

# Adverse Selection in an Efficiency Wage Model with Heterogeneous Agents

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## Abstract

This paper studies efficiency wages in the presence of heterogeneous workers and asymmetric information. It includes an incentive compatibility constraint (ICC) in the efficiency wage model with heterogeneous workers to show that the implementation of efficiency wages in the presence of heterogeneity faces the problem of adverse selection. Employees with a smaller effort aversion supply a smaller level of effort than what is optimal under perfect information due to hidden information. In this vein only a second best solution is obtained.

*Keywords:* Efficiency Wages, Adverse Selection, Asymmetric Information

*JEL Classification:* J41, M59

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## Resumo

Este artigo analisa a utilização de salários de eficiência na presença de trabalhadores heterogêneos e assimetria de informação. Através da inclusão de uma restrição de compatibilidade de incentivos nesse modelo com agentes heterogêneos, é possível mostrar que ocorre o problema de seleção adversa. Trabalhadores com menor aversão ao esforço ofertam um menor nível de esforço do que aquele que é ótimo sob informação perfeita. Assim apenas uma solução de segundo ótima é possível.

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## 1. Introduction

Efficiency wages models predict that homogeneous workers may receive different wages relying on the degree of shirking aversion of firms. In their classical article, Shapiro and Stiglitz (1984) proposed a non-shirking condition which once satisfied avoid workers to incur in shirking. More recently Charness and Kuhn (2005) propose an efficiency wage model with heterogeneous workers (high productivity and low productivity workers) to explain why firms can compress wages.

The main idea of efficiency wage models is that employers are able to set wages up its competitive level to avoid shirking. That is, employer offers a contract to an employee that is able to avoid shirking, this is the non-shirking condition. Since the worker accepts the contract this implies that her utility to work hard is bigger than the utility of incurring in shirking. In other words, the non-shirking condition may be viewed as a participation constraint.

Accepting the non-shirking condition (NSC) as a participation constraint raises the question about the incentive compatibility constraint. While NSC warrants participation, it does not imply that the worker will reveal his true type. In models with homogeneous workers it is not a problem. However, it is hardly defensible that the work force is homogeneous. Difference in abilities, skills, and motivation are simple examples which can be mentioned to refute homogeneity hypothesis. While skills can be observed, motivation and abilities can not be easily observed. However, all of them have impact over productivity.

In the literature of efficiency wages with heterogeneous workers it is usual to assume different types of individuals with particular productivities. Furthermore, it is implicit assumed that the identification of the productivity level of the worker is easy. Since the employer can always verify the worker productivity – and classify him/her as low or high productive – the incentive compatibility constraint is not necessary.

Facing a heterogeneous supply of labor, employers have some difficulties to learn about employee's productivity, mainly during the hiring process. In this scenario, the incentive compatibility constraint (ICC) becomes important and a device to force workers to reveal their true types becomes necessary. In order to tackle this point this paper extends the Shapiro-Stiglitz model to allow for heterogeneity between workers.<sup>1</sup> Assuming that the heterogeneity comes from the disutility to work, we introduce an ICC condition in the efficiency wage model. Our main result shows that the implementation of efficiency wages in the presence of heterogeneous workers gives rise only to a second best solution due to the issue of hidden information: workers with a smaller disutility of effort act as if they had a higher aversion to effort in order to obtain a better contract. This paper is structured as follows: Section 3 presents the formal model and Section 4 concludes.

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<sup>1</sup> Shapiro-Stiglitz original paper suggests that heterogeneous workers could be a source of stigma from losing a job. This stigma could serve as a discipline device. However, they do not deal with the implications of the heterogeneity of workers over wages which is the objective of this paper.

## 2. The Model

Let us assume an one period model in which the utility function is given by:  $u(w, e) = w - Re$ , where  $w$  is the wage paid by the firm,  $e$  is the effort, and  $R$  is a positive constant that measures the disutility of effort (inverse of motivation).<sup>2</sup> The employee has outside opportunities that provide him with a reservation salary  $\bar{w}$  with no effort ( $e = 0$ ). There is a probability  $p$  to lose the job attached to shirking behavior. The expected utility is given by:

$$V_N = w - Re \quad (1)$$

$$V_S = p\bar{w} + (1 - p)w \quad (2)$$

where  $V_N$  is the utility associated with no shirking and  $V_S$  the utility associated with shirking behavior. The efficiency wage is chosen to satisfy the inequality:

$$V_N \geq V_S \quad (3)$$

Due to the maximization behavior of the firm the equality holds, that is:

$$w - Re = p\bar{w} + (1 - p)w \quad (4)$$

Implicit derivation of  $w$  as a function of  $R$  yields:

$$w = \bar{w} + \frac{Re}{p} \quad (5)$$

This seems that wage may be an increasing function of  $R$ . Let us specify the contract under perfect information, which is assumed to be given by:

$$\underset{w, e}{Max} F[e] - w \quad (6)$$

$$\text{s.t. } w - Re \geq p\bar{w} + (1 - p)w \quad (\text{NSC})$$

where the price of the product is normalized to one, and  $F(\cdot)$  is a production function, with  $F'(\cdot) > 0$  and  $F''(\cdot) < 0$ . From the constraint we obtain:  $w = \bar{w} + \frac{Re}{p}$ . This expression may be rewritten as  $e = \frac{p(w - \bar{w})}{R}$ . By inserting this expression into the objective function and deriving with respect to  $w$  we obtain:  $F'(\cdot) = \frac{R}{p}$ . This expression shows that the marginal productivity of effort is an increasing function of the disutility of effort ( $R$ ). By assuming that:  $F[e] = e^\alpha$  then after some algebraic manipulation the expression above yields the following efficiency wage:

$$w = \bar{w} + \alpha^{\frac{1}{1-\alpha}} \left( \frac{p}{R} \right)^{\frac{\alpha}{1-\alpha}} \quad (7)$$

<sup>2</sup> Katz (1986), Akerlof and Yellen (1990), and Charness and Kuhn (2005) also used an effort that is a function of wages.

By taking the partial derivative of  $w$  with respect to  $R$  we conclude that:  $\frac{\partial w}{\partial R} = -\frac{\alpha \frac{1-\alpha}{R}}{\left(\frac{p}{R}\right)^{\frac{\alpha}{1-\alpha}}} < 0$ . This result shows that the optimal strategy for the firm is to set wages to make it a decreasing function of  $R$ . Let us assume now that there are only two types of workers:<sup>3</sup> low aversion to effort,  $R_1$ , and high aversion to effort,  $R_2$ , with  $R_2 > R_1$ . The incentive compatibility constraint for worker type 1 requires that:

$$w_1 - R_1 e_1 \geq w_2 - R_1 e_2 \tag{IC1}$$

where  $w_1$  and  $w_2$  stand for the wages for the workers of type 1 and 2, respectively and IC1 stands for the incentive compatibility constraint for worker of type 1. Expression (IC1) requires that the utility to behave like type 1, when the worker is type 1, should be larger (or at least equal) than the utility to behave like type 2. In the same way, there is an incentive compatibility restriction to worker type 2, namely (IC2), which is written as:

$$w_2 - R_2 e_2 \geq w_1 - R_2 e_1 \tag{IC2}$$

The non-shirking conditions for workers 1 and 2, namely (NS1) and (NS2) respectively, require that:

$$w_1 - R_1 e_1 \geq p\bar{w} + (1 - p)w \tag{NS1}$$

$$w_2 - R_2 e_2 \geq p\bar{w} + (1 - p)w \tag{NS2}$$

They state that each worker prefers the contract that is designed for her. When the firm is hiring the worker or proposing the contract it does not know the type of the employee. Let us assume that the only information the firm has is the distribution of workers of type 1 and 2 in the population and that the probability of hiring a worker of type 1 is  $\sigma$ , and the remainder,  $1 - \sigma$ , is the probability of hiring a worker of type 2. The contract under imperfect information that satisfies the revelation principle is given by:

$$\underset{w_1, w_2, e_1, e_2}{Max} \quad \sigma[F(e_1) - w_1] + (1 - \sigma)[F(e_2) - w_2] \tag{8}$$

$$s.t. \quad w_1 - R_1 e_1 \geq w_2 - R_1 e_2 \tag{IC1}$$

$$w_2 - R_2 e_2 \geq w_1 - R_2 e_1 \tag{IC2}$$

$$w_1 - R_1 e_1 \geq p\bar{w} + (1 - p)w \tag{NS1}$$

<sup>3</sup> The number of types could be extended to consider  $n$  different types but the case in which there are two types is enough to convey the main implications of the model without cumbersome algebraic manipulations.

$$w_2 - R_2e_2 \geq p\bar{w} + (1 - p)w \tag{NS2}$$

A possible way of tackling the above problem is to adopt the Kuhn-Tucker Theorem. Before doing this let us show that one of the restrictions of the problem, namely restriction (NS1), may be ignored. This is the content of the following:

**Lemma 1:** The constraint (NS1) may be ignored in problem (8).

**Proof.** From (IC1) from the fact that  $R_1 < R_2$  one obtains:  $w_1 - R_1e_1 \geq w_2 - R_1e_2 > w_2 - R_2e_2$ . From (NS2) it is possible to conclude that:  $w_1 - R_1e_1 \geq p\bar{w} + (1 - p)w$ .  $\square$

Another useful Lemma is the one that shows that (NS2) holds with equality.

**Lemma 2:** (NS2) holds with equality.

**Proof.** Assume for the sake of contradiction that the optimal contract is the one in which  $w_2 - R_2e_2 > p\bar{w} + (1 - p)w$ . Then the firm may choose another contract by choosing  $w_2 - \epsilon$  that still satisfies the above equations and yields smaller costs. But this is a contradiction to fact that the first contract was optimal.  $\square$

This result may also be proven by solving the maximization problem (8) subject to (IC1), (IC2) and (NS2) through the Kuhn-Tucker method. By using Lemma 1, the problem may be rewritten as:

$$\underset{w_1, w_2, e_1, e_2}{Max} \quad \sigma[F(e_1) - w_1] + (1 - \sigma)[F(e_2) - w_2] \tag{9}$$

$$\text{s.t. } w_1 - R_1e_1 \geq w_2 - R_1e_2 \tag{IC1}$$

$$w_2 - R_2e_2 \geq w_1 - R_2e_1 \tag{IC2}$$

$$w_2 - R_2e_2 \geq p\bar{w} + (1 - p)w \tag{NS2}$$

The Lagrangean function related to this problem may be written as:

$$L = \sigma[F(e_1) - w_1] + (1 - \sigma)[F(e_2) - w_2] + \lambda_1[w_1 - R_1e_1 - w_2 + R_1e_2] + \lambda_2[w_2 - R_2e_2 - w_1 + R_2e_1] + \mu[w_2 - R_2e_2 - p\bar{w} - (1 - p)w]$$

where  $\lambda_1$ ,  $\lambda_2$  and  $\mu$  are the Kuhn-Tucker multipliers related to constraints (IC1), (IC2) and (NS2) respectively. The first order conditions are given by:

$$\frac{\partial L}{\partial w_1} = -\sigma + \lambda_1 - \lambda_2 = 0 \tag{A1}$$

$$\frac{\partial L}{\partial w_2} = -(1 - \sigma) - \lambda_1 + \lambda_2 + \mu = 0 \tag{A2}$$

$$\frac{\partial L}{\partial e_1} = \sigma F'(e_1) + \lambda_1 R_1 - \lambda_2 R_2 = 0 \tag{A3}$$

$$\frac{\partial L}{\partial e_2} = (1 - \sigma)F'(e_2) - \lambda_1 R_1 + \lambda_2 R_2 + \mu R_2 = 0 \tag{A4}$$

The Kuhn-Tucker conditions are given by:

$$\lambda_1 \frac{\partial L}{\partial \lambda_1} = \lambda_1 [w_1 - R_1 e_1 - w_2 + R_1 e_2] = 0, \lambda_1 \geq 0, \frac{\partial L}{\partial \lambda_1} = w_1 - R_1 e_1 - w_2 + R_1 e_2 \geq 0 \quad (\text{A5})$$

$$\lambda_2 \frac{\partial L}{\partial \lambda_2} = \lambda_2 [w_2 - R_2 e_2 - w_1 + R_2 e_1] = 0, \lambda_2 \geq 0, \frac{\partial L}{\partial \lambda_2} = w_2 - R_2 e_2 - w_1 + R_2 e_1 \geq 0 \quad (\text{A6})$$

$$\mu \frac{\partial L}{\partial \mu} = \mu [w_2 - R_2 e_2 - p\bar{w} - (1-p)w] = 0, \mu \geq 0, \frac{\partial L}{\partial \mu} = w_2 - R_2 e_2 - p\bar{w} - (1-p)w \geq 0 \quad (\text{A7})$$

It is possible to show – see the Appendix – that from the eight possible cases just one is feasible, namely case (v), in which  $\lambda_1 = 0$ ,  $\lambda_2 = \sigma > 0$  and  $\mu = 1 > 0$ . In this context the following propositions hold.

**Proposition 1:** The utility of a worker of type 1 is larger than the utility of a worker of type 2, that is:  $w_1 - R_1 e_1 > w_2 - R_2 e_2$

**Proof:** From (IC1) we know that:  $w_1 - R_1 e_1 \geq w_2 - R_1 e_2$ . Since  $R_1 < R_2$  we conclude that:  $w_1 - R_1 e_1 \geq w_2 - R_1 e_2 > w_2 - R_2 e_2$ .  $\square$

This proposition shows that the firm makes the utility of an employee of type 1, with higher willingness to work to be larger than the utility of an employee of type 2. At a first glance the contract provides right incentives since it gives more rewards in terms of utility to workers with higher willingness to work. Proposition 2 shows that the effort of a worker of type 1 is chosen to be larger or at least equal than the effort of a worker of type 2.

**Proposition 2:** The effort of a worker of type 1 is larger or equal than the wage of a worker of type 2. That is:  $e_1 \geq e_2$ .

**Proof:** By summing (IC1) and (IC2) and after some algebraic manipulation we obtain:  $(R_1 - R_2)[e_2 - e_1] \geq 0$ . Since  $R_1 < R_2$  we conclude that:  $e_2 \leq e_1$ . Hence the effort of a worker of type 1 is larger or equal than the salary of a worker of type 2.  $\square$

From Propositions 1 and 2 it is possible to conclude that in general a worker of type 1 has better reward than a worker of type 2. This result is according to what was established as the wage chosen by the firm as the outcome of the profit maximization behavior of the firm under perfect information and it shows that the conventional wisdom that workers with higher willingness to work have to receive larger wages than those which higher effort aversion prevails. However since the worker of type 1 has to provide an effort level higher or at least equal to the one provided by worker or type 2 it may find wrong incentives to announce that she is a worker of type 2 in order to provide less effort than what it is optimal under perfect information.

**Proposition 3:** An employee of type 1 chooses a contract designed for an employee of type 2.

**Proof:** From Proposition 2, we know that:  $e_2 \leq e_1$ . From (IC2) we know that  $w_1 - R_1 e_1 \geq w_2 - R_1 e_2$ . Note that it is not possible to exclude the possibility that:  $w_1 - R_1 e_1 = w_2 - R_1 e_2$ . In fact this result holds as it is shown in case (vi) of the Appendix. In this vein a worker of type 1 by announcing of being a worker of type 2 may provide smaller effort and receives the same utility as if he admitted his own type.  $\square$

By announcing to be a worker of type 2 allows the worker of type 1 to provide less effort while keeping her utility of being a worker of type 1. The consequence of this strategy is that the salary earned by her will be smaller than if she admitted her true type. Of course this fall in wages will be compensated in the utility function by the smaller effort provided. But a further analysis on this outcome shows that it can produce only a second best result for the firm: profits that the firm obtains hiring an employee of type 1 is smaller than what it would obtain under perfect information. This is the content of the next proposition.

**Proposition 4:** Profits of the firm hiring worker of type 1 are smaller than profits it obtains under perfect information.

**Proof:** From the profit maximization problem of the firm it is possible to conclude that the profit of the firm under perfect information by hiring a worker of type 1 is:  $\Pi_1 = F[e_1] - w_1$  where  $w_1 = \bar{w} + \frac{R_1 e_1}{p}$ . This yields:  $\Pi_1 = F[e_1] - \bar{w} - \frac{R_1 e_1}{p}$ . The same result holds for the profit of the firm by hiring a worker of type 2 under perfect information, that is:  $\Pi_2 = F[e_2] - \bar{w} - \frac{R_2 e_2}{p}$ . It is possible to show that the profit of the firm by hiring a worker of type 1 is large or at least equal to the profit of the firm hiring a worker of type 2: as  $R_1 < R_2$  then  $F[e_2] - \bar{w} - \frac{R_1 e_2}{p} > F[e_2] - \bar{w} - \frac{R_2 e_2}{p} = \Pi_2$ . But we know that  $\Pi_1 = F[e_1] - \bar{w} - \frac{R_1 e_1}{p} \geq F[e_2] - \bar{w} - \frac{R_1 e_2}{p}$  since  $e_1$  is the solution for the profit maximization under perfect information if the effort aversion is  $R_1$ . Hence we conclude that  $\Pi_1 > \Pi_2$ . Note that the common term in both inequalities is nothing but the profit of the firm by hiring a worker of type 1, acting as worker of type 2, which is smaller than the profit of a worker of type 1 under perfect information and higher than the profit of a worker of type 2 under perfect information.  $\square$

This result is not the first best since the effort provided by a worker of type 1 is smaller than her effort under perfect information which leads to smaller profits. Hence the contract is not optimal since it produces only a second best solution. The model shows that after the contract is signed and productivity is revealed it is possible to identify workers of type 1 and 2 by the efforts they provide. Hence a possible way to avoid the second best solution due to hidden information is to establish a probation period in which the productivity of workers is screened. In this case, workers of type 1, by knowing that they are identified during the probation period find right incentives to announce his true productivity when the contract is signed. But it is important to bear in mind that this device works only in the case in which the probation period does not create incentives to workers of type 1 to remain only during this period and quit the job after that and that they are aware of their own productivity.

Pouyet et al. (2008) assuming that principals compete for attracting heterogeneous agents by offering contracts have found that when the agents' types are publicly observed then competitive equilibria are efficient, a result similar to the one obtained here. When types are privately observed these authors continue to obtain the result that efficiency holds provided that principals do not directly care about the agents' private information. Here we have shown that only a second best solution is possible in the case of hidden information.

### 3. Concluding Remarks

In this paper we get apart from homogeneous work hypothesis and derive an incentive compatibility constraint (ICC) for the efficiency wage approach. Supposing that the heterogeneity arises from different disutility to work (motivation) between workers, we show that the efficiency wage contract provides only second best solution: employee of type 1 supplies a smaller level of effort than what is optimal under perfect information due to hidden information.

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## Appendix

Let us analyze the eight possible cases, which arise from the possible combination of the Kuhn-Tucker multipliers:

- (i)  $\lambda_1 = \lambda_2 = \mu = 0$ . From expression (A1) it yields that  $\sigma = 0$ , a contradiction.
- (ii)  $\lambda_1 = \lambda_2 = 0$  and  $\mu > 0$ . For the same reason of case (i) we exclude this possibility.
- (iii)  $\lambda_1 = \mu = 0$  and  $\lambda_2 > 0$ . From expression (A1) it yields  $\lambda_2 = -\sigma < 0$ , a contradiction.
- (iv)  $\lambda_2 = \mu = 0$  and  $\lambda_1 > 0$ . From expression (A1) it yields  $\lambda_1 = \sigma$ . By substituting this result into expression (A2) we conclude that  $\mu = 1$ , a contradiction.
- (v)  $\lambda_1 = 0$ ,  $\lambda_2 > 0$  and  $\mu > 0$ . From expression (A1) it yields  $\lambda_2 = -\sigma < 0$ , a contradiction.
- (vi)  $\lambda_1 > 0$ ,  $\lambda_2 = 0$  and  $\mu > 0$ . From expression (A1) it yields  $\lambda_1 = \sigma > 0$ . By substituting this result into expression (A2) we conclude that  $\mu = 1$ . Note that in this case, from expression (A6) it is possible to conclude that  $w_2 - R_2 e(w_2) = w_1 - R_2 e(w_1)$ , which is the content of Proposition 3, meaning that a worker of type 1 is indifferent between her contract and a contract for the type 1. Besides as  $\mu = 1 > 0$  then (NS2) holds with equality which is the content of Lemma 2.
- (vii)  $\lambda_1 > 0$ ,  $\lambda_2 > 0$  and  $\mu = 0$ . By substituting (A1) into (A2) we conclude that  $\mu = 1$ , a contradiction to the fact that  $\mu = 0$ .
- (viii)  $\lambda_1 > 0$ ,  $\lambda_2 > 0$  and  $\mu > 0$ . From (A1)  $\lambda_1 - \lambda_2 = \sigma$ . By inserting this expression into expression (A2) one obtains  $\mu = 1$ . As all the multipliers are different from zero then the constraints are binding. By summing up (A3) and (A4) and after some algebraic manipulation we conclude that:  $\sigma F'(e_1) + (1 - \sigma)F'(e_2) + R_2 = 0$ . All the terms in the left hand side of this equality are larger than zero then it is not possible to hold.