A Stock and Flow Consistent Post Keynesian Model for an Open Economy with Imported Intermediary Inputs and Ex-Ante Portfolio Allocation

Alexandre M. F. Sarquis[*]

José Luis Oreiro[**]

Abstract: We build up on the model of Oreiro and Ono (2005), Oreiro and Lemos (2006) and, more thoroughly, of Passos and Oreiro (2008) in order to introduce a portfolio allocation function to the Post Keynesian Macrodynamic Model developed there, to make the model Stock and Flow Consistent as suggested by dos Santos and Zezza (2008) and to build the model in a more flexible computational platform. We evaluate the model performance for major aggregates and evaluate directions for future research.

Keywords: Post-Keynesian Growth, Stock-flow Consistency, Simulation Models, Real-financial Interactions.

JEL Classification – E12, F43, E37.

July, 2009.

[*] Graduate Student in Economics at Universidade de Brasília. E-mail: alexandre.sarquis@camara.gov.br.

[**] Associate Professor of Economics at Universidade de Brasília (UnB) and Researcher of National Scientific Council. E-mail: jlcoreiro@terra.com.br.

1. Introduction

The main feature of capitalist dynamics is the occurrence of growth-cum-fluctuations, that is, the occurrence of fluctuations of real GDP along a stable but not necessarily constant "trend" in the long run. These fluctuations are, in general, irregular but non-explosive; i.e. there is no tendency to increase the magnitude of business cycles in the long-term.

Heterodox economists usually think the 'problem of capitalist dynamics´ in terms of *linear and non-linear* models of differential or difference equations that have definite (general) solutions. The linear dynamic models as those of Samuelson (1939) and Kalecki (1954) are only capable to produce regular fluctuations of economic activity along an exogenous determined trend of growth of output for a very restrictive set of parameters values.

However, the fluctuations of output observed in the real world are essentially irregular fluctuations. Non-linear models with definite (general) solutions – as those of Hicks (1950) and Goodwin (1967) – are, in general, based on ad-hoc 'ceils´ and ´floors´ or generate solutions of the "limit-cycle" type that do not reproduce the irregular character of real world fluctuations.

Because of the limitations of the dynamical models with general solutions, we can observe in recent years an increasing interest for dynamical models designed to be simulated in computer. These models have, in general, a non-linear structure, but the high number of equations and the complexity of inter-relations between the endogenous variables make impossible the determination of a general, definite, solution. These models can be solved only with the assistance of computer simulations; and the solution assumes the form of time-paths, instead of equilibrium positions, for the endogenous variables. The solution is obtained by computer after setting numerical values for the parameters of the equations and the initial conditions of the model. These values have to be as realistic as possible in order to assure a robust result in terms of time-paths.

One of the main limitations of the post-Keynesian paradigm in economics, as stressed several years ago by Solow (1979), is the inexistence of a *common-unifying framework* to analyze the capitalist dynamics. In a certain sense, post Keynesian economics is more a collection of alternative theories about growth, income distribution, inflation, business cycles than a theoretical approach to all these problems[1]. There are several, and not necessarily compatible, theories to deal with the same problems. Take investment theory, for example. The approach of Davidson (1978) and Minsky (1975), although fully compatible with John Maynard Keynes ideas on the subject, is very different from the approach of Kalecki (1954) that stress the importance of the acceleration principle for investment decisions.

An emerging line of economic research in Post Keynesian Economics tries to integrate the several partial models of this tradition into a Stock and Flow consistent-SFC model (dos Santos 2005; Godley and Lavoie, 2007). The resulting model might have several non-linear equations, several exogenous variables and parameters and several endogenous variables. Its results are better presented as realizations of the time series they define.

[1] For a critique of Solow (1979), see Carvalho (1992).

In this paper, we follow the schematic approach of the SFC methodology described in dos Santos and Zezza (2008), adapting Passos and Oreiro (2008) Post Keynesian model to this scheme.

The computational simulations of the model reproduces some important features of capitalist dynamics as the occurrence of irregular, but non-explosive, fluctuations of the growth rate of real output; the stability of the profit rate in the long-term; the maintenance of idle capacity, the occurrence of a single episode of great reduction in the level of real output over the entire simulation period with is in accordance with the rare character of "Great Depressions" in the history of capitalism, the increasing importance of financial wealth for the dynamics of the wealth of capitalists and the irrelevance of real exchange rate for the dynamics of balance of payments.

To obtain this goal we established the SFC accounting in the following section. In section 3, we establish relevant behavioral relationships and present our developments to Passos and Oreiro (2008). In section 4 we perform "comparative dynamics" exercises with the help of MathWorks® Matlab® computer simulations. Finally, in section 5 we point out concluding remarks.

2. Accounting Structure and Theoretical Assumptions

In our model, the economy is assumed to consist of workers, firm rentiers, bank rentiers, firms, banks, a central bank, government and external sector. The balance sheet of these sectors are summarized in table 1 below.

Table 1: Aggregate Balance Sheets of the Institutional Sectors

Assets/Sectors	Workers	Firm Rentiers	Bank Rentiers	Firms	Banks	Central Bank	Govern.	External Sector	Total
High powered money	0	Hc	Hf	He	Hb	(-) H	0	0	0
Bank deposits	0	Mc	Mf	Me	(-) M	0	0	0	0
Central Bank advances	0	0	0	0	(-) A	A	0	0	0
Domestic bonds	0	Bc	Bf	0	Bb	Bbc	(-) B	0	0
External bonds	0	0	0	(-) EB*f	(-) EB*b	0	0	EB*	0
Foreign currency	0	0	0	0	E. Rb	E. Rbc	0	(-) ER	0
Bank loans	0	0	0	(-) L	L	0	0	0	0
Capital infrastructure	0	0	0	pK	0	0	0	0	pK
Net Worth	0	Vc	Vf	Ve	0	0	(-) B	E(B*-R)	pK-E(B*-R)

Note: Positive variables are assets, while negative ones are liabilities; p stands for the price of the single good produced by firms.

Table 1 summarizes some of our theoretical hypothesis. Before analyzing every sector of the economy, however, we need to point out that null valued cells make clear we are using very simplifying assumptions, such as: banks lend only to firms, domestic bonds are issued just by government and external bonds are only issued by firms and banks.

Workers are assumed to have zero wealth, i.e. they spend all their wages in consumption. Both productive and financial capitalists are assumed to hold high powered money, bank deposits and domestic bonds. Firms are assumed to hold high powered money, bank deposits and capital goods financed with external bonds issues and bank loans. Banks uses bank deposits, central bank advances and external bonds issues to hold high powered money, foreign currency, domestic bonds and to loan to firms.

Banks distribute all their profits to financial capitalists, hence their net worth is zero. Central Bank uses high powered money issued to lend advances to banks, hold domestic bonds for monetary policy purposes and to maintain Forex (foreign exchange reserves). Its net worth is zero for it distributes all its profits to government. Government does not hold any asset and uses domestic bond issues to finance its deficits. External sector uses its foreign currency to buy external bonds.

The composition of capital described above causes flows of funds among sectors, which are described in Table 2.

Table 2. "Current" Transactions in our Artificial Economy

Flows	Workers	Firm Rentiers	Bank Rentiers	Firms		Banks	Central Bank	Government		External Sector	Total
				Current	Capital			Current	Capital		
Cons.	(-) pCw	(-) pCc	(-)pCf	pC + pGc + (EX - Ep*M)	0	0	0	(-)pGc	0	Ep*M - EX	0
Invest.	0	0	0	pI+pGk	(-) pI	0	0	0	(-)pGk	0	0
Wages	W	0	0	(-) W	0	0	0	0	0	0	0
Taxes	(-)pTw	(-)pTc	(-)pTf	(-)pTe	0	(-)pTb	0	pT	0	0	0
Imp. Inputs	0	0	0	(-)RmEp*	0	0	0	0	0	RmEp*	0
Interest loans	0	0	0	(-) il (-1) L(-1)	0	il(-1)L(-1) - ia(-1)A(-1)	ia(-1)A(-1)	0	0	0	0
Interest bonds	0	ib(-1)Bc(-1)	ib(-1)Bf(-1)		0	ib(-1)Bb(-1)	ib(-1)Bbc(-1)	(-)ib(-1)B(-1)	0	0	0
Interest ext. bonds	0	0	0	(-) i*(-1)EB*e(-1)	0	(-)i*(-1)EB*b(-1)	0	0	0	i*(-1)EB*	0
Dividends	0	Fe	Fb	(-) Fe	0	(-)Fb	(-)Fbc	Fbc	0	0	0
Current Saving	0	Sc	Sf	Se	0	0	0	Sg	0	Sext	S

Note: A (+) sign leading a variable denotes a receipt, while a (-) sign denotes a payment

Workers expend their income (wages) in consumption goods and taxes. Productive and Financial capitalists receive interest on domestic bonds and dividends to buy consumption goods, pay taxes and save. Firms use sales income from consumption goods to pay wages, taxes, imported inputs, interest on their inherited stock of loans and external bonds issued, and dividends to productive capitalists, retaining the rest to help finance investment.

Banks revenue is composed by interest on loan to firms and domestic bonds coupons, its expenses are composed of taxes, interest on external bonds issued and dividends to bank rentiers. Central Bank receive interest on banks advances and coupons on domestic bonds, its profit is entirely transferred to the government. The government, in turn, receives taxes and Central Bank profit and spent in consumption and capital goods from firms, and pay coupons and interest on its stock of domestic debt.

External Sector revenue is composed by international trade and coupons on external bonds issued by banks and firms. There's a net flow of funds, loosely referred to as external savings.

While table 2 summarizes the "current" flows of funds resulting from the stocks presented in table 1, table 3 below does the same with the "capital flows," i.e., those that change the balance sheets of the sectors. In this sense, its columns can be interpreted (when its components are viewed as ex-ante variables) as aggregated budget constraints of their respective sectors.

In other words, table 3 demonstrates how stocks (that will constrain the flow behavior in the next period) are modified by current flows. Indeed, in our model the net worth of a sector is increased by its current savings during the period, plus capital gains arising from changes in the market value of the single good produced in the economy during the period.

Table 3. Flows of Funds in the Economy

Assets	Workers	Firm Rentiers	Bank Rentiers	Firms	Banks	Central Bank	Govern.	External Sector	Total
Current Saving	0	Sc	Sf	Se	0	0	Sg	Sext	S
Δ High power money	0	- ΔHc	- ΔHf	- ΔHε	- ΔHb	+ ΔH	0	0	0
Δ Bank Deposits	0	- ΔMc	- ΔMf	- ΔMε	+ ΔM	0	0	0	0
Δ Central Bank Advances	0	0	0	0	+ ΔA	- ΔA	0	0	0
Δ Domestic bonds	0	- ΔBc	- ΔBf	0	- ΔBb	- ΔBbc	+ ΔB	0	0
Δ External bonds	0	0	0	+ ΔEB*e	+ ΔEB*b	0	0	- ΔEB*	0
Δ Bank loans	0	0	0	+ ΔL	- ΔL	0	0	0	0
Δ Foreign currency	0	0	0	0	- ΔERb	- ΔERbc	0	+ ΔER	0
Δ Capital goods	0	0	0	- pΔK	0	0	- pGk	0	- pΔK-pGk
Total	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**	**0**

Note: Positive figures denote sources of funds, while negative ones denote uses of funds

Capitalists current savings are directed to high powered money, bank deposits and/or domestic bonds, according to the Portfolio Allocation Mapping described below. Government deficits are necessarily financed with the issuing of domestic bonds. External sector saving implies changes in its holdings external bonds and/or foreign currency. Finally, investment is necessarily financed by a combination of retained earnings, domestic bond issues by government and bank loans.

3. Model Behavioral Structure

Social accounting is not capable of predicting, by itself, the path taken by the economy. One agent decision may be free in one end, but the composition of actions will produce economy wide output, growth and price levels. In what follows we use several Post Keynesian ideas, in a multi-sector – including real side and financial side – open economy that leverages on two homogeneous factors of production: Labor and Capital.

Initial levels and calibration are worked to be compatible with Passos and Oreiro (2008). From t=3 on we let the dynamics in the model build a path of global variables that is latter presented in graphs of this path-dependent stochastic dynamic macroeconomic model.

Some general elements of the post Keynesian framework to be incorporated in the model include:

(i) The effective demand principle;
(ii) Differing marginal savings propensities for workers and capitalists;
(iii) Mark-up based pricing;
(iv) Minskyan investment decision ("two prices" theory);
(v) Firm's capital structure influences pricing and investment decisions (as opposed to the Modigliani-Miller Theorem);
(vi) Distributive conflict between capitalists and workers causes inflation;
(vii) Endogenous money supply;
(viii) Kaldorian endogenous technological growth.

We present our model in five modules:

1 – Prices, Wages and Monetary Policy

2 – Production and Technological Progress;

3 – Consumption and Investment;

4 – Portfolio Choice;

5 – External Sector.

3.1 – Prices, Wages and Monetary Policy.

Firms set their prices as to top direct costs by a factor z^e:

$$P_t = (1 + z_t^e).[a_t^0.w_t + a_t^1.E_t.P_t^*] \quad (1)$$

Where a_t^0 is labor requisite to produce one unit of output, which is valued 1 monetary unit at t=1, and a_t^1 is imported intermediary input requisite to produce one unit of output. Wages are represented by w_t and the price of intermediary goods are represented as the nominal exchange rate, E_t, multiplied by the foreign price level, P_t^*.

The mark-up is decided based on the "utilization rate, u^e, and by the debt index:

$$z_t^e = z_0^e + z_1^e.u_{t-1}^e + z_2^e.\left[\frac{L_{t-1}+E_{t-1}.B_{t-1}^{*}{}^{e}}{K_{t-1}+M_{t-1}^e+H_{t-1}^e}\right] \quad (2)$$

We suppose that the nominal wages are the result of a bargaining process among firms and unions. In this negotiation process, unions demand higher wages to cover the inflation losses of the previous period and to increase the real wage rate to a certain *target rate*, $\bar{V}$. This target rate is determined by the conditions prevailing in the labor market and by productivity growth, since unions desire to incorporate productivity gains in real wages. Greater is the bargaining power of unions, greater will be the target real wage in the determination of nominal wages.

$$\frac{w_t - w_{t-1}}{w_{t-1}} = \pi_{t-1} + \Phi.(\bar{V}_t - V_{t-1}) \quad (3)$$

Target Real wage is altered based on the evolution of productivity of labor, $\frac{1}{a^0}$, and unemployment rate, u^w:

$$\bar{V}_t = \phi_0 - \phi_1.u_{t-1}^w + \phi_2.\frac{1}{a_t^0} \quad (4)$$

Population at time t, N_t, grows at an stochastic defined rate g_N.

Regarding the operation of monetary policy we will suppose that monetary policy is conducted in a *Inflation Targeting Regime*[2] by means of fixing the short-term interest rate according to a *Taylor's Rule* (Taylor, 1993). Inflation targeting is performed by smoothing the path towards a long term target:

$$\pi_t^* = \pi_{t-1}^* + \alpha.(\pi^{lt} - \pi_{t-1}^*) \quad (5)$$

Base interest rate is established based on the following Taylor rule:

$$i_t^{BC} = (1-\lambda).i_{t-1}^{BC} + \lambda.[\beta_0.(\pi_{t-1} - \pi_t^*) + \beta_1.(g_{t-1} - g_{t-1}^*)] \quad (6)$$

Banks mark-up their interest rate based on past inflation, utilization rate of industry and a solvency index.

$$i_t^b = (1 + z_t^b).i_t^{BC} \quad (7)$$

$$z_t^b = z_0^b + z_1^b.\pi_{t-1} + z_2^b.\left[\frac{L_{t-1}^e + E_{t-1}.B_{t-1}^{*\,e}}{K_{t-1}^e + M_{t-1}^e + H_{t-1}^e}\right] \quad (8)$$

Where the "e" superscript represents productive firms stocks.

3.2 - Production and Technological Progress.

Regarding technical progress, we will suppose the existence of *dynamical economies of scale*, as, for example, "learning by doing". This means that the rate of change of labor productivity is determined by the rate of change of real output and/or the rate of change in the level of capital stock. Here we adopted a Kaldorian technical progress function (Kaldor, 1957) as the one written above. In the equation that follows, ζ is an stochastic shock to productivity and ψ is depreciation of capital assets.

$$a_t^0 = a_{t-1}^0 - \rho_0.\left[\frac{(1-\psi).K_{t-1} + I_t + G_t^i}{(1-\psi).K_{t-2} + I_{t-1} + G_{t-1}^i} - 1 + \zeta\right].a_{t-1}^0 \quad (9)$$

According to the principle of effective demand, the production level is determined by effective demand for goods and services (Pasinetti, 1997, p.99). This occurs if and only if there is no full utilization of productive capacity (i.e. there is "idle" capacity). We suppose that firms meet all changes in the demand for their products with changes in the output level until they reach the potential output.

The level of potential output is determined by two restrictions: (i) the manpower available (equation 11); (ii) the maximum level of capacity utilization (equation 10). We have also to consider another restriction. Real output cannot increase between periods at any rate. In fact, there is a maximum rate of increase of real output between periods due to the existence of adjustment costs. These costs are related to selecting, contracting and training new workers. Therefore, we will consider the existence of a maximum real GDP growth rate between periods, g^{max} (equation 12).

$$Y_t^{max,k} = u^{max}.\sigma.K(t-1) \quad (10)$$

$$Y_t^{max,l} = \frac{N_t}{a_t^0}.(1 - u^{frict}) \quad (11)$$

$$Y_t^{max,g} = g^{max}.Y_{t-1} \quad (12)$$

In equation (11), u^{frict}, is a purely frictional unemployment rate. Based on this exposition we come to output as a constrained function:

---

[2] For the compatibility between Inflation Targeting and Post-Keynesian economics see Setterfield (2006).

$$Y_t = \min\{Y^{max,k}\,; Y^{max,l}\,; Y^{max,g}\,; z\,\} \quad (13)$$

3.2 - Consumption and Investment

Let us assume an exogenous rate of growth for government consumption expenditures.

$$G_t^c = (1 + g_t^c).\,G_{t-1}^c \quad (14)$$

Regarding government investment we will suppose that public sector will invest in a way to achieve a target proportion of public capital stock on total capital as bellow:

$$\widetilde{K}_t = \frac{K_t^G}{K_t} \quad (15)$$

$$G_t^k = \gamma.\left(\widetilde{K}_t^* - \widetilde{K}_{t-1}\right).\,K_{t-1}^G \quad (16)$$

Regarding private investment expenditures by the private sector, we will suppose that private investment expenditure is determined by a two-stage process. In the first one, businessmen determines the desired level of capital stock for that period and, given the stock of capital inherited from the past, the level of investment that they want to undertake. The level of desired investment will depend on the state of long-term expectations and the liquidity preference of businessman. In the second stage, businessmen match the desired investment with the *financial restriction to invest.* This restriction will be determined by the maximum level of banking debt that businessmen are willing to accept. If desired investment is superior to the financial restriction, than firms will only be able to invest the amount permitted by their financial restriction. On the other hand, if desired investment is lower than the feasible investment, the firm will be able to invest all that it wants.

The capital stock that is desired by businessmen at each period has two components. The first one is: $\frac{\alpha_0 Y_{t-1}}{\sigma}$, where σ denotes the output-capital ratio and α_0 is the coefficient of sales projection that is used by entrepreneurs to form their expectations about the future level of production and sales from last period level of production. This is a simple formalization of a *conventional behavior* of expectations formation where the present situation is considered to be a guide for the future (Possas, 1993).

The second component of the desired stock of capital is: $\left[\alpha_1\left(\frac{P_t^D}{P_t^S} - 1\right)\right]$, where P_t^D is the demand price of capital assets in period t; P_t^S is the supply price of this equipment in the same period and α_1 is a positive constant. This component incorporates the investment decision in to a general theory of portfolio choice, since by this mechanism investment is capital assets is compared in terms of profitability and liquidity with alternative forms of wealth accumulation. More specifically, businessmen have always the alternative of using money instead of capital assets as a time vehicle to accumulate wealth (Davidson, 2002, p.71). A demand price of capital assets higher then the supply price of capital goods is a signaling for entrepreneurs that investment in capital assets is *superior* than hoarding money as a strategy for wealth accumulation.

Desired investment and desired capital stock can be expressed by:

$$I_t^d = K_t^d - K_{t-1}^d \quad (17)$$

$$K_t^d = \min\left\{\alpha_0.\frac{Y_{t-1}}{\sigma} - \alpha_1.\left(\frac{P_t^D}{P_t^S} - 1\right) \;\; ; \;\; \alpha_0.\frac{Y_{t-1}}{\sigma}\right\} \quad (18)$$

Assuming again a conventional behavior in expectations formation, one can compute the present value of expected revenues from capital equipment (the *demand price of capital equipment).* By means of a simple projection of last period profits to the future (Possas, 1993), we arrive at:

$$P_t^D = \frac{(1-\tau).m_{t-1}.P_{t-1}.Y_{t-1}}{d_t} \quad (19)$$

Where: τ is Firms Income Tax, m is the share of capital on income, and P is the general level of prices.

The equipment replacement cost (or the supply *price of equipment)* is equal to the value of capital stock evaluated at current prices of this equipment. Since output is a homogenous good, the supply price of capital goods is equal to the general level of prices. So, we have:

$$P_t^S = P_{t-1}.K_{t-1} \quad (20)$$

The discount rate, applied to expected revenues of capital equipment, depends on two elements: (i) The rate of interest of bank loans (a *proxy* for opportunity cost of investment projects, i_{t-1}); and (ii) the borrower's risk, a weighted mean of the in*solvency risk* (δ_{t-1}) and *liquidity risk* (f_{t-1}).

$$d_t = i_{t-1} + \theta.\delta_{t-1} + (1-\theta).f_{t-1} \quad (21)$$

Where: θ is the weight factor of insolvency and liquidity risk (it indicates the degree of managerial aversion to the risk of insolvency vis-à-vis to liquidity risk); L_t is the total amount of debts with the banking sector; γ is the amortization coefficient of debts; δ_{t-1} is the ratio between firms` total debts and their capital stock; and f_t is a coefficient of *financial fragility*, given by the ratio between firms´ financial debts and their operational profits.

$$\delta_{t-1} = \frac{L_{t-1}}{P_{t-1}.K_{t-1}} \quad (22)$$

$$f_{t-1} = \frac{i_{t-1}.L_{t-1}}{m_{t-1}.P_{t-1}.Y_{t-1}} \quad (23)$$

Once determined the expected investment, firms should match their investment intentions with their financial restriction. The financial restriction to investment expenditures will depend on: (i) the amount of new loans that they can contract with the banking sector, taking in consideration the maximum level of debts that businessmen are willing to accept to have with banks; and (ii) the amount of retained profits that are at hand for the financing of investment decisions. We suppose that government does not tax retained profits. Thus, the *financial restriction to investment* is the sum of the increase in the level of indebtedness that firms are willing to accept and retained profits. With effect, the investment, at the t period, that firms can support is determined by:

$$F_t = \delta_{max}.P_{t-1}.K_{t-1} - L_{t-1} + \vartheta.[P_{t-1}.Y_{t-1} - w_{t-1}.N_{t-1} - (i_{t-1} + \gamma).L_{t-1}] \quad (24)$$

The equation (25) details the realized investment at the period t

$$I_t = min\{I_t^D \;\; ; \;\; F(t)\} \quad (25)$$

Regarding the consumption expenses, we suppose the existence of different propensities to consume on, respectively, wages and profits (Kaldor, 1956 and Pasinetti, 1962). Specifically, we consider that "workers spend all they get" (their propensity to save is equal to zero). On the other hand, we assume that productive capitalists (in other words, the non-financial company owners) have a propensity to save out of operational profits equal to s_c. Finally, we suppose that financial capitalists (i.e., the banks owners) have a propensity to save over net receipts of financial intermediation equal to s_f . Consumption expenditures is given by:

$$P_t.C_t = W_{t-1}.N_{t-1} + (1+s_c).(1-\tau).(1-\vartheta).(P_{t-1}.Y_{t-1} - w_{t-1}.N_{t-1} - i_{t-1}.L_{t-1} - i^*_{t-1}.E_t.B^{*,e}_{t-1}) + (1-s_f).(1-\tau)(L_{t-1}.i_{t-1} - i^*_{t-1}.E_t.B^{*,B}_{t-1}) \quad (26)$$

3.4 - Portfolio Choice

Economic units (firms, banks and capitalists) hold their wealth in three different assets: money, bonds, and foreign currency. Private wealth is allocated between these assets based on four variables: real expected domestic bond interest rate ($i^e_d - \pi^e$), real expected foreign interest rate ($i^* - \pi^{e*}$), real expected depreciation of national currency ($\hat{e}^e - \pi^e$), and expected level of output (X^e). Table 4 summarizes the effects of these variables on asset demand.

Table 4: Effects of Variations of Asset Demand Determinants

	m^d_1	b^d_p	$e.r_p$
$i^e_d - \pi^e$	-	+	-
$i^* - \pi^{e*}$	+	+	-
$\hat{e}^e - \pi^e$	-	-	+
X^e	+	-	+

Furthermore, we need our mapping to give positive alocative factors m^d_1, b^d_p, and ▯r_p, in the (0, 1) interval, and they must attend the budget constraint with equality, that is, they must add up to 1, so as to deplete wealth:

$$\overbrace{m'_1.w_p}^{assets\ allocated\ in\ liquidity} + \overbrace{b'_p.w_p}^{assets\ allocated\ in\ bonds} + \overbrace{er'_p.w_p}^{assets\ allocated\ in\ currency} = w_p$$

Our portfolio allocation function is a $\mathbb{R}^4$ mapping into the three dimensional simplex $S^3 = (0,1) \times (0,1) \times (0,1)$. As the table above suggests, we need 12 sensitivities, 6 positive and 6 negative, so as to be able to fully determine the behavior of the portfolio allocation. The functional form proposed is:

$$Portolio: \mathbb{R}^4 \longrightarrow S^3$$

$$Portfolio(i^e_d - \pi^e;\ i^* - \pi^{e*};\ \hat{e}^e - \pi^e;\ X^e) = (m'_1; b'_p; 1 - m'_1 - b'_p)$$

$$m_1'\left(\overbrace{i_d^e-\pi^e}^{x}\ ;\ \overbrace{i^*-\pi^{e*}}^{y}\ ;\ \overbrace{\hat{e}^e-\pi^e}^{z}\ ;\ \overbrace{X^e}^{w}\right)=$$

$$\frac{\exp(\delta_1^x.x+\delta_1^y.y+\delta_1^z.z+\delta_1^w.w)}{\exp(\delta_1^x.x+\delta_1^y.y+\delta_1^z.z+\delta_1^w.w)+\exp(\delta_2^x.x+\delta_2^y.y+\delta_2^z.z+\delta_2^w.w)+\exp(\delta_3^x.x+\delta_3^y.y+\delta_3^z.z+\delta_3^w.w)} \qquad (27)$$

Here we use delta or the sensitivities refered above. We can simplify exposition by defining $\Delta^i(x,y,z,w) = \exp(\delta_i^x.x + \delta_i^y.y + \delta_i^z.z + \delta_i^w.w)$, so as to have:

$$m_1'(x,y,z,w) = \frac{\Delta^1}{\Delta^1+\Delta^2+\Delta^3}\ ;\ b_p'(x,y,z,w) = \frac{\Delta^2}{\Delta^1+\Delta^2+\Delta^3}\ ; er_p'(x,y,z,w) = \frac{\Delta^3}{\Delta^1+\Delta^2+\Delta^3}$$

Clearly $m_1' + b_p' + er_p' = 1$ and elasticities are proportional to the parameters referred at table 4.

3.5 - External Sector

Foreign inflation is given by π^*, a rate that fluctuates around some mean $\overline{\pi^*}$:

$$\pi_t^* = \overline{\pi^*} + \epsilon(t) \qquad (28)$$

$$P_t^* = (1+\pi_t^*).P_{t-1}^* \qquad (29)$$

External growth rate also randomly fluctuates around some mean $\overline{g^*}$:

$$g_t^* = \overline{g^*} + \epsilon(t) \qquad (30)$$

$$Y_t^* = (1+g_t^*).Y_{t-1}^* \qquad (31)$$

The quantum imported is given by:

$$M_t = j\left(\frac{E_{t-1}.P_{t-1}^*}{P_{t-1}}\right)^{\chi}.Y_{t-1}^{\varepsilon} \qquad (32)$$

The quantum exported:

$$X_t = x\left(\frac{E_{t-1}.P_{t-1}^*}{P_{t-1}}\right)^{\Omega}.{Y_{t-1}^*}^{\nu} \qquad (33)$$

Finally, effective demand can be expressed as:

$$Z(t) = C(t) + I(t) + G_c(t) + G_i(t) + X(t) - e(t).M(t) \qquad (34)$$

4. Comparative Dynamics and Calibration

We calibrate our model to perform as Oreiro and Passos (2008), using the same parameters whenever possible. Other parameters were revaluated or simply altered, and yet others are new.

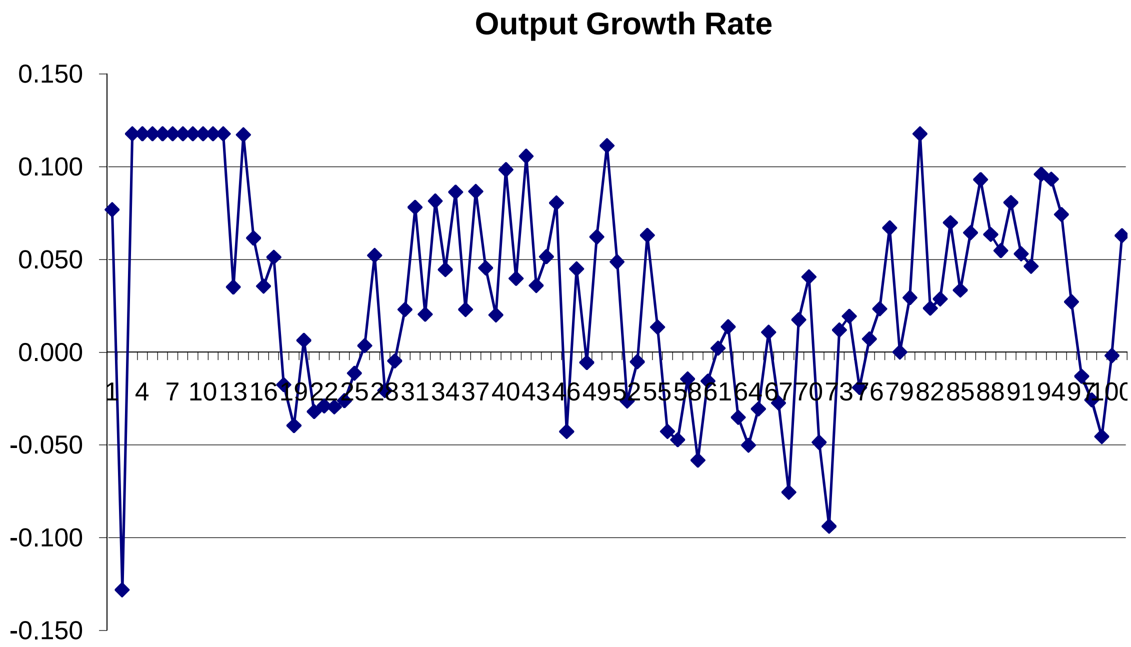
Table 5: Values of the Parameters

Parameter	Description and value used in simulation
g_t^*	World growth rate. Normal distribution following random variable centered in 2% and with 0.5% standard deviation.
i_t^*	World interest rate. Normal distribution following random variable centered in 4% and with 0.5% standard deviation.

π_t^*	World inflation rate. Normal distribution following random variable centered in 2% and with 0.1% standard deviation.
ψ	Capital assets depreciation. Normal distribution following random variable centered in 10% and with 0.5% standard deviation.
τ^w	Tax rate on wages, assumed to be 15%.
τ	Income tax, assumed to be 10%.
h^w	Population growth rate. Normal distribution following random variable centered in 1.5% and with 0.1% standard deviation.
π^{lt}	Long term inflation target. Assumed to be 4.5%.
g_Y^*	Target growth rate. Assumed to be 3%.
ζ	Random shock to the Kaldorian productivity function. Normal distribution following random variable centered in 0 and with 0.3 standard deviation.
g_t^c	Government expending growth rate. Normal distribution following random variable centered in 5% and with 1% standard deviation.
χ	Demand for imports price-elasticity. Normal distribution following random variable centered in -0.6 and with 0.1 standard deviation.
Ω	Demand for exports price-elasticity. Normal distribution following random variable centered in 0.4 and with 0.1 standard deviation.
ν	Demand for exports income-elasticity. Normal distribution following random variable centered in 0.9 and with 0.05 standard deviation.
ϵ	Demand for imports income-elasticity. Normal distribution following random variable centered in0.8 and with 0.05 standard deviation.

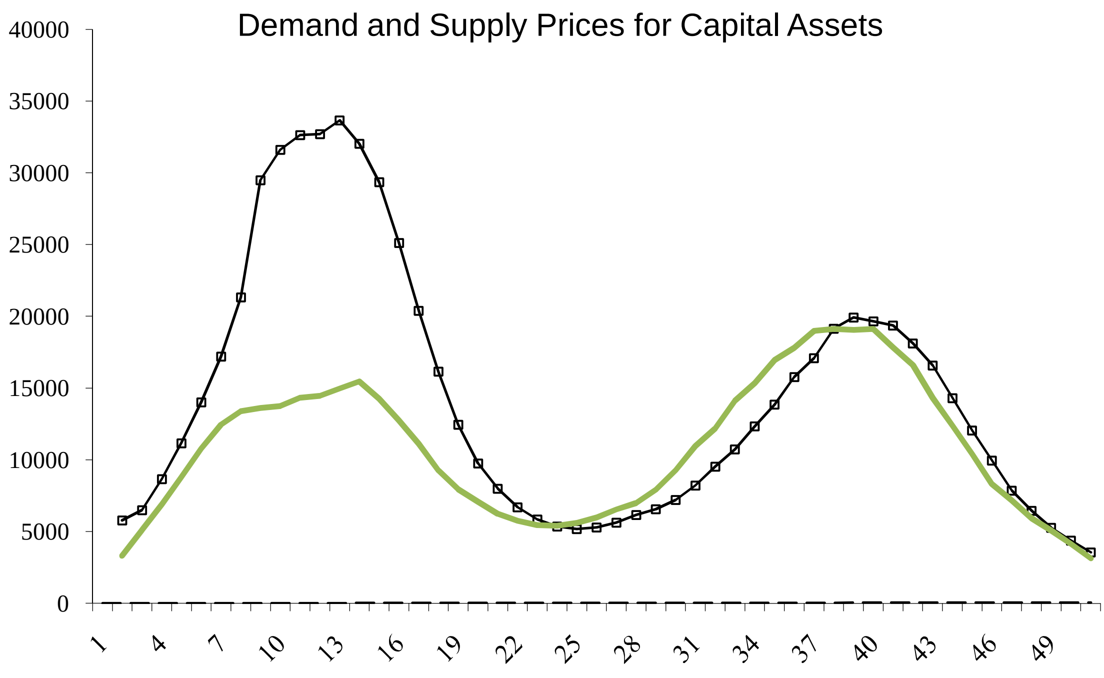
The first simulation performed is the growth rate for 100 periods. We can observe that the rate presents a positive long term trend, not explosive, but not smooth either. There's an irregular succession of recessions and growth periods, complying to Blanchard and Fisher (1989) stylized facts on the variable. Deep recessions are, in the other hand, rather rare events (Leijonhfvud 1996)

Figure 1: Output Growth Rate



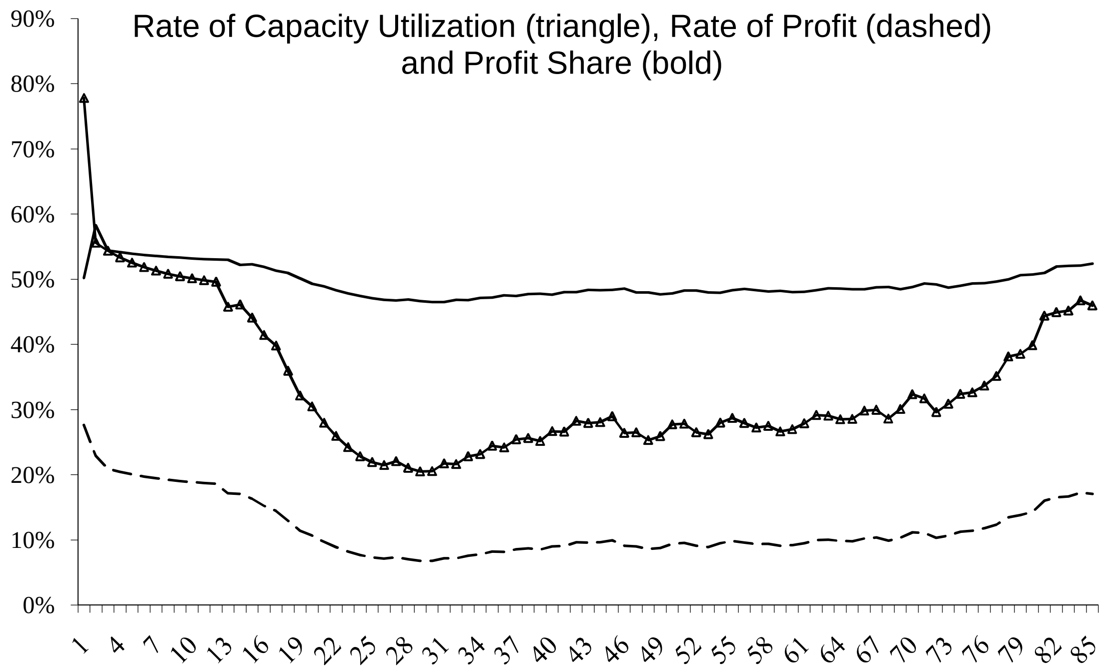
Next we analyze Minskys two prices (P^D marked with squares and P^S solid). There's no clear long term trend, specially no clear decreasing capital productivity trend (Kaldor 1957).

Figure 2: Price of Demand for (squares) and Supply of (solid) Equipment



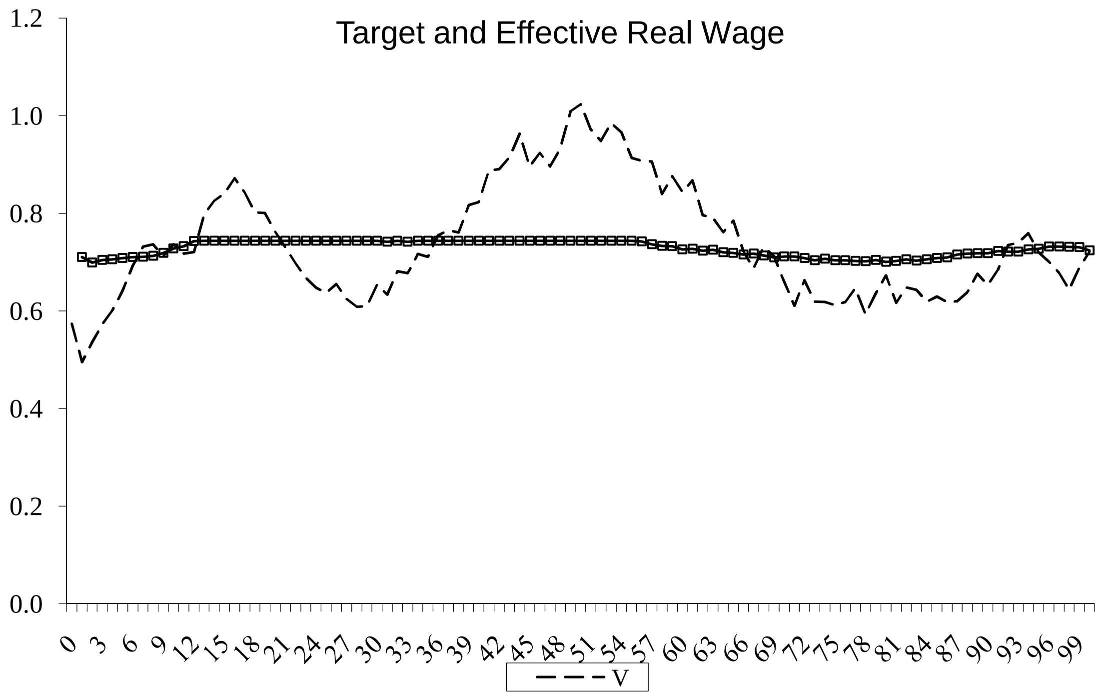
Below we depicted profits share in output (bold line), rate of capacity utilization (marked with triangles), and remuneration of capital (dashed). Past transient state we can see a general tendency to increase in profits share, and remuneration of capital is loosely related to utilization rate.

Figure 3: Capacity Utilization and Profit Rates and Profit Share



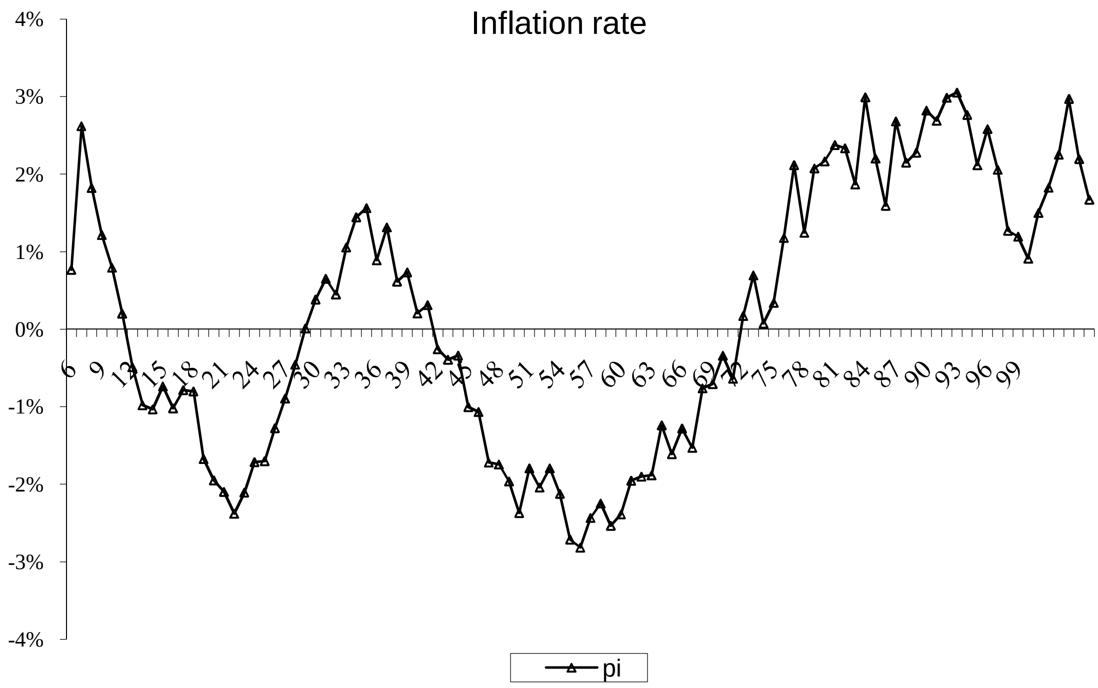
Real wages present an increasing trend as labor productivity grows. In the graph below target real wage is depicted with squares and effective real wage is marked by the dashed line.

Figure 4: Target (squares) and Effective (dashed) real wage



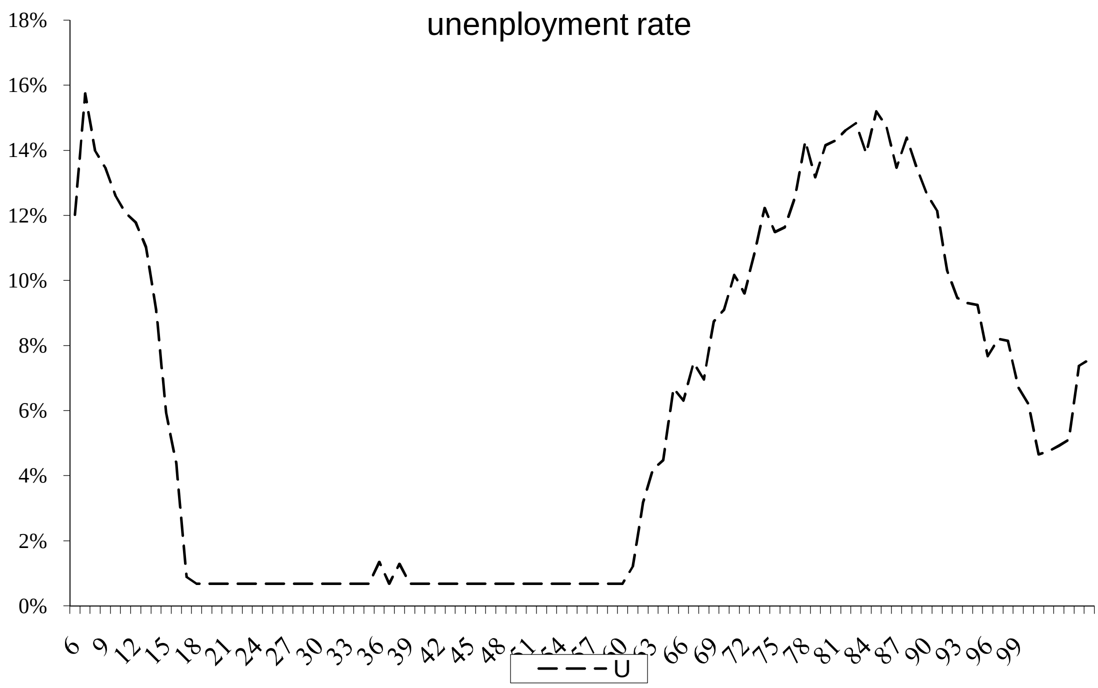
Inflation presents fluctuations typical of business cycles, in what firms mark-up varies greatly.

Figure 5: Inflation Rate



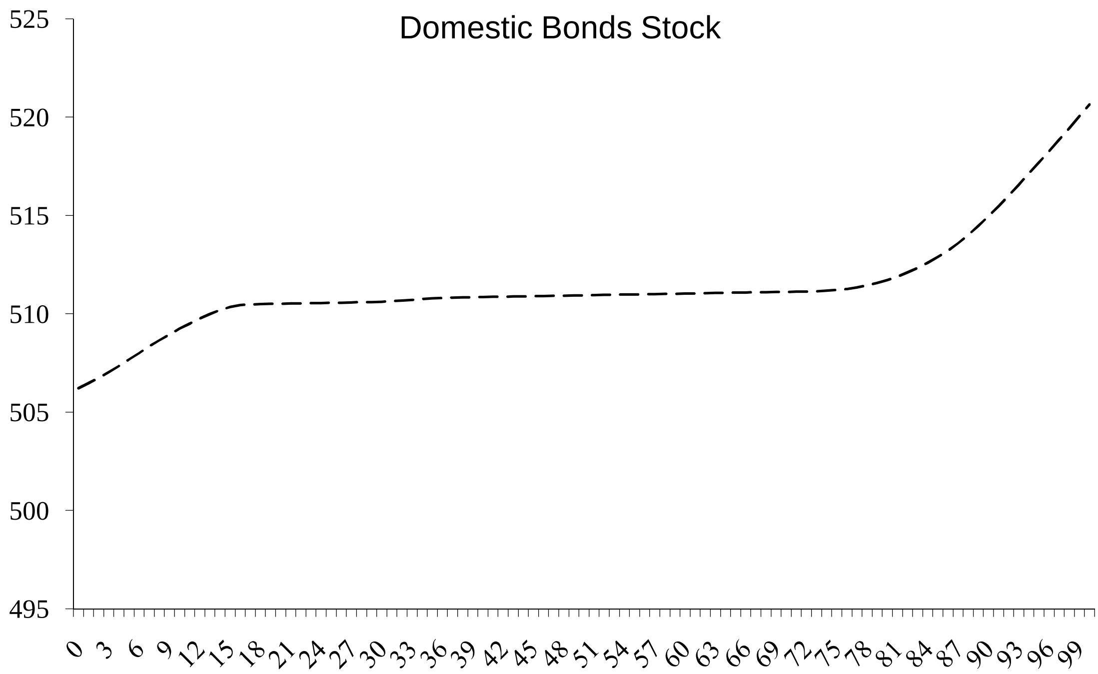
Unemployment rate also presents fluctuations, never going below its frictional rate.

Figure 6: Unenployment Rate



Graphing the stock of government bonds we can realize a trend towards persistent nominal deficit in governmental accounts.

Figure 7: Domestic Bonds Stock



5. Concluding Remarks

In this paper, we made some extensions to the Post Keynesian Macrodynamic model of growth and business cycles developed in Oreiro and Ono (2007), Oreiro and Lemos (2006) and Oreiro and Passos (2008). In particular:

i) We consider that firms and banks funding necessities can be financed by external bonds issues, and that there is an active market of imported intermediary goods and foreign currency;
ii) Government investment is not purely pro-cyclical, but rather depends on a target ratio between public and private capital stock;
iii) The investment function was developed to better conform to Minsky two prices theory;
iv) Government engages in active inflation targeting monetary policy;
v) Leading economy wide decisions and prices of every period, there's a basic portfolio allocation decision that leverages on past information on interest and exchange rates.

Besides these extensions, we have applied the stock and flow consistent - SFC schematic approach described in dos Santos and Zezza (2008). We have clearly established the SFC accounting and relevant behavioral relationships. The conformation of the model to the SFC scheme posed various challenges:

i) Non-transactionary flows, like exchange rate appreciation related revenues and expenses, had to be accounted for;
ii) Every aggregated expense, like aggregated investment, imports or exports, had to be individualized by agent.

Finally we adapted the model to operate as a recursive set of MathWorks® Matlab® methods. This approach provided a set of auxiliary methods for graphing results and storing intermediary variables.

Methods were made general enough to ease implementation of more complex decision processes, as the “look ahead”, or the “sliding window average and standard deviation” decision making algorithms. We hope that bringing this generation of models into a flexible platform (Matlab®) can further develop a line in research.

Further research can attach financial fragility, sovereign risk or capital structure indexes to diagnose realizations of the series, and to infer on causes of financial fragility. Monte-Carlo estimation can very easily be used. Other trend could relate SFC variables to national accounts and/or balance of payments accounts so as to have direct counterpart, and measure quality of prediction.

Appendix A. Computer Variable Names to Model Variable Names Mapping

Agents
w - workers
c - real side capitalists
f - financial capitalists
e - firms
b - banks
bc - central bank
g - government
star - rest of the world

Statements
bs - Balance Sheet
is - Income Statement

Balance Sheet Accounts
H - Hard Money Held
M - Check Deposits Held
A - Central Bank Loans Held
B - Domestic Bonds Held
B_star - Foreign Bonds Held
R - Foreign Currency Held
L - Bank Loans Held
pK - Capital Stock Held
V - Owners Equity

Income statement Accounts (Revenues positive and expenses negative in either case)
C - Consumption
I - Investment
W - Wages
T - Taxes

[Digite texto]

R_m - Imported Intermediate Goods
i_l_L - Interest on Bank Loans
i_B_B - Interest on Domestic Bonds
i_star_B_star - Interest on Foreign Bonds
F - Dividends
S - Current Savings

References

BLANCHARD, O. and FISHER, S. (1989). "Lectures on Macroeconomics". MIT Press: Cambridge.

CARVALHO, F.C. (1992). *Mr Keynes and the Post Keynesians: principles of macroeconomics for a monetary production economy*. Edward Elgar: Aldershot

DAVIDSON, P. (1968). "Money, Portfolio Balance, Capital Accumulation, and Economic Growth". *Econometrica*, 367, abril, p. 291-321.

DAVIDSON, P. (1972). "Money and the Real World". Armonk, NY: M.E. Sharpe.

____________ (2002). *Financial Markets, Money and the Real World*. Edward Elgar: Alderhot.

DOS SANTOS, C. H. (2005). "A stock-flow consistent general framework for formal Minskyan analyses of closed economies", *Journal of Post-Keynesian Economics*, 27(4): 11–35.

DOS SANTOS, C. H. and ZEZZA, G. (2008). "A Simplified "Benchmark" Stock-flow Consistent (SFC) Post-Keynesian Growth Model". *Metroeconomica*, 59: 441-478.

DOS SANTOS, C. H. and SILVA, A. C. M. (forthcoming). "Revisiting (and connecting) Marglin-Bhaduri and Minsky: a SFC look at financialization and profit-led growth".

GOODWIN, R. (1967). "A Growth Cycle" In: *Essays in Economic Dynamics*. MacMillan: London.

KALDOR, N. (1956). "Alternative Theories of Distribution". *Review of Economic Studies*, v. 23, p.83-100, 1956

--------------- (1957). "A Model of Economic Growth". *The Economic Journal*, vol. 67, n. 268, p591-624.

KALECKI, M. (1954). "Theory of Economic Dynamics – an essay on cyclical and long run changes in capitalist economy". *London: Allen & Unwin*.

LEIJONHFVUD, A. (1996) Towards a Not Too Rational Macroeconomics. In: COLLANDER, D. (ed.) Beyond Microfundations: post walrasian macroeconomics. Cambridge: Cambridge University Press.

MINSKY, H. P. (1975). "John Maynard Keynes", New York, Columbia University Press.

OREIRO, José Luís; ONO, Fábio Hideki. (2007). "Um Modelo Macrodinâmico Pós-Keynesiano de Simulação". *Revista de Economia Política*, Vol. 27, N.1.

OREIRO, J. L. C., e LEMOS, B.(2006). "Um modelo pós-keynesiano de crescimento e distribuição de renda aplicado a dinâmica das economias capitalistas desenvolvidas e em desenvolvimento." *Economia e Sociedade* (UNICAMP), v. 15, p. 475-514.

OREIRO, J. L. C. ; PASSOS, M. O. (2008) "Um modelo macrodinâmico estocástico de simulação para uma economia aberta". *Econômica* (Niterói), v. 10, p. 79-122.

PASINETTI, L. (1962). **"**Rate of Profit and Income Distribution in Relation to the Rate of Economic Growth**".** *Review of Economic Studies*, 29, p. 267-79, 1962.

_________ (1997). "The Principle of Effective Demand" In: HARCOURT, G.; RIACH, P.A. *A Second Edition of The General Theory*. Routledge: London.

POSSAS, M. (1993). "Racionalidade e Regularidades: Rumo à Integração Micro-Macrodinâmica" *Economia e Sociedade*, 2, p. 59-80.

SAMUELSON, P. (1939). "Interactions between the Multiplier and the Principle of Acceleration". *Review of Economic Studies*, 21.

SETTERFIELD, M. (2006). "Is Inflation Targeting Compatible with Post Keynesian Economics?". *Journal of Post Keynesian Economics*, 28

SOLOW, R. (1979). "Alternative Approaches to Macroeconomic Theory: a partial view". *Canadian Journal of Economics*, 12, pp.339-354.

TAYLOR, J. (1993)."Discretion versus Policy Rules in Practice". *Carnegie-Rochester Conference Series on Public Policy*, 39, 195-214.