ \frac{\beta}{\ln(1-t) - \ln(t) - \ln((1/y^*) - 1)} = \frac{\beta}{h(t)}; & \text{if } t < \hat{t} \end{cases}$$

where $\hat{t} = [1 + ((1/y^*) - 1)e^{\beta}]^{-1}$. It is important to notice that in this example the transition function of types depends on the probability distribution function F. The function t^{new} is continuous, increasing and $(t^{new})'(0) = +\infty$. Depending on the parameter β the graphic of the function t^{new} may be one of the three possibilities showed in Figure 3.

Fig. 3. The characteristics change function in the Insurance Market Model

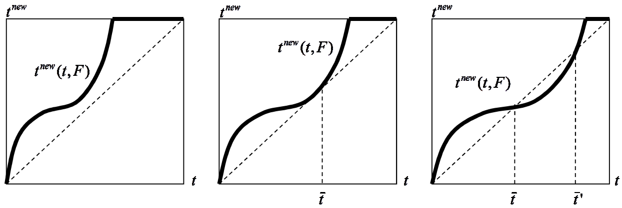


Figure 3.1: $\beta > \bar{\beta}$ Figure 3.2: $\beta = \bar{\beta}$ Figure 3.3: $\beta < \bar{\beta}$

As we can see, there exists $\bar{\beta} > 0$ such that if $\beta > \bar{\beta}$ then t^{new} has the shape depicted in Figure 3.1, if $\beta = \bar{\beta}$ then t^{new} is like in Figure 3.2 and if $\beta < \bar{\beta}$ then t^{new} is like in Figure 3.3. To find $\bar{\beta}$ we have to solve the following equations:

$$\frac{\bar{\beta}}{h(\bar{t})} = \bar{t} \text{ and } -\frac{\bar{\beta}h'(\bar{t})}{h^2(\bar{t})} = 1$$

Solving these equations we find that $\bar{\beta}$ is the only solution of the following equation: $\ln(\bar{\beta}) = -1 - \ln((1/y^*) - 1) - \bar{\beta}$. Finally, we will show the stationary distributions that may result in this model.

For the case corresponding to Figure 3.1, there is no stationary equilibrium. This is because the measures δ_0 and δ_1 are not stationary since the demand is not defined for these values. Therefore if there exists stationary measure its support must be inside the interval [0,1]. However if the function t^{new} is strictly increasing and greater than the identity function, it moves the support of the stationary measure toward the right side of the interval. That would violate the invariance of such support.

For the case corresponding to Figure 3.2 the stationary equilibrium is given by $F^* = \alpha\delta_i + (1-\alpha)\delta_1$, where $\bar{t} = \beta/(1+\beta)$ and $\alpha = (1+\beta)/(1+\beta e^{1+\beta})$. The stationary premium per dollar of coverage in this stationary equilibrium is $y^* = 1 - (1+\beta e^{1+\beta})^{-1}$. It is worth noting that the asymmetric information problem persists in the stationary distribution.

For the case in Figure 3.3 there exists multiplicity of equilibria. The stationary distribution is $F^* = \alpha_1\delta_{\bar{t}} + \alpha_2\delta_{\bar{t}'} + (1-\alpha_1-\alpha_2)\delta_1$ where the parameters are found in the following way. Given $\bar{\beta} > \beta$ solve $\bar{\beta} + \ln(\bar{\beta}) = -1 - \ln((1/y^*) - 1)$, where y^* is the expected value of types with respect to the stationary distribution. Next, take the two solutions of the equation:

$$\frac{1}{t} - 1 = \left(\frac{1}{t} - 1\right) e^{\beta/t}$$

these solutions $\bar{t}$ and $\bar{t}'(\bar{t} < \bar{t}')$ are the intersections of the function t^{new} with the diagonal (see Figure 3.3). Finally, choose $\alpha_1 \in (0,1)$ and $\alpha_2 \in (0,1)$ satisfying:

$$\alpha_1\bar{t} + \alpha_2\bar{t}' + (1-\alpha_1-\alpha_2) = y^*$$

As a numerical example, we report $\beta = 0.8, \bar{\beta} = 1.1, y^* = 0.9, \bar{t} = 0.238, \bar{t}' = 0.758, \alpha_1$ and α_2 satisfying $0.238\alpha_1 + 0.758\alpha_2 + (1 - \alpha_1 - \alpha_2) = 0.9$. Therefore there exists multiplicity of equilibria and for $\alpha_1 > 0$ and $\alpha_2 > 0$ the asymmetric information problem prevails in the stationary distribution. This case is particularly interesting because there exists indeterminacy of the population profile of characteristics which may justify the presence of a central planner in order to attain more efficient allocations.

4. Stationary Equilibrium and the Equilibrium in the Classical Approach of Asymmetric Information

In classical models of asymmetric information it is usual to suppose that the less informed individuals know the preferences and reservation utilities of the more informed individuals. In addition, adverse selection models suppose that the types which determine the hidden information in the contract are invariant. Our framework defined in Section 2 relaxes those two assumptions generating a model with a sequence of short-run equilibria that may converge to a long-run equilibrium. In this section we compare both equilibria (short and long run) with the classical notion of equilibrium for models with asymmetric information.

In fact, our model can be seen as a mixed model of adverse selection (AS) followed by moral hazard (MH), where the hidden information variable is the same in both problems. Classical mixed models of asymmetric information are presented in the literature (Laffont and Martimort 2002), however none of them deals with the same variable for both problems; although that identification emerges as a reasonable assumption in some cases (as discussed in the examples of the Section 3).

Recall the timing of our model: First, the principal or contractor (or group of contractors) offers a contract menu based on the distribution of characteristics (F) and on the contract price (y) in order to maximize his profits. Simultaneously each agent or contractee chooses her individual demand based on the price of the contract and her individual type in order to maximize her payoffs. Second, the market adjusts the contract price in order to equalize supply and the total demand. Until this point, the AS problem with competitive principals took place. Finally the MH problem appears changing the characteristics of the individuals in order to attain a greater payoff; this modifies the characteristics distribution profile and another round for negotiations emerges. Observe that principals can not foresee the possible change in the distribution of characteristics since they do not know the preferences of the agents.

It what follows, we are going to prove that both, the short-run and the long-run equilibria defined in Section 2 are also equilibrium in the classical model of AS. Let us write our model in the classical framework of AS with competitive principals. Given the price of the contract $y \in Y$, a contract menu satisfying the compatibility of incentives (CI) is a $(x_t)_{t \in T}$ such that for all $t \in T$ we have that:

$$U\left(t, x_t, y\right) \geq U\left(t, x_{t'}, y\right); \forall t' \in T \tag{CI}$$

If $(\bar{U}_t)_{t \in T}$ defines the reservation utilities of the agents, the menu satisfies the participation constraint (PC) if for all $t \in T$:

$$U\left(t, x_t, y\right) \geq \bar{U}_t \tag{PC}$$

Therefore, the principal problem is to find:

$$\hat{x}_t(y) = \underset{(x_t)}{\operatorname{argmax}}\, B\left(\int_T x_t dF(t), y, F\right)$$

such that (CI) and (PC) are satisfied

From that definition of equilibrium of classical AS models we can state the last theorem whose proof is given in the Appendix.

Theorem 5. *Suppose that for all* $t \in T, \bar{U}_t \leq \min_x U(t, x, y)$. *Then any equilibrium (short-run or long-run) of the model given in Section 2 is equilibrium in the classical AS model.*

The theorem above asserts that in each contract revision the evolution of the system is trough equilibria (according to the classical definition of AS). This is a remarkable result since it asserts that assuming less hypotheses than in classical models (principals do not know the utilities of the agents), our equilibrium notion coincides with the classical one. Moreover, the MH problem that arises right after the signature of the contract triggers an evolution of the characteristics distributions. If the sequence of distributions converges, our Theorem 3 claims that the limit distribution is invariant and the MH problem vanishes whereas the AS problem may prevail if the stationary distribution is not degenerated (i.e. if it is not a Dirac measure).

5. Conclusions

In this article we describe the evolutionary dynamics followed by the population in an asymmetric information framework. In this structure both the adverse selection problem and the moral hazard problem are present and they occur consecutively, keeping the same variable (called the characteristic of the individual) as the hidden information variable of the model. In addition, we suppose that the non-informed party of the contract (contractor or principal) knows the distribution of characteristics of the population of the informed party (contractees or agents), but he does not know their individual utility functions. This is the novel ingredient of our framework and the reason for not implementing a contract menu in our models of moral hazard or a non-linear pricing in our models of adverse selection. In addition, after signing the contract, the contractees may change their characteristics in order to maximize their utilities. The contractor cannot foresee that change since he ignores the contractees' utilities. As a consequence we obtain a group effect (changes in the distribution of characteristics of the group of contractees) resulting purely from the asymmetric information problem. This type of group effect has not been considered in the literature of social interactions and may amplify the classical peer-group effect resulting from peer interactions.

The revision of the contracts resulting from the update of the types distribution creates a sequence of temporary equilibria, which coincide with the classical notion of equilibrium in adverse selection models. We provide some sufficient conditions to obtain the convergence of that sequence. In such a case, the "limit contract" is stationary and we show that in this equilibrium, the moral hazard problem disappears and the adverse selection problem may remain. We believe that this result contributes to the better understanding of asymmetric information mixed models where the hidden information variable is the same in both problems. For example, in regulation problems, where the regulator has information about the distribution of characteristics of the agents (or firms) that he wants to regulate, but he does not know their specific costs, our result has a very simple and important implication: revision of contracts may lead the contractor to eliminate moral hazard problems in the long-run.

Finally we provide three examples where our framework can be applied. For each one, the equilibria are calculated and those examples show the diversity of possibilities for the long-run equilibrium: uniqueness or multiplicity may arise. Future research might find conditions on the fundamentals in order to characterize the types of equilibria.

The framework and examples are closely linked to the contract theory, but they are not restricted to it. In fact, we can also use this structure in sociology to try to explain the changes in the behavior or characteristics of some social groups resulting from the treatment they receive from the society as a whole. Individuals in a social group try to adapt to the treatment they receive in order to maximize their own welfare. Besides this, the individuals understand that their personal decisions about the new characteristics will not change the treatment. The new characteristics will define the new profile for the group and a new treatment will emerge. This evolutionary dynamic may stop in a stationary group profile and treatment and, as shown in one of our examples, it may depend on the initial distribution of characteristics.

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Appendix

Proof of Theorem 1 (part i): By the strict concavity of B and U, the solutions $x^S(y,F)$ and $x^D(t,y)$ of problems (1) and (2) are unique and by the Theorem of the Maximum they are continuous functions of their variables. To prove that Z is continuous it is sufficient to show that $X^D(y,F)=\int_T x^D(t,y)dF(t)$ is a continuous function. Let $(y_n)_n$ and $(F_n)_n$ be two sequences such that $y_n \to y$ and $F_n \to F$ (the last convergence is in the weak topology). Then we have:

$$\left|X^D(y,F)-X^D(y_n,F_n)\right| = \left|\int_T x^D(t,y)dF(t) - \int_T x^D(t,y_n)dF_n(t)\right|$$
$$\leq \left|\int_T x^D(t,y)dF(t) - \int_T x^D(t,y)dF_n(t)\right|$$
$$+\left|\int_T \left(x^D(t,y)-x^D(t,y_n)\right) dF_n(t)\right|$$

The first term goes to zero as $n \to +\infty$ because x^D is a continuous function and $F_n \to F$. The second term is lower than $||x^D(.,y)-x^D(.,y_n)||$ which goes to zero as $n \to +\infty$ because continuity of x^D in a compact set implies uniform continuity.$\square$

Proof of Theorem 3: It will be sufficient to prove that the function $\Phi : \triangle(T) \to \triangle(T)$ is a continuous function. From definition of $\Phi(F)$ given in (3), we have that for each measurable function f:

$$\int_T f(z)\Phi(F)(dz) = \int_T f\left(t^n(t,F)\right)F(dt) \tag{A1}$$

Let $(F_k)_k$ be a sequence of probability distributions, which converges to F^* in the weak topology. Then for any $f \in C^0(T)$:

$$\begin{aligned}\lim_{k\to+\infty}\int_T f(z)\Phi(F_k)(dz) &= \lim_{k\to+\infty}\int_T f(t^{new}(t,F_k))F_k(dt)\\ &= \int_T f(t^{new}(t,F))F(dt)\\ &= \int_T f(z)\Phi(F)(dz)\end{aligned}$$

where the first and the last equalities result from (A1) and the second one from the continuity of f and t^{new} and the weak convergence of the sequence $(F_k)_k$.

Proof of Theorem 4:
a) Suppose that $t^{new}(t,F) \geq t, \forall t \in T$. For each $z \in T$ we have that:

$$\Phi(F)(z) = Prob\left[t^{new}(\tilde{t},F) \leq z\right] \leq Prob[\tilde{t} \leq z] = F(z)$$

Therefore $\Phi(F)$ dominates F in the sense of the first stochastic degree dominance ($\Phi(F) \geq_{FSD} F$). It implies the following ordering in the sequence of distributions: $F_0 \leq_{FSD} F_1 \leq_{FSD} F_2 \leq_{FSD} \ldots$. Using the Helly theorem (Billingsley 1979) the sequence $(F_k)_k$ weakly converges to a $F^* \in \triangle(T)$.

b) Let $\bar{t} \in T$ be the global attractor of t^n. In this case it will be better to analyze the probability measure of types rather than the probability distributions. Let $(\mu_k)_k$ be the probability measures associated to $(F_k)_k$. We will show that the support of μ_k converges to $\{\bar{t}\}$. Since $F_1(z) = Prob[t^{new}(t) \leq z] = \mu_0((t^{new})^{-1}(]-\infty, z]))$, we conclude that $\mu_1(A) = \mu_0((t^{new})^{-1}(A))$ for each measurable set $A \subseteq T$. By induction we conclude that $\mu_k(A) = \mu_0((t^{new})^{-k}(A))$ for all k. By using the property $(t^{new})^{-k}((t^{new})^k(A)) \supseteq A$ we obtain that for all $k, \mu_k((t^{new})^k(A)) \geq \mu_0(A)$. Taking $A = T$ we conclude that the support of μ_k is included in $(t^{new})^k(T)$ which converges to $\{\bar{t}\}$.

Proof of Theorem 5: Let $((\bar{x}_t), \bar{y}, F)$ be equilibrium in our framework. Then for all $t \in T$:

$$U(t, \bar{x}_t, \bar{y}) \geq U(t, x, \bar{y}); \forall x \in X,$$

Putting $x = x_{t'}$ we obtain the CI restriction.

Since $\bar{U}_t \leq \min_x U(t, x, y)$ for all t and y, then $(\bar{x}_t)$ also holds the PC.

Finally, let $\bar{x} = \int_T \bar{x}_t dF(t)$ and for any other contract menu which satisfies CI and PC let $x = \int_T x_t dF(t)$. Since $\bar{x}$ maximizes the contractor's profit, we obtain that:

$$B\left(\int \bar{x}_t dF(t), \bar{y}, F\right) \geq B\left(\int x_t dF(t), \bar{y}, F\right)$$

Thus, the menu maximizes the contractor profit in the classic model.