The Effects of Fiscal Policy and its Interactions with Monetary Policy in Brazil

Elcyon Caiado Rocha Lima
Universidade Estadual do Rio de Janeiro (UERJ) and Instituto de Pesquisa Econômica Aplicada (IPEA), Rio de Janeiro, Brazil

Alexis Maka
Instituto de Pesquisa Econômica Aplicada (IPEA), Rio de Janeiro, Brazil

Amadeu Pumar
Universidade Estadual do Rio de Janeiro (UERJ), Rio de Janeiro, Brazil

Abstract This paper analyzes the effects of fiscal policy shocks on the dynamics of the economy and the interaction between fiscal and monetary policy using structural vector autoregressions (SVARs). We test the Fiscal Theory of the Price Level for Brazil, analyzing the response of public sector liabilities to primary surplus shocks. For the hybrid identification we find that it is not possible to distinguish empirically between Ricardian (Monetary Dominance) and non-Ricardian (Fiscal Dominance) regimes. However, using sign restrictions there is some evidence that the government followed a Ricardian (Monetary Dominance) regime from January 2000 to June 2008.

Keywords: Structural VAR, Hybrid Identification, Fiscal Policy, Monetary Policy
JEL Classification: E62, E63, C32, E31

Resumo Este artigo analisa os efeitos de choques na política fiscal sobre a dinâmica da economia e a interação entre as políticas fiscal e monetária usando modelos SVARs. Testamos a Teoria Fiscal do Nível de Preços para o Brasil analisando a resposta do passivo do setor público a choques no superávit primário. Para a identificação híbrida, encontramos que não é possível distinguir empiricamente entre os regimes Ricardiano (Dominância Monetária) e não-Ricardiano (Dominância Fiscal). Entretanto, utilizando a identificação de restrições de sinais, existe evidência que o governo seguiu um regime Ricardiano (Dominância Monetária) de janeiro de 2000 a junho de 2008.

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E-mail addresses: elcyon.rocha-lima@ipea.gov.br, alexis.maka@ipea.gov.br, amadeu.pumar@ipea.gov.br

1. Introduction

Recently, there has been a worldwide movement toward the adoption of a policy regime in which the central bank is assigned the task of achieving an inflation target. At the same time, the independence of central banks to pursue this goal has also increased, suggesting that the choice of monetary policy to achieve the inflation target is a problem that can, and in fact ought to be, separated from the choice of fiscal policy or any other public policy. As pointed out by Sims (1994), in a rational expectations, market-clearing equilibrium model with a costlessly-produced fiat money that is useful in transactions, the following is true:

i) the existence and uniqueness of the equilibrium price level cannot be determined from knowledge of monetary policy alone;
ii) the determinacy of the price level under any policy depends on the public's beliefs about what the policy authority would do under conditions that are never observed in equilibrium.

Therefore, fiscal policy plays an important role, and the choice of monetary policy to achieve the inflation target should not be separated from the fiscal policy adopted by the government.

This paper aims to uncover some stylized facts related to the effects of fiscal policy shocks on the dynamics of the Brazilian economy and the interaction between fiscal and monetary policy in Brazil. To achieve our goal we use a structural vector autoregression (SVAR) model and the test proposed by Canzoneri et al. (2001). The SVAR is identified by two alternative methodologies. The first methodology uses sign restrictions on impulse responses of the exogenous disturbances. The second methodology, developed by Lima et al. (2009) [LMA], combines sign restrictions with restrictions on the contemporaneous causal interrelationships among variables, derived by Directed Acyclic Graphs (DAGs). LMA analysis is concerned mainly with the identification of the effects of monetary policy and exchange rate shocks, so no attention was given to fiscal policy. In this paper, we extend LMA analysis introducing a set of fiscal variables (budget surplus and public sector liabilities) in their VAR model. The hybrid identification strategy pursued in this article consists of two steps. In the first step, we use DAGs to select over-identifying restrictions on the contemporaneous coefficients based on the conditional independence relations between the variables. These over-identifying restrictions allow us to identify monetary policy and demand shocks, and to restrict the covariance matrix of the reduced-form residuals. In the second step, maintaining restricted the covariance matrix of reduced-form residuals, we keep the identified monetary policy and demand shocks, and impose sign restrictions on the impulse response functions on other three shocks to identify the fiscal policy, supply, and exchange rate shocks. Analyzing the case of Brazil, we observe for both identification strategies that in response to positive ("contracionist") fiscal shocks there is a significative and long-lasting reduction in the price level, and a short-lived reduction on economic activity. There is no evidence of significative response of the exchange rate to fiscal innovations.

Dungey and Fry (2009) [DF] propose a different hybrid identification approach that combines traditional short-run restrictions, sign restrictions and long run restrictions. The hybrid methodology adopted here has some similarities with the one used by DF. However, we do not use long run restrictions and instead of traditional short-run restrictions, we use DAGs to impose restrictions on the contemporaneous causal interrelationships among variables. As for the sign restrictions, we use the QR decomposition to generate the candidate shocks, while DF use the Givens rotation.[1]

The motivation for our hybrid strategy comes from the fact that the DAG and sign restrictions approaches complement each other, so that their combination may be superior than each methodology taken isolated. While the DAG approach imposes restrictions that may identify exogenous shocks, the response of variables to these shocks may indicate that they are not the ones we are trying to identify. They may be linear combinations of the shocks we are interested on or parameter uncertainty may be responsible for the distortions in the responses. On the other side, Sign restrictions have economic justification but may not impose enough restrictions to identify the shocks (as described in the previous paragraph). We believe that a combination of the available methodologies increases the chance that all shocks of interest are identified.

According to the traditional monetarist view, a necessary and sufficient condition for achieving price stability is a fully credible commitment of the central bank to stable prices. This traditional analysis has been challenged by the Fiscal Theory of the Price Level (FTPL), which links price determination to the government present value budget constraint, i.e. the equality of the public debt with the present discounted value of future expected primary surpluses.[2] The key intuition of the FTPL is that, if current and future fiscal policies are set without concern about sustainability, the general price level will "jump" in order to fulfill the present value budget constraint. This idea contrasts with the conventional monetarist theory of price determination, according to which the stock of money (and thus the central bank) is the sole determinant of the price level and fiscal policy is (often implicitly) assumed to passively adjust primary surpluses to guarantee solvency of the government for any price level.[3] Such fiscal policy is called Ricardian. The FTPL reverse the argument above: if the fiscal authority chooses primary surpluses independently of government debt, then it is the price level that has to adjust to satisfy the present value government budget constraint. This alternative regime is

[1] Fry and Pagan (2007) show that the QR decomposition and the Givens rotation are equivalent. However, as the model grows in size the QR decomposition is expected to be superior in terms of computational speed.

[2] For an introduction to the FTPL see Carlstrom and Fuerst (2000), Christiano and Fitzgerald (2000), and Canzoneri et al. (2001).

[3] This view can be summarized in Milton Friedman's dictum that "inflation is always and everywhere a monetary phenomenon".

called non-Ricardian.[4]

Two main features distinguish our work from the related empirical literature that tested the FTPL for Brazil. First, in contrast to the existing studies that applied Canzoneri et al. (2001) [CCD] test for Brazil,[5] which restrict their analysis to a 2-3 variable closed economy VAR model usually containing only the primary surplus and government liabilities, our investigation involves much more variables (9), including key variables like the exchange rate and interest rates, that allow us to better evaluate the impact of fiscal policy shocks and its interaction with other economic variables. Second, the identification strategies adopted in this article, depart from the Cholesky decomposition usually followed in the literature, and represent an effort to overcome the limitations of the available identification methodologies. We test the assumption, held by the Fiscal Theory of the Price Level, that the policy regime is non-Ricardian (Fiscal Dominance), applying the test proposed by CCD that analyzes the response of public sector liabilities to primary surplus shocks. This response depends on the identification adopted. For the hybrid identification we find that it is not possible to distinguish empirically between Ricardian (Monetary Dominance) and non-Ricardian (Fiscal Dominance) regimes. However, using sign restrictions there is some evidence that the government followed a Ricardian (Monetary Dominance) regime from January 2000 to June 2008.[6]

We also check if the identified exogenous monetary policy shocks show a "stepping on a rake" effect (tighter monetary policy leads to a higher inflation rate in the long run), as described by Sims (2008) in a theoretical framework designed for understanding the effects of fiscal uncertainties on monetary policy. According to our results, there is no evidence that a tighter monetary policy would lead to higher inflation in the long run.

The article is organized as follows. Section 2 presents an overview of the FTPL. Section 3 describes the empirical model and the results. Its first part presents the hybrid identification procedure that combines short-run restrictions on the contemporaneous coefficients with sign restrictions on the impulse response functions. Its second part shows an alternative identification procedure based on sign restrictions only. Finally, Section 4 offers some concluding remarks.

2. An Overview of the Fiscal Theory of the Price Level[7]

The government budget constraint is an accounting identity linking monetary and fiscal policies at each point in time and across time. The government budget

[4] Some authors refer to the Ricardian regime as "Monetary Dominance" and to the non-Ricardian regime as "Fiscal Dominance".

[5] See, for example, Tanner and Ramos (2002), Rocha and Silva (2004), and Fialho and Portugal (2005).

[6] The results of the studies that applied the CCD test for Brazil are mixed. Tanner and Ramos (2002) found evidence of non-Ricardian regime for the 1991-2000 period using monthly data. Rocha and Silva (2004) and Fialho and Portugal (2005) instead, found evidence of Ricardian regime for the 1966-2000 (with annual data) and 1995-2003 (with monthly data) period, respectively.

[7] The presentation of the FTPL presented in this section follows CCD closely.

constraint for period j can be written in nominal terms as [8]

$$B_t = (T_t - G_t) + (M_{t+1} - M_t) + B_{t+1}/(1 + i_t) \tag{1}$$

where M_t and B_t are the stocks of base money and government debt at the beginning of period t, $T_t - G_t$ is the primary surplus during period t, and i_t is the interest rate for period t.

Expressing the budget constraint in terms of total government liabilities, $M + B$, and scaling the fiscal variables on GDP, we have that

$$\frac{M_t + B_t}{P_t y_t} = \left[\frac{T_t - G_t}{P_t y_t} + \left(\frac{M_{t+1}}{P_t y_t}\right)\left(\frac{i_t}{1 + i_t}\right)\right] + \left(\frac{y_{t+1}/y_t}{(1 + i_t)(P_t/P_{t+1}}\right)\left(\frac{M_{t+1} + B_{t+1}}{P_{t+1} y_{t+1}}\right) \tag{2}$$

Equation (2) can be written synthetically as

$$w_t = s_t + \alpha_t w_{t+1} \tag{3}$$

where w_t is the liabilities to GDP ratio, s_t is the surplus (including seigniorage) to GDP ratio, and α_t is the discount factor represented by the ratio of the real growth in GDP to the real interest rate.

Iterating equation (3) forward from the current period, j, and taking expectations conditional on information available in period j, we obtain the present value budget constraint

$$w_j = s_j + E + t \sum_{t=j+1}^{+\infty} \left(\Pi_{k=j}^{t-1} \alpha_k\right) s_t \tag{4}$$

The difference between the conventional view and the FTPL lies on the way in which the government's present value budget constraint (equation (4)) is satisfied. The conventional view holds that this equation is a constraint on the government's tax and expenditure policies. According to this view, when equation (4) is disturbed, the government must alter its expenditures or its taxes to restore equality. FTPL advocates, however, argue that present value budget constraint is not a constraint on policy, but instead it is an equilibrium condition: when something threatens to disturb the equation, the market-clearing mechanism moves the price level, P, to restore equality.

The policy regime is said to be *Ricardian* (R) if the sequence $\{s_t\}$ is chosen so that the intertemporal budget equation (4) is satisfied no matter what P is realized. In contrast, if $\{s_t\}$ is chosen in a way that does not guarantee that equation (4) is satisfied for all possible prices, the policy regime is said to be *non-Ricardian* (NR). The assumption that the policy regime is non-Ricardian is what distinguishes the FTPL from the conventional view.

[8] We are assuming the government issues nominal liabilities (M and B); while the nominal values of these liabilities are fixed at the beginning of the period, their real values depend on the price level.

The Ricardian and non-Ricardian regimes are observationaly equivalent as they use the same equations to explain a given data set. It is not possible to test whether the government has chosen to follow a Ricardian or a non-Ricardian policy regime because the FTPL per se has no testable implications. The budget constraint (4) holds in equilibrium for both regimes. The issue is whether, in determining or adjusting towards equilibrium, the price level adjusts to expected future surpluses, or whether the path of surpluses adjusts in response to the price level. All we observe is an equilibrium; we do not observe who adjusted to bring about that equilibrium. However, one way of assessing the empirical value of the FTPL is viewing the non-Ricardian assumption as a starting point for a set of testable auxiliary assumptions that restrict the time series data, and then test those restrictions.

CCD proposed to differentiate between R and NR regimes studying the response of public liabilities to positive surplus shocks in a bivariate VAR. In an R regime, the surplus positive innovation pays off some of the debt, and w_{t+1} falls. In a NR regime, given a positive s_t innovation, there are three possibilities:

(i) corr $(s_t, s_{t+k}) = 0$ and corr $(s_t, \alpha_{t+k}) = 0; w_{t+1}$ constant

(ii) corr $(s_t, s_{t+k}) > 0$ and corr $(s_t, \alpha_{t+k}) > 0; w_{t+1}$ increases

(iii) corr $(s_t, s_{t+k}) < 0$ and corr $(s_t, \alpha_{t+k}) < 0; w_{t+1}$ decreases

In cases (i) and (ii) it should in principle be possible to differentiate between R and NR regimes. For example, the impulse response function from a VAR in s_t and w_t would tell us how w_{t+1} responds to an innovation in s_t. If w_{t+1} falls, we have an R regime; if it does not, we have an NR regime. However, in case (iii) w_{t+1} would fall in either an R regime or an NR regime, and we have an identification problem.

3. Data, Model Specification and Estimation

The model is estimated using monthly data and it is composed of the following variables: the short-term interest rate (SELIC), the nominal exchange rate (EXCHRATE), the price index (IPCA), the medium-term interest rate (SWAP), output, a monetary aggregate (M1), public sector net liabilities over GDP (LIABILITIES), primary surplus over GDP (SURPLUS), a discount factor based on nominal GDP (TXDESCS), a constant, and seasonal dummies.[9] We use the primary surplus as a measure of fiscal stance to avoid the problem of separately identifying tax revenue and government expenditure exogenous innovations.

Our sample period starts on 2000:01 and goes until 2008:06. The lag length chosen is six months. The model identifies five independent sources of exogenous disturbances: fiscal policy, monetary policy, demand, supply, and exchange rate shocks.

[9] A detailed description of the data and its sources can be found in Appendix I.

We use the Gibbs Sampling algorithm developed by Waggoner and Zha (2002) to estimate the model. A detailed description of the application of the methodology in our case is described in Appendix II.

4. Model Identification

Over the last years there has been a growing interest on graphical models and in particular on those based on DAGs as a general framework to describe and infer causal relations, exploring the connection between causal structure and probability distributions. These methods have been used in a variety of fields but are unfamiliar to most economists. Swanson and Granger (1997) were the first to apply graphical models to identify contemporaneous causal order of a SVAR, although they restrict the admissible structures to causal chains. Bessler and Lee (2002) use error correction and DAGs to study both lagged and contemporaneous relations in late 19th and early 20th century U.S. data. S. and Hoover (2003) evaluate the PC algorithm employed by TETRAD in a Monte Carlo study and conclude that it is an effective tool of selecting the contemporaneous causal order of SVARs. Awokuse and Bessler (2003) use DAGs to provide over-identifying restrictions on the innovations from a VAR and compare their results with the ones of Sims (1986). Moneta (2004) use DAGs and the data set of Bernanke and Mihov (1998) to identify the monetary policy shocks and their macroeconomic effects in the U.S.

4.1. *The hybrid approach*

The hybrid identification strategy pursued in this article consists of two steps. In the first step, we use DAGs to select over-identifying restrictions on the contemporaneous coefficients based on the conditional independence relations between the variables. These over-identifying restrictions allow us to identify monetary policy and demand shocks, and to restrict the covariance matrix of the reduced-form residuals. In the second step, maintaining restricted the covariance matrix of reduced-form residuals, we keep the identified monetary policy and demand shocks, and impose sign restrictions on the impulse response functions of the other three shocks to identify fiscal policy, supply, and exchange rate shocks.

Step 1: Selection of the Over-Identifying Restrictions to Identify Monetary Policy Shocks [10]

Spirtes et al. (2000) [SGS] developed algorithms for inferring causal relations from data that are embodied in a computer program used in this article, called TETRAD. [11] The program assumes a multivariate normal distribution and takes as input the covariance matrix of the variables of the model, [12] converting it into a correlation matrix and performing hypothesis tests in which the null hypothesis is a zero partial correlation.

Conditional independence is a key notion in multivariate analyses such as graphical modelling, where two vertices are connected if and only if the corresponding variables are not conditionally independent. To confirm the conditional independence, it is a common practice to check whether or not the partial correlation is close enough to zero. This is done because it is assumed that zero partial correlation suggests that the variables are conditionally independent, or nearly so. Under the assumption of multivariate normality, a test of zero correlation or zero partial correlation is also a test of independence or conditional independence. Moreover, if X, Y and Z are normally distributed, the partial correlation coefficient ρ_{XYZ} is zero if and only if X is independent of Y conditional on Z.

TETRAD begins with a 'saturated' causal graph, where any pair of nodes (variables) is joined by an undirected edge. [13] If the null hypothesis of zero partial correlation cannot be rejected – at, say, the 5% level, using Fisher's z test – the edge is deleted. [14] After examining all pair of vertices, TETRAD move on to triples, and so forth, orienting the edges left in the graph through the connection between probabilistic independence and graph theory. The final output of TETRAD is a set of observationally equivalent DAGs containing the proposed causal structure(s) of the model.

Robins et al. (2003) showed that the asymptotically consistent procedures of SGS are pointwise consistent, but not uniform consistent. [15] Furthermore, they also showed that there exists no causality test, based on associations of non-experimental data under the conditions assumed by SGS, which is uniform

[10] For an introduction on how to use DAGs to identify VARs, see Lima et al. (2008).

[11] The program is available for download at `www.phil.cmu.edu/projects/tetrad/index.html`. We used TETRAD III in this paper.

[12] In our application, the input of TETRAD is the covariance matrix of the reduced form VAR residuals.

[13] An edge in a graph can be either directed (marked by a single arrowhead on the edge) or undirected (unmarked). Arrows represent causal relationships: if there is an arrow pointing from X_i to X_j it means that X_i has a direct causal effect on X_j.

[14] In the case of the normal distribution, the partial correlation coincides with the conditional correlation, which is another measure of conditional independence of two random variables. See Baba et al. (2004) for further details.

[15] A pointwise consistent test is guaranteed to avoid incorrect decision if the sample size can be increased indefinitely. However, pointwise consistency is only a guarantee about what happens in the limit, not at any finite sample size. A stronger form of consistency, uniform consistency, guarantees that it is possible to bound the decisions error rates with a finite number of observations.

consistent. Therefore, for any finite sample, it is impossible to guarantee that the results of the SGS causality tests (or any other causality test) will converge to the asymptotic results.

Under the SGS model, it is sufficient to have a sample covariance between two variables, say, v_1 and v_2, exactly equal to zero to deduce that v_1 is not a cause of v_2. However, if the sample correlation between v_1 and v_2 is not exactly zero (as will almost always happens in finite samples) and the true model is unknown, as Robins et al. (2003) have shown, the acceptance or rejection of the null hypothesis of zero partial correlation is not unequivocally tied to the absence of causality. In other words we don't know, in any finite sample, how close to zero a partial correlation has to be to indicate non-causality. When the sample correlation is not exactly zero, it is not possible to determine which significance level should be used to test for zero partial correlation when attempting to test for the presence of causality. The "significance level", used by Tetrad, cannot be interpreted as the probability of type I error for the pattern output, but merely as a parameter of search. The higher is this parameter of the search, the smaller is the absolute value of the partial correlation that is taken as an indication of absence of causality. Intuitively, we are assuming that small partial correlations indicate small direct causal effect but we don't know how small the absolute value of the correlation has to be to obtain the correct causal inferences for the sample data we are using. Nevertheless, we can test the sensibility of the impulse response function of the model to different discrete values of this "parameter". [16]

Applying the software TETRAD on a 20% "significance level" [17] (our search parameter) and imposing the restriction that the SWAP rate [18] affects contemporaneously the SELIC rate set by the Central Bank we obtain a graphical representation of the DAG containing the contemporaneous causal ordering of the variables, displayed on Figure 1. [19, 20]

It is interesting to notice that the introduction of fiscal variables and discount factor changes completely the contemporaneous ordering obtained by LMA (see Figure 1 of their article). According to Figure 1, none of the policy variables affect contemporaneously the price level. The SELIC rate does not affect any variable contemporaneously, while the stock of money (M1) has an effect over the level of economic activity. LIABILITIES and SURPLUS have a contemporaneous effect only over M1 and the discount factor.

[16] This is a bit more data oriented than the usual procedure of changing the order of the Cholesky decomposition of reduced form VAR residuals to identify the model.

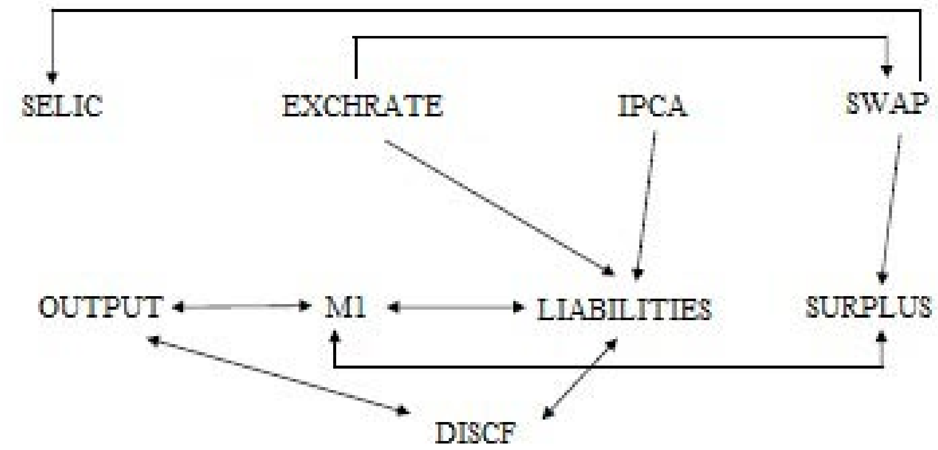
[17] We tested different discrete values for this parameter in the neighborhood of the chosen level (20%) and the model's impulse response function didn't change much.

[18] The 180 days SWAP rate is partially affected by the expectations of future SELIC rates.

[19] If we do not assume that the central bank takes into account the SWAP rate when setting the SELIC rate, we observe that the price level temporarily increases in response to a positive SELIC shock, a result known in the literature as the "price puzzle".

[20] In reality TETRAD puts an undirected edge between exchange rate and swap, meaning that there is causality in one of the two directions, but not on both. In what follows we restrict our attention on the causality going from exchange rate to the swap rate. However the results discussed next doesn't change much when the alternative causal ordering is used.

Fig. 1. Contemporaneous causal ordering based on DAGs



The causal ordering between the variables of the VAR can be represented by matrix A that establishes a relationship between reduced form and structural form residuals. The DAG pictured on Figure 1 can be represented by the following matrix:

$$A = \begin{bmatrix} A_{11} & 0 & 0 & A_{14} & 0 & 0 & 0 & 0 & 0 \\ 0 & A_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & A_{42} & 0 & A_{44} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & A_{55} & A_{56} & 0 & 0 & A_{59} \\ 0 & 0 & 0 & 0 & A_{65} & A_{66} & A_{67} & A_{68} & 0 \\ 0 & A_{72} & A_{73} & 0 & 0 & A_{76} & A_{77} & 0 & A_{79} \\ 0 & 0 & 0 & A_{84} & 0 & A_{86} & 0 & A_{88} & 0 \\ 0 & 0 & 0 & 0 & A_{95} & 0 & A_{97} & 0 & A_{99} \end{bmatrix}$$

where A_{ij} are parameters to be estimated and the vector of endogenous variables that multiplies A is given by [SELIC, exchange rate, IPCA, SWAP, output, M1, LIABILITIES, SURPLUS, TXDESCS].

The contemporaneous causal ordering resulting from the application of DAGs implies restrictions on the covariance matrix of the reduced-form residuals, meaning that we now have an overidentified model. Structural VAR models that are overidentified can be consistently estimated only by Bayesian estimation methods that introduce these restrictions on the covariance matrix of reduced form residuals. These restrictions are considered when Bayesian estimation methods are applied to the parameters of a structural VAR (and not to the parameters of a reduced form VAR). The method developed by Sims and Zha (1998), and adopted in this article, is one of these methods.

Using the contemporaneous causal ordering of Figure 1 to identify the SVAR, we obtained the impulse response functions of economic variables to exogenous and independent shocks, displayed on Figure 2. We identify SELIC shocks as monetary policy shocks and output shocks as demand shocks, leaving fiscal policy shocks to be identified by sign restrictions in the next step, when we identify also exchange rate and supply shocks in order to better identify fiscal policy disturbances.

According to Figure 2, after a positive monetary policy (SELIC) innovation that correspond to an increase in the SELIC rate, the stock of M1 falls and output decreases temporarily, taking near 12 months to recover. The direction of the exchange rate response is not clear, but it is more likely that it will depreciate slightly in the short-run. The price level goes down, but it takes near six months until the price level starts to fall despite the contraction of economic activity. The public sector net liabilities temporarily increase, probably as a result of a fall in the primary surplus and the larger interest payments. In response to a positive demand (output) innovation we observe an increase in prices and a possible exchange rate appreciation.

Step 2: Imposing Sign Restrictions to Identify Fiscal Policy, Supply and Exchange Rate Shocks

Having identified monetary policy shocks and demand shocks, and restricted the covariance matrix of the reduced-form residuals using the contemporaneous causal order suggested by TETRAD for the consolidated public sector, now we impose sign restrictions on the remaining impulse response functions in order to identify fiscal policy, supply and exchange rate shocks. The fiscal policy sign restrictions are based on the model developed by Sims (2008), while the supply and exchange rate restrictions are based on LMA and can be justified by the short-run dynamics of a stochastic open-economy macroeconomic model. Table 1 summarizes the sign restrictions on the IRFs used to identify the fiscal policy, supply, and exchange rate shocks.[21] The sign restrictions are supposed to hold for two months.

According to Table 1, a positive ("contracionist") fiscal shock does not reduce the primary surplus, does not increase the SELIC rate, the price level, output, the SWAP rate. A positive supply shock implies that prices do not increase, while output and primary surplus do not go down. An unexpected depreciation of the nominal exchange rate is supposed to imply changes in the same direction of the

[21] The (log) real exchange rate is defined as $q_t = s_t + p_t^* - p_t$, where s_t is the (log of) nominal exchange rate, $p_t(p_t^*)$ is the (log of) domestic (foreign) price level. We assume that the foreign price level is constant, so that a restriction on the real exchange rate translates into a restriction on $s_t - p_t$.

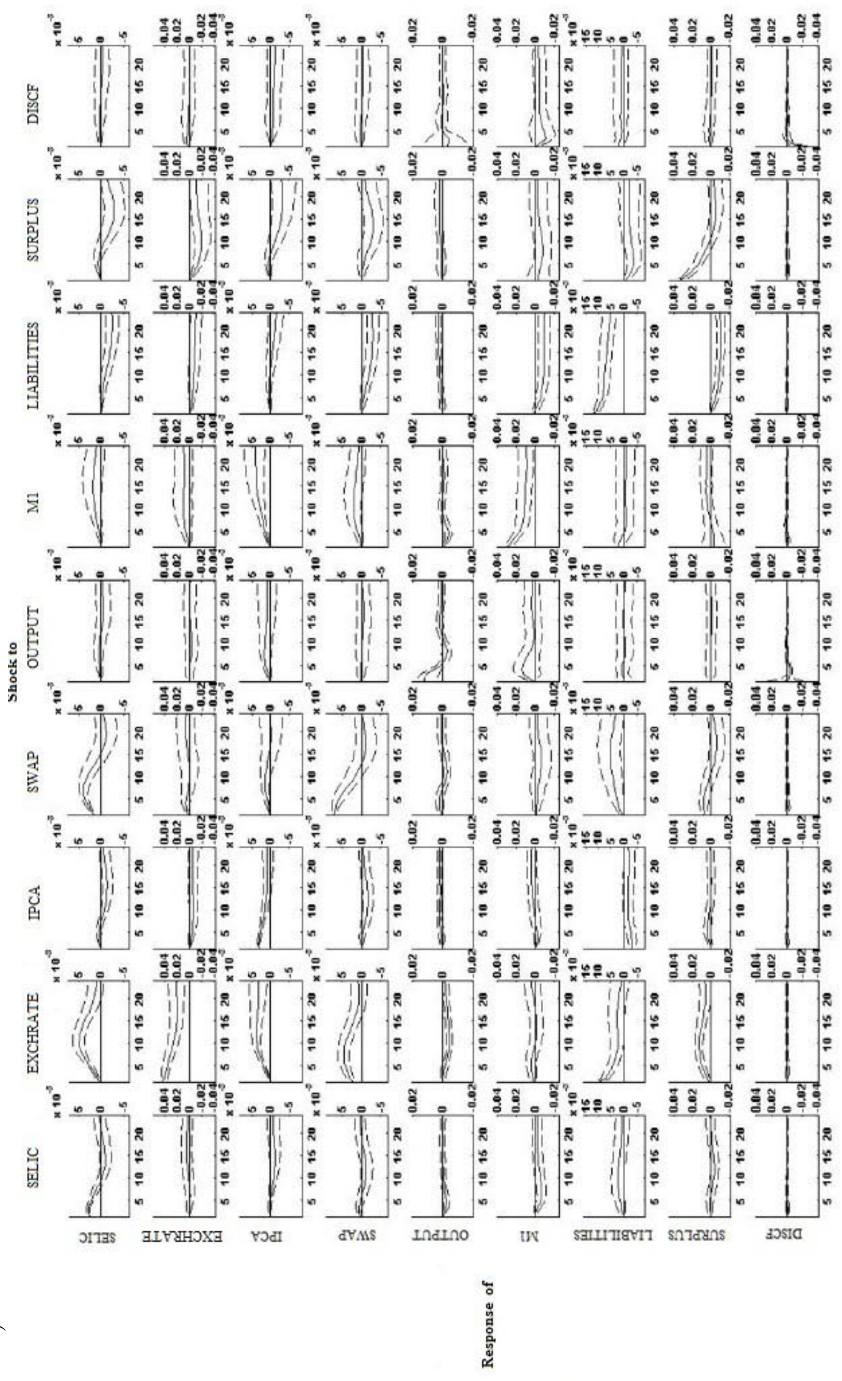


Fig. 2. IRFs with 68% probability bands, using the contemporaneous causal ordering of Figure 1 to identify the SVAR (24 months ahead)

real exchange rate, and that the short-term interest rate, prices, output, and the surplus do not go down after the exchange rate shock.

Table 1
Sign restrictions used to identify the SVAR model

Response of / Type of shock	SELIC	IPCA	Output	SWAP	Real exchange rate	Surplus
Fiscal Policy	≤ 0	≤ 0	≤ 0	≤ 0		≥ 0
Supply		≤ 0	≥ 0			≥ 0
Exchange rate shock	≥ 0	≥ 0	≥ 0		≥ 0	≥ 0

A blank entry indicates that no restrictions have been imposed.

The IRFs that result from the imposition of sign restrictions are presented in figures 3-4, showing the median as well as the 68% probability bands for a horizon of 24 and 60 months following the shocks, respectively.

In response to positive ("contracionist") fiscal shocks we observe a significative and long-lasting reduction in the price level and a short-lived reduction on economic activity. There is no evidence of significative response of the exchange rate to fiscal innovations. The primary surplus increases but the direction of the response of public liabilities to positive fiscal policy innovations is not clear. Therefore, applying CCD's test to the results of the hybrid identification we are unable to distinguish empirically between Ricardian and non-Ricardian regimes. Monetary policy does not control the long run rate of inflation, as shown by response of prices to SELIC shocks 60 months ahead (Figure 4). However, there is no evidence of what Sims (2008) calls "step on a rake" effect, where increases in the interest rate increase, rather than decrease, the inflation rate. [22]

4.2. *The sign restrictions approach*

We consider now an alternative identification where we impose sign restrictions on the IRFs to all shocks, including monetary policy and demand shocks. We maintain the previous restrictions summarized in Table 1, and impose additional restrictions on monetary policy and demand shocks based on LMA and Sims (2008). We assume that in response to a "contractionary" monetary policy shock, interest rates does not fall, and that output, prices, M1, and the real exchange rate do not increase. We assume further that positive demand shocks do not decrease the SELIC rate, the price level, output, the primary surplus, and do not imply a depreciation of the real exchange rate. Table 2 shows the sign restrictions on the IRFs used

[22] Loyo (1999) refers to this situation as "tight money paradox".

Fig. 3. IRFs based on the hybrid identification (24 months ahead), with 68% probability

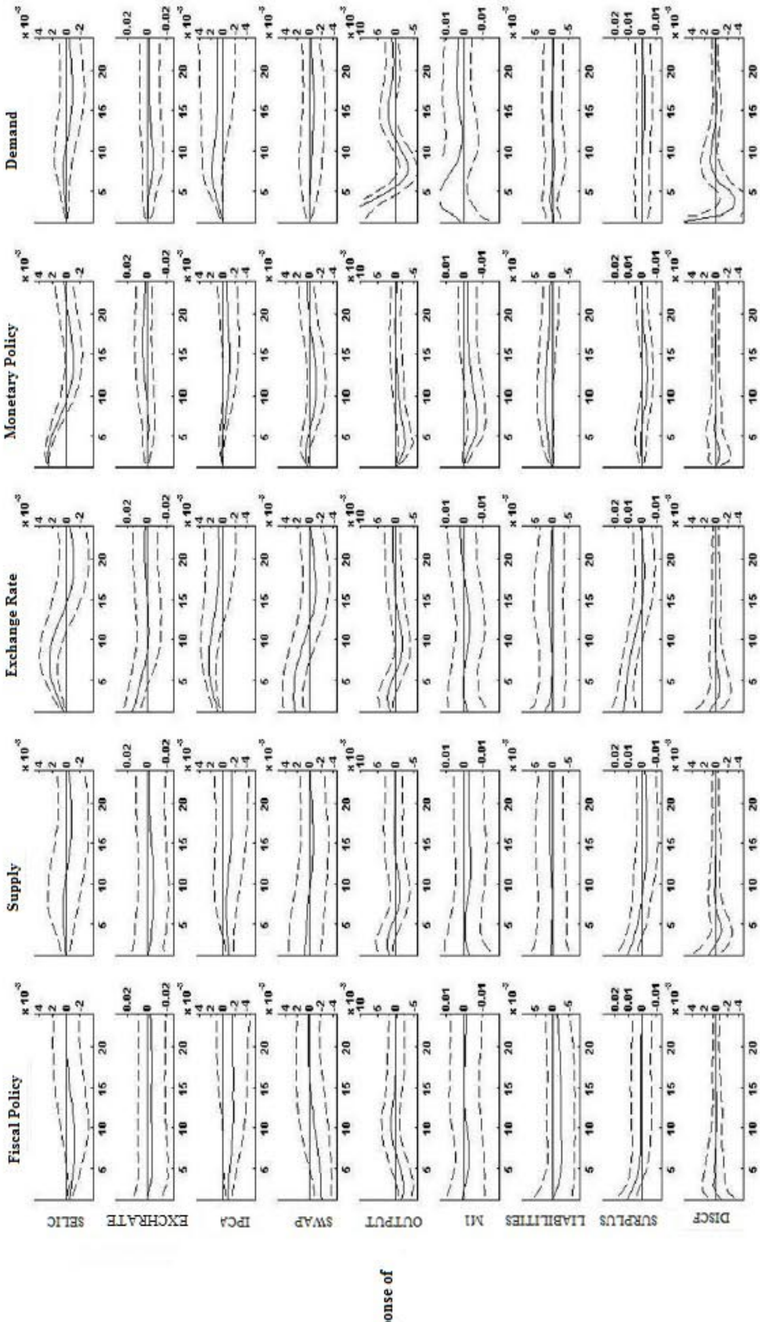
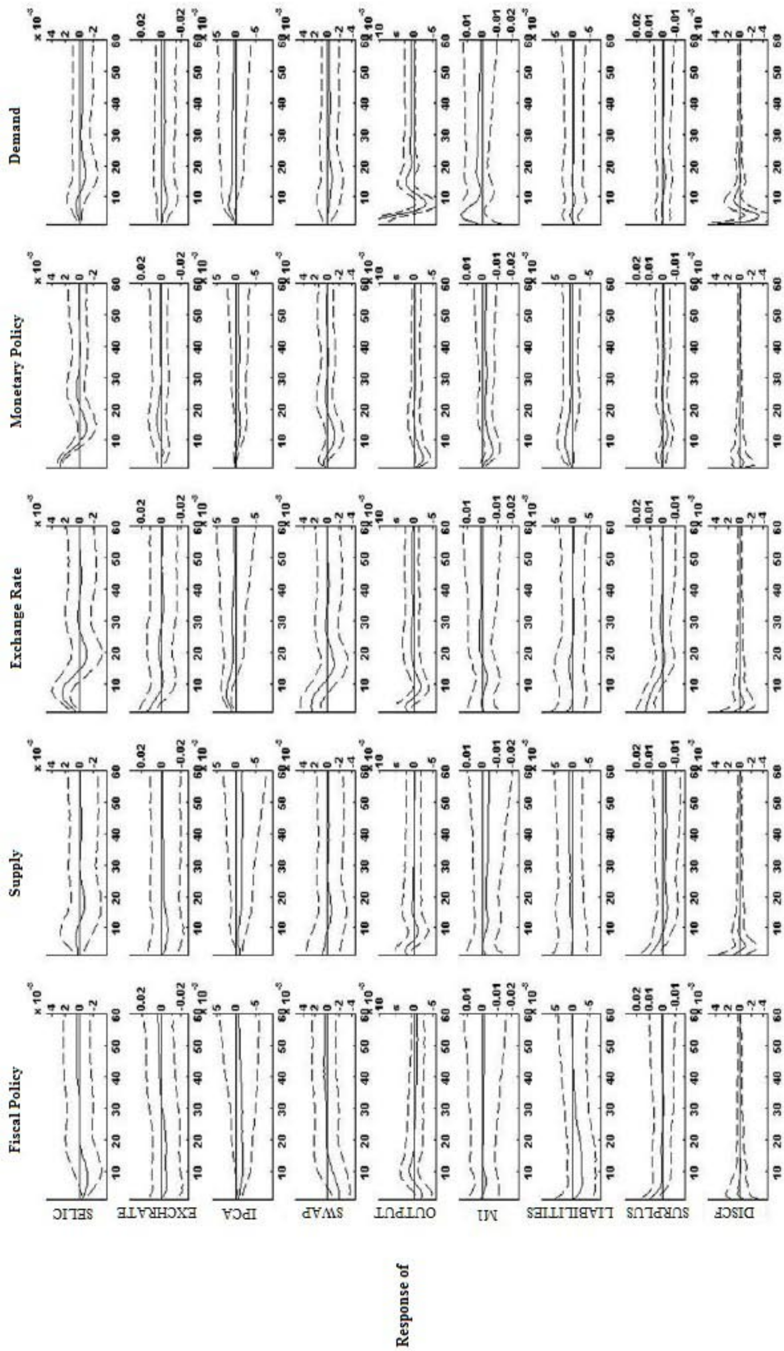


Fig. 4. IRFs based on the hybrid identification (60 months ahead), with 68% probability bands



to identify the fiscal policy, supply, exchange rate, monetary policy, and demand shocks. We impose the sign restrictions for a two months window.

Table 2
Sign restrictions used to identify the SVAR model

Response of / Type of shock	SELIC	IPCA	Output	SWAP	Real exchange rate	Surplus	M1
Fiscal Policy	≤ 0	≤ 0	≤ 0	≤ 0		≥ 0	
Supply		≤ 0	≥ 0			≥ 0	
Exchange Rate	≥ 0	≥ 0	≥ 0		≥ 0	≥ 0	
Monetary Policy	≥ 0	≤ 0	≤ 0		≤ 0		≤ 0
Demand	≥ 0	≥ 0	≥ 0		≤ 0	≥ 0	

A blank entry indicates that no restrictions have been imposed.

For the IRFs based on the alternative identification that uses only sign restrictions to identify all shocks are presented on figures 5-6, showing the median as well as the 68% probability bands for a horizon of 24 and 60 months following the shocks, respectively. The main differences with respect of the IRFs based on the hybrid identification rely on the responses to fiscal and monetary policy shocks. Using sign restrictions only, we observe a reduction on government's liabilities in response to fiscal shocks, which according to CCD's test is evidence of a Ricardian regime. Monetary policy now has an important role as a source of short-run fluctuations on output, prices, and the exchange rate. Monetary policy now controls the long run rate of inflation and there is still there is no evidence of the "step on a rake" effect.

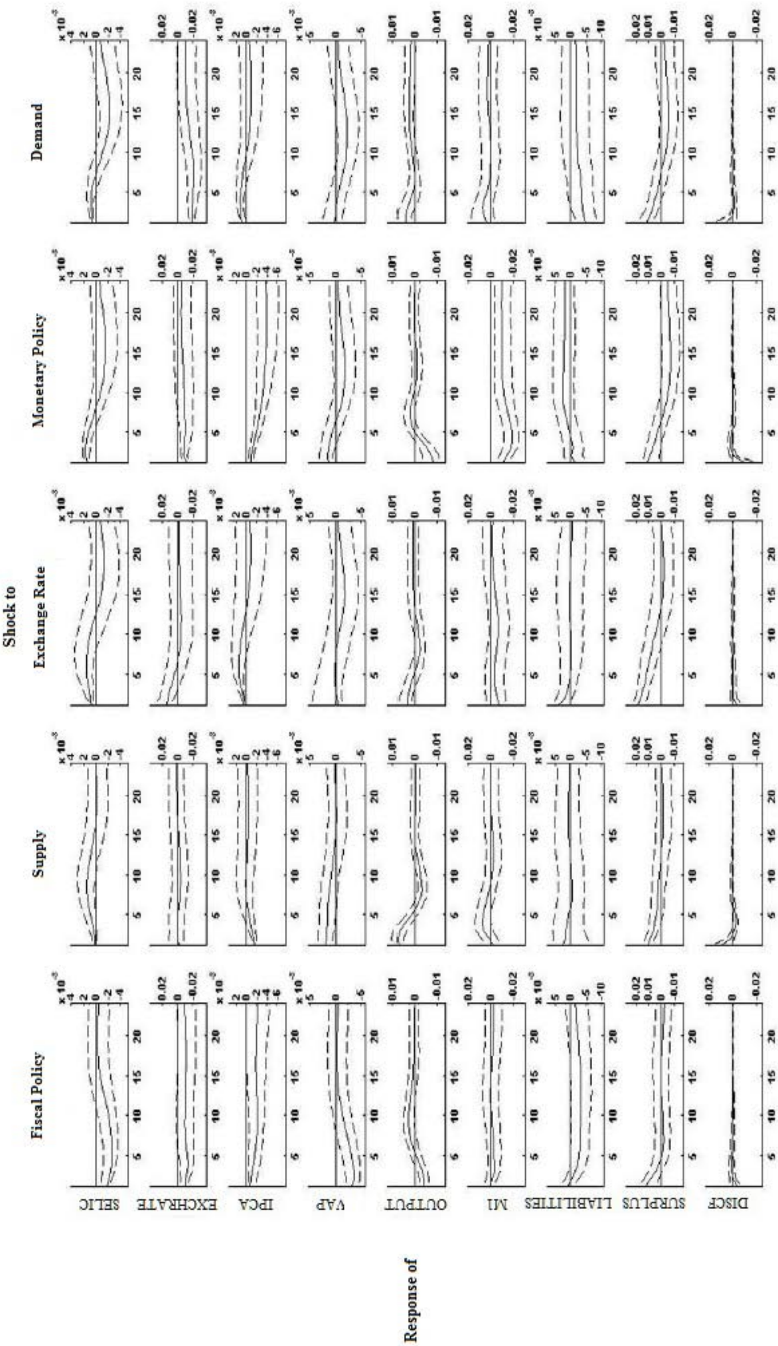


Fig. 5. IRFs based on the sign restrictions identification (24 months ahead), with 68% probability bands

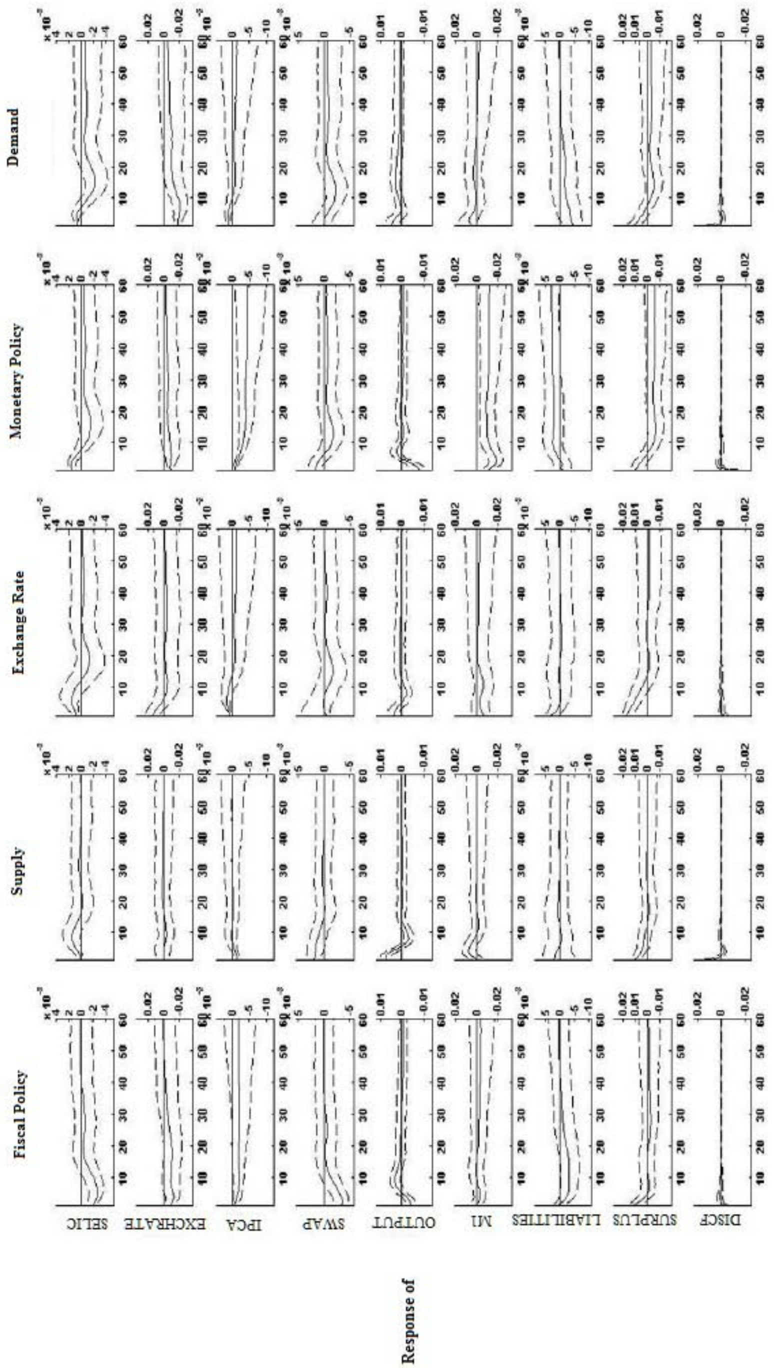


Fig. 6. IRFs based on the sign restrictions identification (60 months ahead), with 68% probability bands

5. Concluding Remarks

While there is an agreement between most economists regarding the effects of monetary policy shocks, the empirical literature has struggled so far to provide robust stylized facts on the effects of fiscal policy shocks. In particular, there is no agreement on even the qualitative effects of fiscal policy shocks on macroeconomic variables.

This paper analyzed the effects of fiscal policy shocks and the interaction between fiscal and monetary policy. To achieve our goals we use a structural vector autoregression (SVAR) model and the test proposed by Canzoneri et al. (2001). The SVAR is identified by two alternative methodologies. The first methodology used sign restrictions on impulse responses of the exogenous disturbances. The second methodology (hybrid) combined sign restrictions with restrictions on the contemporaneous causal interrelationships among variables, derived by Directed Acyclic Graphs (DAGs). Analyzing the case of Brazil, we observed for both identification strategies that in response to positive ("contracionist") fiscal shocks there is a significative and long-lasting reduction in the price level, and a short-lived reduction on economic activity. There is no evidence of significative response of the exchange rate to fiscal innovations.

Monetary and fiscal policy have two main objectives: controlling inflation and stabilizing the ratio of government debt to GDP. "Controlling inflation" means avoiding deviations of inflation from target and "stabilizing government debt" means maintaining the value of the ratio of the debt to GDP and preventing it from growing unsustainably. The conventional assignment gives monetary policy responsibility for controlling inflation and fiscal policy the role of stabilizing government debt (Monetary Dominance) ratio. In this case, since fiscal policy is assigned to stabilize debt, monetary policy is free to target inflation. However, the assignments can be reversed: fiscal policy can determine inflation, while monetary policy prevents debt from becoming unstable. This second regime can arise in crises or states of fiscal stress, and is the distinguishing assumption held by the Fiscal Theory of the Price Level (FTPL) [Fiscal Dominance]. The FTPL is a specific case of monetary-fiscal interaction and it challenges conventional-purely monetary-explanations of price level determination.

We tested for Brazil the assumption, held by the Fiscal Theory of the Price Level, that the Brazilian policy regime is non-Ricardian (Fiscal Dominance), applying the test proposed by CCD that analyzes the response of public sector liabilities to primary surplus shocks. This response depends on the identification adopted. For the hybrid identification we found that it is not possible to distinguish empirically between Ricardian (Monetary Dominance) and non-Ricardian (Fiscal Dominance) regimes. However, using sign restrictions there is some evidence that the government followed a Ricardian (Monetary Dominance) regime from January 2000 to June 2008.

We also checked if the identified exogenous monetary policy shocks show a "stepping on a rake" effect (tighter monetary policy leads to a higher inflation rate in the long run), as described by Sims (2008) in a theoretical framework designed for understanding the effects of fiscal uncertainties on monetary policy. According to our results, there is no evidence whatsoever that a tighter monetary policy would lead to higher inflation in the long run.

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Appendix I: Data Description

Short-term interest rate (SELIC): SELIC interest rate – adjusted average rate of daily financing guaranteed by federal government securities, calculated in the Special Settlement and Custody System (SELIC) and published by the Central Bank of Brazil (BCB) – annualized rate.

Nominal exchange rate: R$ / US$ – end of period buying rate – source: BCB.

Price index (IPCA): IPCA price index – source: IBGE.

Medium-term interest rate (SWAP): 180 days SWAP rate (PRE $\times$ CDI) – source: Brazilian Mercantile & Futures Exchange – annualized rate.

Output: the industrial production index – three month moving average – source: IBGE.

Monetary Aggregate: M1 – working days average – source: BCB.

Surplus: primary surplus of the consolidated public sector (includes central government, state and municipal governments, and public enterprises), as a ratio of the GDP – 12 months accumulated – source: BCB.

Public Sector Net Liabilities: consolidated public sector debt plus monetary base, as a ratio of the GDP – 12 months accumulated – source: BCB.

Discount factor $= \frac{Y_{t+1}/Y_t}{(1+i_t^*)}$, where Y is monthly GDP reported by the BCB and i^* is calculated as (nominal) interest payments – excluding the effect of exchange rate fluctuation – over public sector borrowing requirements.

Appendix II: Methodology

Let y_t be the data vector – there are 9 variables in the model, therefore y_t has dimension $n \times 1 (n = 9)$ for each period t:

$$y_t = [y_{1t} y_{2t} \cdots y_{nt}]$$

where
$y_{1t} = \log$ (Gross annualized Selic interest rate),
$y_{2t} = \log$ (Nominal exchange rate(R$/US$)),
$y_{3t} = \log$ (IPCA index),
$y_{4t} = \log$ (180 days Swap rate (PRE × CDI – annualized considering 252 working days)),
$y_{5t} = \log$ (Industrial Production Index),
$y_{6t} = \log$ (M1),
$y_{7t} = \log$ (government liabilities as ratio of the GDP),
$y_{8t} = \log$ (primary surplus as ratio of the GDP) and
$y_{9t} = \log$ (Discount factor).

The structural VAR model has the general form:

$$y_t' A' = \sum_{t=1}^{p} y_{t+1}' A_t' + z_t' D' + \varepsilon_t', \quad \text{for} \quad t = 1, \cdots, T \tag{1}$$

where
y_t is an $n \times 1$ column vector of endogenous variables at time t,
A and A_t are $n \times n$ parameter matrices,
D is an $n \times h$ parameter matrix,
z_t is an $h \times 1$ column vector of seasonal dummies and constant term at time t,
ε_t is an $n \times 1$ column vector of structural disturbances at time t;
p is the lag length, and T is the sample size ($p = 6$ and $T = 103$).

The parameters of individual equations in (1) correspond to the columns of A', A_t' and D'.

The structural disturbances have a Gaussian distribution with $E(\varepsilon_t | y_t, \ldots, y_{t-1}, z_1, \ldots, z_T) = 0_{n\times 1}$ and $E(\varepsilon_t \varepsilon_t' | y_1, \ldots, y_{t-1}, z_1, \ldots, z_T) = I_{n\times n}$ and then are normalized to have an identity covariance matrix. Right multiplying the structural form (1) by $(A')^{-1}$, we will obtain the usual representation of a reduced-form VAR with the reduced-form variance matrix being $\Omega = (A'A)^{-1}$

Unlike typical unrestricted VAR models, Ω will be restricted when the contemporaneous parameter matrix A is overidentified.

The structural VAR models (1) can be rewritten in the compact form:

$$y_t' A' = x_t' F' + \varepsilon_t'$$

where

$$x_{t_{1\times k}}' = \left[y_{t-1}' \cdots y_{t-p}' z_t' \right], \qquad F_{n\times k} = [A_1 \cdots A_p D]$$

and $k = np + h$. We will refer to F' as lagged parameters even though F' may also contain exogenous parameters.

For $1 \leq i \leq n$ let a_i be the i'th column of A', let f_i be the i'th column of F' and let T_i be an $n \times n$ matrix of rank q_i. The linear restrictions of interest can be summarized as follows:

$$T_i a_i = 0, \qquad i = 1, \cdots, n \tag{2}$$

The restrictions given by (2) are said to be non-degenerate if there exists at least one non-singular matrix A' satisfying them. In this paper, all restrictions are assumed to be non-degenerate.

When VAR models are large and degrees of freedom are low, the likelihood function itself can be ill behaved and there is the well-known tendency of estimates to become unreliable. To deal with these problems, Litterman (1986) introduces a widely used Bayesian prior distribution for reduced-form models to down-weight models with large coeficients on distant lags and explosive dynamics. Sims and Zha (1998) incorporate Litterman's idea in the structural framework by specifying the prior distribution of a_i and f_i as

$$a_i \sim N\left(0, \bar{S}_t\right) \text{ and } f_i|a_i \sim N\left(\bar{P}_i a_i, \bar{H}_i\right) \tag{3}$$

where $\bar{H}_i$ is defined as an $k \times k$ diagonal, symmetric and positive definite (SPD) matrix:

$$\bar{H}_i = \left[\begin{array}{cc} \begin{array}{cccc} \frac{\lambda_0\lambda_1}{\sigma_i} & 0 & \cdots & 0 \\ 0 & \frac{\lambda_0\lambda_1}{\sigma_i} & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & \frac{\lambda_0\lambda_1}{\sigma_i 6^{\lambda_3}} \end{array} & 0_{54\times 12} \\ 54\times 54 & \\ 0_{12\times 54} & \begin{array}{cccc} \frac{\lambda_0\lambda_4}{\sigma_i} & 0 & \cdots & 0 \\ 0 \quad \frac{\lambda_0\lambda_4}{\sigma_i} & 0 & \vdots & \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & \frac{\lambda_0\lambda_4}{\sigma_i} \end{array} \\ & 12\times 12 \end{array}\right]_{(66\times 66)}$$

The standard deviation of the conditional prior of f_i (subset of parameters of equation 1) for the coefficient on lag l of the variable j, is given by

$$\frac{\lambda_0\lambda_1}{\sigma_i l^{\lambda_3}}$$

where the hyperparameter λ_0 controls the tightness of beliefs on A'; λ_1 controls what Litterman called overall tightness of beliefs around the random walk prior; λ_3 controls the rate at which prior variance shrinks for increasing lag length; λ_4 is the tightness for the constant term and seasonal dummies, i.e., for the last 12 rows of each column of F'. We give it a conditional prior mean of zero and a standard deviation controlled by $\lambda_0\lambda_4$.

The vector of parameters $\sigma_1, \ldots, \sigma_n$ (one for each equation) are scale factors, allowing for the fact that the units of measurement or scale of variation may not be uniform across variables. The scale factors are taken as the sample standard deviations of residuals from univariate autoregressive models, with lag length p, fit to the individual series in the sample.

The diagonal matrix S_i is an $n \times n$ positive semidefinite matrix, the individual elements in the i'th column of A' are assumed independent, with prior standard deviations set to $\lambda_0/\hat{\sigma}_i$ (parameters defined above):

$$\bar{S}_{i_{(n\times n)}} = \begin{bmatrix} \frac{\lambda_0}{\hat{\sigma}_1} & 0 & 0 & \cdots & 0 \\ 0 & \frac{\lambda_0}{\hat{\sigma}_2} & 0 & \cdots & \vdots \\ \vdots & 0 & \frac{\lambda_0}{\hat{\sigma}_3} & & \\ & \vdots & & \ddots & 0 \\ 0 & 0 & \cdots & 0 & \frac{\lambda_0}{\hat{\sigma}_n} \end{bmatrix}$$

We use the following values for the hyperparameters:

Hyperparameter	Value
λ_0	0.5
λ_1	0.25
λ_3	1
λ_4	0.5

$\bar{P}_i$is a $k \times n$ matrix defined as:

$$\bar{P}_i = \begin{bmatrix} I_{9\times 9} \\ 0_{57\times 9} \end{bmatrix}$$

The prior form summarized above represents a class of existing Bayesian priors that have been widely used for structural VAR models. Combining the prior form (3) with the restriction (2), we wish to obtain the functional form of the conditional prior distribution:

$$q\left(a_i, f_i | T_i a_i = 0\right) \tag{4}$$

In our case, the following matrices are the restricted A and A' matrices obtained by the application of the TETRAD software, together with the assumption that the swap rate affects the selic rate contemporaneously (swap $\rightarrow$ selic):

$$A = \begin{bmatrix} A_{11} & 0 & 0 & A_{14} & 0 & 0 & 0 & 0 & 0 \\ 0 & A_{22} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{33} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & A_{42} & 0 & A_{44} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & A_{55} & A_{56} & 0 & 0 & A_{59} \\ 0 & 0 & 0 & 0 & A_{65} & A_{66} & A_{67} & A_{68} & 0 \\ 0 & A_{72} & A_{73} & 0 & 0 & A_{76} & A_{77} & 0 & A_{79} \\ 0 & 0 & 0 & A_{84} & 0 & A_{86} & 0 & A_{88} & 0 \\ 0 & 0 & 0 & 0 & A_{95} & 0 & A_{97} & 0 & A_{99} \end{bmatrix}$$

$$A' = \begin{bmatrix} A_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & A_{22} & 0 & A_{42} & 0 & 0 & A_{72} & 0 & 0 \\ 0 & 0 & A_{33} & 0 & 0 & 0 & A_{73} & 0 & 0 \\ A_{14} & 0 & 0 & A_{44} & 0 & 0 & 0 & A_{84} & 0 \\ 0 & 0 & 0 & 0 & A_{55} & A_{65} & 0 & 0 & A_{95} \\ 0 & 0 & 0 & 0 & A_{56} & A_{66} & A_{76} & A_{86} & 0 \\ 0 & 0 & 0 & 0 & 0 & A_{67} & A_{77} & 0 & A_{97} \\ 0 & 0 & 0 & 0 & 0 & A_{68} & 0 & A_{88} & 0 \\ 0 & 0 & 0 & 0 & A_{59} & 0 & A_{79} & 0 & A_{99} \end{bmatrix}$$

Then, we can obtain the T_i's matrices which satisfy the constraints for each column i of A':

$$T_{i_{q_i \times n}} a_{i_{n \times 1}} = 0_{q_i \times 1}$$

Each matrix T_i reproduces the restrictions present in column i of A', given by TETRAD. All element of T_i off the diagonal are zero. At the diagonal, there are zeros in the position of free parameters and ones in the position of parameters restricted to be equal zero. Therefore, for example

$$T_1 = \begin{bmatrix} 0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&1 \end{bmatrix} \quad \text{and} \quad T_9 = \begin{bmatrix} 1&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

Let U_i be an $n \times q_i$ matrix whose columns form an orthonormal basis for the null space of T_i. The column a_i will satisfy the restriction (2) if and only if there exist a $q_i \times 1$ vector b_i (q_i = number of free parameter at column i of matrix A') such that

$$a_i = U_i b_i \tag{5}$$

The column vector b_i contains the free parameters of column i of matrix A' given by TETRAD. For this matrix A' the U_i's are given by,

$$U' = \begin{bmatrix} U_1' \\ U_2' \\ U_3' \\ U_4' \\ U_5' \\ U_6' \\ U_7' \\ U_8' \\ U_9' \end{bmatrix} = \begin{bmatrix} 1&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&1 \\ 0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&1&0 \\ 0&1&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&1 \\ 0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&1 \end{bmatrix}$$

For example,

$$a_4 = \begin{bmatrix} 0 \\ A_{42} \\ 0 \\ A_{44} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = U_4 b_4 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} A_{42} \\ A_{44} \end{bmatrix}$$

The distributions of b_i and f_i are given by

$$b_i \sim N\left(0, \tilde{S}_i\right) \text{ and } f_i | b_i \sim N\left(\tilde{P}_i b_i, \tilde{H}_i\right) \tag{6}$$

where

$$\tilde{H}_i = \bar{H}_i, \qquad \tilde{P}_i = \bar{P}_i U_i, \qquad \text{and} \qquad \tilde{S}_i = \left(U_i' \bar{S}_i^{-1} U_i\right)^{-1}$$

Note that $\tilde{S}_i$ is a $q_i \times q_i$ positive semidefinite matrix, $\tilde{H}_i$ is an $r_i \times r_i$ positive semidefinite matrix, and $\tilde{P}_i$ is a $r_i \times q_i$ matrix. It can be verified that the prior distribution (6) for bi is equivalent to the prior distribution (4) for a_i. For the most part of this paper, we work directly with bi with the understanding that the original parameters ai can be easily recovered via the linear transformations U_i.

Let $b = [b_1' \dots b_n']'$, $f = [f_1' \dots f_n']'$, $X = [x_1 \dots x_T]'$, and $Y = [y_1 \dots y_T]'$.

The likelihood function for b and f $(l((b, f)|X, Y))$ is proportional to

$$|\det[U_1 b_1| \cdots |U_n b_n]|^T \exp\left(-\frac{1}{2}\sum_{i=1}^{n} b_i' U_i' Y' Y U_i b_i - 2 f_i' X' Y U_i b_i + f_i' X' X f_i\right) \tag{7}$$

Combining the priors on b and f given by (6) with the likelihood function given by (7) leads to the following joint posterior probability distribution function for b and f:

$$p(b_1, \cdots, b_n | X, Y)\, \Pi_{i=1}^{n} p(f_i | b_i, X, Y)$$

where

$$p(b_1, \dots, b_n | X, Y) \propto |\det[U_1 b_1| \cdots |U_n b_n]|^T \exp\left(-\frac{T}{2}\sum_{i=1}^{n} b_i' S_i^{-1} b_i\right) \tag{8}$$

$$p(f_i | b_i, X, Y) = \phi(P_i b_i, H_i) \tag{9}$$

with

$$H_i = \left(X'X + \tilde{H}_i^{-1} \right)^{-1}$$

$$S_i = \left(\frac{1}{T} \left(U_i'Y'YU_i + \tilde{S}_i^{-1} + \tilde{P}_i'\tilde{H}_i^{-1}\tilde{P}_i - P_i'H_i^{-1}P_i \right) \right)^{-1}$$

Since (8) has an unknown distribution, we must take draws from the posterior distribution of b by Gibbs Sampling and, and given each draw of b, take draws of f from the Gaussian conditional distribution (9). The notation $\phi(P_i b_i, H_i)$ in (9) denotes the Gaussian density with mean $P_i b_i$ and covariance matrix H_i.

In many works with VARs, only the likelihood function (i.e., proportional to the posterior density under a flat prior for b and f) is considered. Because (7) is the same as (8) and (9) when the prior variances (diagonal elements in $\tilde{S}_i$ and $\tilde{H}_i$) approach infinity, the posterior density specified in (8) and (9) includes the likelihood as a special case.

To obtain small-sample inferences of b and f or for functions of them (e.g., impulse responses), it is necessary to simulate the joint posterior distribution of b and f. This simulation involves two consecutive steps. First, simulate draws of b from the marginal posterior distribution (8). Second, given each draw of b, simulate draws of f from the conditional posterior distribution (9). The second step is straightforward because it requires draws only from a multivariate normal distribution. The first step, as mentioned earlier, can be challenging when linear restrictions on A imply a restricted reduced-form covariance matrix.

The following algorithm was designed to obtain a sample of the impulse response functions, which satisfy the sign restrictions.

Algorithm: The following steps compose the algorithm for simulating draws from the posterior distribution of b, f and, given these draws, draws of the impulse responses that satisfy the sign restrictions.

1. Get the values at the peak of the posterior density function.
2. For $s = 1, \ldots, N_1$ and given $b_1^{(s-1)}$ obtain $b_1^{(s)}, \ldots, b_n^{(s)}$ by
   a. simulating $b_1^{(s)}$ from the distribution $b_1 | b_2^{(s-1)}, \ldots, b_n^{(s-1)}$,
   b. simulating $b_2^{(s)}$ from $b_2 | b_1^{(s)}, b_3^{(s-1)}, \ldots, b_n^{(s-1)}$,
   $\vdots$
   c. simulating $b_n^{(s)}$ from $b_n | b_1^{(s)}, \ldots, b_{n-1}^{(s)}$.
3. Keep $b_1^{(N_1)}, \ldots, b_n^{(N_1)}$.
4. For $s = N_1 + 1, N_2$ and given $b_1^{(s-1)}, \ldots, b_n^{(s-1)}$, obtain $b_1^{(s)}, \ldots, b_n^{(s)}$ by
   d. simulating $b_1^{(s)}$ from the distribution $b_1 | b_2^{(s-1)}, \ldots, b_n^{(s-1)}$,
   e. simulating $b_2^{(s)}$ from $b_2 | b_1^{(s)}, b_3^{(s-1)}, \ldots, b_n^{(s-1)}$,
   $\vdots$
   f. simulating $b_n^{(s}$ from $b_n | b_1^{(s)}, \ldots, b_{n-1}^{(s)}$.
   g. Given $b_1^{(s)}, \ldots, b_n^{(s)}$ simulate $f_1^{(s)}, \ldots, f_n^{(s)}$ from the conditional normal

distribution described in equation (9).

h. Given $b_1^{(s)}, \ldots, b_n^{(s)}$ and $f_1^{(s)}, \ldots, f_n^{(s)}$ obtain $A^{(s)}$ and $B^{(s)} = F^{(s)}A^{(s)-1}$ (A and F were described previously – B contains the reduced form parameters).

i. Draw an independent standard normal $n \times n$ matrix $\tilde{X}$ and let $\tilde{X} = \tilde{Q}\tilde{R}$ be the QR decomposition of $\tilde{X}$ with the diagonal $\tilde{R}$ normalized to be positive.

j. Let $P = \tilde{Q}$ and generate the impulse responses $IRF^{(s)}$ from $A^{(s)}P$ and $B^{(s)}P = F^{(s)}A^{(s)-1}P$.

k. If $IRF^{(s)}$ satisfies the sign restrictions keep it, otherwise discard it.

l. If the number of accepted IRF is equal to 1000 stop.

5. Collect all the IRF that were not discarded in step 4.

In step 2 and 4 of the Algorithm, all simulations are carried out according Theorem 2 of Waggoner and Zha (2002). The central result of Theorem 2 states that drawing from the distribution of b_i conditional on $b_1, \ldots, b_{i-1}, b_n$ is equivalent to drawing from a multivariate Gaussian distribution and a special univariate distribution.

For a fixed $i*$, where $1 \leq i* \leq n$. Let w be an non-zero $n \times 1$ vector perpendicular to each vector in $\{U_i b_i | i \neq i*\}$. Since the restrictions are assumed to be non-degenerate, the $n-1$ vectors $U_i b_i$ for $i \neq i*$ will almost surely be linearly independent and $U'_{i*}w$ will be non-zero. Define $w_1 = T'_{i*}U'_{i*}w / ||T'_{i*}U'_{i*}w||$, where T_{i*} is a $q_{i*} \times q_{i*}$ matrix such that $T_{i*}T'_{i*} = S_{i*}$, and choose $w_2, \ldots, w_{q*}$ so that $w_1, w_2, \ldots, w_{q*}$ form an orthonormal basis for R^{q*}. Then the random vector b_i conditional on $b_1, \ldots, b_{i-1}, b_{i+1}, \ldots, b_n$ can be represented as

$$b_i = \beta_1 U'_i T_i^{-1} w_1 + \sum_{j=2}^{qi} \beta_j U'_i T_i^{-1} w_j$$

The random variable β_j, for $2 \leq j \leq q_i$, is normally distributed with mean zero and variance $1/T$ and is straightforward to simulate. The density function for β_1, the special univariate distribution, is proportional to $|\beta_1|^T \exp(-T\beta_1^2/2)$. Waggoner and Zha (2002) show how to simulate from this latter distribution.

Hybrid Identification[23]

Suppose we want to keep the identification of the first shock obtained by TETRAD (the monetary policy shock). Then we have to modify matrix P employed in step 4-j of the previous algorithm. It will take the hybrid form:

[23] A discussion of the differences between our hybrid identification methodology and that of Dungey and Fry (2009) is presented on Section 1.

$$P = \tilde{Q} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & \tilde{Q}_{22} & \tilde{Q}_{32} & \cdots & \tilde{Q}_{92} \\ 0 & \tilde{Q}_{23} & \tilde{Q}_{33} & \cdots & \tilde{Q}_{93} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \tilde{Q}_{29} & \tilde{Q}_{39} & \cdots & \tilde{Q}_{99} \end{bmatrix}$$

where the submatrix,

$$Q_s = \begin{bmatrix} \tilde{Q}_{22} & \tilde{Q}_{32} & \cdots & \tilde{Q}_{92} \\ \tilde{Q}_{23} & \tilde{Q}_{33} & \cdots & \tilde{Q}_{93} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{Q}_{29} & \tilde{Q}_{39} & \cdots & \tilde{Q}_{99} \end{bmatrix}$$

is obtained by a draw of an independent standard normal $(n-1) \times (n-1)$ matrix $\tilde{X}$, and Qs is obtained by the QR decomposition of $\tilde{X}(\tilde{X} = Q_s\tilde{R}$, with the diagonal $\tilde{R}$ normalized to be positive).