Optimal Investment Specific Technological Progress Allocation in a Two Sector Model

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Abstract

In this paper the impact of investment specific technical progress on investment allocation is studied by using an extended version of the Feldman's two sector model that takes into account embodied technical progress. The aim of the paper is to analyze the impact of investment specific technical progress on the optimal allocation of investment, economic growth and structural change.

Keywords: Investment Allocation, Two Sector Models, Investment Specific Technical Progress

JEL Classification: E22, E32, O40

Resumo

Neste artigo, o impacto do progresso tecnológico investimento específico sobre a alocação ótima de investimento é estudada a partir de uma versão estendida do modelo de Feldman de dois setores. O objetivo do artigo consiste em determinar a alocação ótima de investimentos quando o progresso tecnológico está incorporado nos bens de capital.

1. Introduction

Feldman's two-sector growth model (1928,1953) is widely used as a benchmark to study the effects of the investment allocation on economic growth. This is a model with a consumption and an investment sector in which capital goods can be used to increase the capacity of either sectors. This model of planned development is an approach of accelerated accumulation in which capital goods feeds upon itself and consumption is temporarily compressed.

A number of authors such as Bose (1968), Weitzman (1971), Araujo and Teixeira (2002) and Araujo (2004), departing from the seminal contribution of Feldman

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(1928) have shown that the decisions of investment allocation play an important role in economic growth since the rate of investment allocation determines the growth rate of output in a closed economy. But a common characteristic of these analyses and of the original Feldman's model is that they do not take into account technical progress embodied in capital goods.

If larger portions of technical progress are embodied into capital goods then the decisions on investment allocation are also decisions on the allocation of technical progress. In this paper this point is analyzed by introducing investment specific technical progress in a two sector model departing from the Feldman's contribution to study the impact of embodiment on the optimal rate of investment allocation.

To accomplish this task we consider investment specific technical progress in the lines suggested by Solow (1957) and Greenwood et al. (1997) who assume that it is implemented through the sectoral equation of investment. Two cases are analyzed here: the first considers that each vintage of capital goods embodies a constant level of technical progress. In this case it is possible to find a constant rate of optimal allocation of investment and closed form solutions for the model. The structural change between the capital and consumption goods sectors occurs only in the short run until the balanced growth path is reached.

The second case considers that embodied technological progress grows according to an exponential function of time. In such a case, the optimal rate of investment allocation is not a constant but varies according to the rate of technical progress. This implies a constant process of structural change between the capital goods and the consumption goods sectors even in the long run which depends on the rate of embodiment.

This paper is structured as follows. In Section 2, the basic model is presented and the law of motion of stock of capital goods is derived for both sectors. In Section 3, the optimal rate of investment allocation is established for a centrally planed economy. Section 4 concludes.

2. The Model

Feldman's model of investment allocation assumes a closed economy with labour surplus and two sectors. The capital goods sector is denoted by subscript 1, and the consumption goods sector, is denoted by 2. The capital goods are used by both sectors but, once investment is made, they cannot be transferred from one sector to the other (irreversibility assumption). A proportion λ of the current production of the investment sector is allocated to itself while the remaining, $1-\lambda$, is allocated to sector 2. ($1 \geq \lambda \geq 0$). The technology is Leontief in both sectors and since by assumption labour is surplus the output of sector 1 is given by:

$$X_1 = A_1 K_1 \tag{1}$$

where X_1 stands for the production of capital goods, A_1 is the output-capital ratio and K_1 refers to the stock of capital in the investment sector. In the same vein the output of sector 2 is given by:

$$X_2 = A_2 K_2 \tag{2}$$

where X_2 refers to the production of consumption goods, A_2 is the output-capital ratio and K_2 is the stock of capital in the consumption goods sector. The investment goods cannot be imported and the production of capital goods does not depend on the production of consumption goods sector.

The literature on embodiment presents some alternative ways of introducing investment specific technical progress in growth models. A possible one is accomplished through the sectoral equation of investment in the lines suggested by Greenwood et al. (1997). According to this approach the law of motion of stock of capital in each of the sectors is given by:

$$\dot{K}_1(t) = \sigma\lambda(t) A_1 K_1(t) - \delta K_1(t) \tag{3}$$

$$\dot{K}_2(t) = \sigma\left[1 - \lambda(t)\right] A_1 K_1(t) - \delta K_2(t) \tag{4}$$

where $\sigma > 1$ captures the investment specific technical progress. Let us consider first the case in which σ is constant. In this case the variation in the stock of capital in sector 1 is a function of the fraction of investment allocated to this sector which embodies technical progress minus depreciation. The law of motion for the stock of capital in sector 2 follows a similar rationale.

Another possible approach is due to Solow (1957) and was followed by a number of authors such as Phelps (1962) and Nelson (1964). This approach considers that each vintage of capital goods is the result of investment in period v and has a rate m of embodied technical progress and depreciates at a rate δ:

$$K_1(v,t) = \lambda(v) I(v) e^{mv + \delta(v-t)} \tag{5}$$

The stock of capital goods in sector 1 in period t is given by the integral over the ages of different vintages of capital goods that are installed in this sector.

$$K_1(t) = \int_0^t K_1(v,t)dv = \int_0^t \lambda(v) I(v) e^{mv+\delta(v-t)} \tag{6}$$

Since $I(v) = X_1(v) = A_1 K_1(v)$ expression (6) may be rewritten as:

$$K_1(t) = \int_0^t K_1(v,t)dv = \int_0^t \lambda(v) A_1 K_1(v) e^{mv+\delta(v-t)} dv \tag{7}$$

By differentiating both sides of this expression and applying the Fundamental Theorem of Calculus we conclude that the variation in the stock of capital goods in sector 1 is given by:

$$\dot{K}_1(t) = \lambda(t) A_1 K_1(t) e^{mt} - \delta K_1(t) \tag{8}$$

By considering that $\sigma(t) = e^{mt}$ captures the investment specific technical progress the above expression may be written as:

$$\dot{K}_1(t) = \sigma(t)\lambda(t)A_1K_1(t) - \delta K_1(t) \tag{9}$$

The law of motion for the stock of capital obtained from the Solow's specification is similar to the previous one with σ constant but in the former $\sigma(t)$ is an exponential function of time. By adopting the same procedure in relation to K_2 we have that:

$$\dot{K}_2(t) = \sigma(t)\left[1 - \lambda(t)\right]A_1K_1(t) - \delta K_2(t) \tag{10}$$

One of the properties of this model is that in the short run the higher the rate of investment allocation the higher the growth rate of the capital goods sector and smaller the growth rate of the consumption goods sector. But in the long run a higher rate of investment allocation implies a higher growth rate of both sectors since the consumption goods sector feeds upon the capital goods one. Hence an optimal value of the rate of investment allocation maximizes the intertemporal production of consumption goods.

3. The Optimal Rate of Investment Allocation

The aim of this section is to establish the optimal value of the rate of investment allocation that maximizes intertemporal consumption under the embodiment hypothesis. This is one of the issues addressed by the original Feldman's model. Domar (1957, p. 254) wrote on this point that "Feldman's task was to explain to the Soviet planners the basic principles of economic growth and to furnish them with several alternative patterns of development, depending on the magnitudes of the rate of investment allocation and of the capital coefficients. It was up to the planners to choose the optimum path, depending on their own objective, and on their evaluation of existing economic and political conditions and possibilities. Such an evaluation of 'the state of the mind of the masses' was in a sense a search for a discount function, but what exactly would be gained by an attempt to formalize it?"

However, this task was accomplished by Bose (1968) and Weitzman (1971). The former put the Feldman's model in an intertemporal dynamic model a la Ramsey and the later extended it by including a third sector of intermediate goods and services used indirectly in producing both consumption and investment goods. Their analyses were carried out assuming that there is no technical progress along the lines of the original models. In order to determine the optimal rate of capital accumulation let us assume that the central planner solves the following problem:

$$\max_{\lambda} \int_0^{\infty} \ln\left(A_2K_2\right)e^{\rho t}dt \tag{11}$$

$$s.t.\dot{K}_1 = \sigma\lambda A_1K_1 - \delta K_1 \tag{7}$$

$$\dot{K}_2 = \sigma\left(1 - \lambda\right)A_1K_1 - \delta K_2 \tag{8}$$

$$0 \leq \lambda \leq 1 \tag{12}$$

where $\rho > 0$ is the social rate of pure time discount. Introducing two co-state variables $q_1(t)$ and $q_2(t)$ related to the investment in sectors 1 and 2 respectively, allows us to write the corresponding Hamiltonian as follows:

$$H = \ln(A_2K_2) + q_1 [\sigma\lambda A_1K_1 - \delta K_1] + q_2 [\sigma(1-\lambda)A_1K_1 - \delta K_2] + \theta_1\lambda + \theta_2(1-\lambda) \tag{13}$$

where θ_1 and θ_2 are respectively the Kuhn-Tucker multipliers associated to the inequality constraints $-\lambda \leq 0$ and $\lambda - 1 \leq 0$. The first order condition is given by:

$$H_\lambda = 0 \Rightarrow \sigma A_1K_1 [q_1 - q_2] = \theta_1 - \theta_2 \tag{14}$$

The Kuhn-Tucker conditions are given by:

$$\theta_1\lambda = 0 \tag{15}$$

$$\theta_2(1-\lambda) = 0 \tag{16}$$

The Euler equations are given by:

$$\dot{q}_1 = (\rho + \delta - \sigma A_1)\, q_1 \tag{17}$$

$$\dot{q}_2 = (\rho + \delta)\, q_2 - K_2^{-1} \tag{18}$$

The transversality conditions are given by:

$$\lim_{t\to\infty} q_1(t)e^{\rho t}K_1(t) = 0 \tag{19}$$

$$\lim_{t\to\infty} q_2(t)e^{-\rho t}K_2(t) = 0 \tag{20}$$

From expressions (15) and (16) there are three possibilities for the optimal rate of investment allocation: $\lambda = 0$, $\lambda = 1$ or $0 < \lambda < 1$. Let us analyze first the case in which the solution is interior, that is $0 < \lambda < 1$. From expressions (15) and (16) we conclude that $\theta_1 = \theta_2 = 0$. By substituting this result into expression (14) it yields: $q_1 = q_2$, which implies that $\dot{q}_1 = \dot{q}_2$. By equalizing (17) to (18) we obtain:

$$\sigma A_1 q_1 = K_2^{-1} \tag{21}$$

Until this point the constancy or not of σ has made no difference since it was not necessary to differentiate any of the previous expressions in relation to time. But now it is necessary to do it in relation to expression (21) and hence to consider the two different possibilities for σ. If σ is constant, after differentiation of (21) the relationship between the growth rate of stock of capital in sector 2 and the shadow price of investment in sector 1 is given by:

$$\frac{\dot{q}_1}{q_1} = -\frac{\dot{K}_2}{K_2} \tag{22}$$

On the other hand if $\sigma(t) = e^{mt}$ then differentiation of (21) yields:

$$m + \frac{\dot{q}_1}{q_1} = -\frac{\dot{K}_2}{K_2} \tag{23}$$

Then it is necessary to treat these two different cases and analyze their implications in terms of growth paths. First let us consider the case in which σ is constant. From expression (17) the growth rate of shadow price of investment in sector 1 is given by:

$$\frac{\dot{q}_1}{q_1} = \rho + \delta - \sigma A_1 \tag{24}$$

By substituting expression (24) into (22), we conclude that the growth rate of capital goods in sector 2 is given by:

$$\frac{\dot{K}_2}{K_2} = \sigma A_1 - \rho - \delta \tag{25}$$

In order to keep the structure of the model it is necessary to assume that $\sigma A_1 - \rho - \delta < n$ in the long run where n is the growth rate of population. With this assumption it is possible to ensure that the labour is surplus along the process of economic growth and the model keeps its properties. Further research has to be made in order to understand the transition of this model, which describes well the growth experience of underdeveloped countries, to a model that captures the experience of developed economies. From expression (3) the growth rate of stock of capital in sector 1 is given by:

$$\frac{\dot{K}_1}{K_1} = \sigma\lambda A_1 - \delta \tag{26}$$

In steady state the growth rates of stock of capital goods in both sectors are the same. By equalizing expressions (25) to (26) allows us to find the optimal rate of investment allocation for both cases.

$$\delta^* = \frac{\sigma A_1 - \rho}{\sigma A_1} \tag{27}$$

In order to meet the constraint on λ, namely $0 < \lambda < 1$, it is necessary that $\sigma A_1 > \rho$. It is easy to see that the higher the pace of embodied technological progress the higher the rate of investment allocation, which shows that it is optimal to concentrate capital goods in the capital goods sector if the pace of embodied technological progress is high. The meaning of this result may be better understood by considering an increasing pace of embodied technological progress which will be made below.

By substituting expression (27) into expression (8) evaluated in steady state it is possible to obtain the optimal assignment of capital across sectors at the balanced growth path:

$$\frac{K_1}{K_2} = \frac{\sigma A_1 - \rho}{\rho} \tag{28}$$

Since sectoral capital is irreversible, if expression (28) does not hold at the initial time for any feasibility conditions, the optimal solution is to set gross investment equal to zero in the sector with too much capital. It would imply that λ is zero or one (and at least one of the inequality constraints is binding). During this phase the model display structural change until the long run optimal solution is reached. When the steady state is reached it is possible to find closed form solutions for differential equations (7) and (17) respectively as:

$$K_1(t) = K_1(t^*) \exp\left[(\sigma A_1 - \rho - \delta)(t - t^*)\right] \tag{29}$$

$$q_1(t) = q_1(t^*) \exp\left[(-\rho - \delta + \sigma A_1)(t - t^*)\right] \tag{30}$$

Where $K_1(t^*)$ is the stock of capital in sector one when the steady state is reached by the first time and $q_1(t^*)$ has the same interpretation in relation to $q_1(t)$. By substituting expressions (25) and (26) into expression (16) it is possible to conclude that the transversality condition hold in this case. Since in steady state $q_1 = q_2$ and the growth rates of stock of capital goods in both sectors are equal we conclude that the transversality condition (17) also holds. Now let us consider the case in which $\sigma(t) = e^{mt}$. By substituting (24) into (23) yields:

$$\frac{\dot{K}_2}{K_2} = \sigma A_1 - \rho - \delta - m \tag{31}$$

By equalizing expression (31) to (7) the optimal rate of investment allocation is given by:

$$\lambda^* = \frac{\sigma(t)A_1 - \rho - m}{\sigma(t)A_1} \tag{32}$$

Now it is necessary that $\sigma A_1 \geq \rho + m$. By taking the limit of (32) when t tends to infinity we obtain an indetermination since both the numerator and the denominator tends to infinity. By applying the L'Hôpital's rule we conclude that $\lim_{t\to\infty} = 1$. This solution shows that once the efficiency of vintages of capital goods are increasing at an exponential rate it is necessary to allocate a smaller fraction of investment to the consumption goods sector. This does not mean that less investment will be made in that sector. The idea is that the production of capital goods will be so large that in absolute grounds the amount of investment that is made in the consumption goods sector is probably higher in each period even though this sector receives a smaller fraction of total investment.

4. Concluding Remarks

In this paper departing from the Feldman's seminal contribution a two sector model of investment allocation is extended to consider investment specific technological progress. The main result of Feldman's analysis was verified here, that is the growth rate of a closed economy is determined by the rate of investment allocation between the capital and consumption goods sectors. By adopting this approach, it was possible to extend Feldman's results concerning physical capital allocation to the case of investment specific technical progress.

The optimal rate of investment allocation that maximizes intertemporal consumption was established. The result for the case that the embodied technical progress is constant for every vintage shows that the higher the investment specific technological progress the higher the optimal rate of investment allocation. Besides the structural change in this case are limited to the short run, when the balanced growth path is not reached.

Embodied technical progress was also introduced in the model by assuming that the every vintage embodies technical progress that grows at an exponential rate. In this case the optimal rate of investment allocation tends to one in the long run, reflecting the fact that if the efficiency of new vintages of capital goods tends to infinity then the optimal strategy consists in concentrate higher fractions of total investment in the capital goods sector.

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