1. Introduction

The main feature of capitalist dynamics is the occurrence of growth-cum-fluctuations, that is, the occurrence of fluctuations of real GDP along a stable but not necessarily constant “trend” in the long run. These fluctuations are, in general, irregular but non-explosive; i.e. there is no tendency to increase the magnitude of business cycles in the long-term. Heterodox economists usually think the ‘problem of capitalist dynamics’ in terms of linear and non-linear models of differential or difference equations that have definite (general) solutions. The linear dynamic models as those of Samuelson (1939) and Kalecki (1954) are only capable to produce regular fluctuations of economic activity along an exogenous determined trend of growth of output for a very restrictive set of parameters values. However, the fluctuations of output observed in the real world are essentially irregular fluctuations. Non-linear models with definite (general) solutions – as those of Hicks (1950) and Goodwin (1967) – are, in general, based on ad-hoc ‘ceils’ and ´floors´ or generate solutions of the “limit-cycle” type that do not reproduce the irregular character of real world fluctuations.

Because of the limitations of the dynamical models with general solutions, we can observe in recent years an increasing interest for dynamical models designed to be simulated in computer. These models have, in general, a non-linear structure, but the high number of equations and the complexity of inter-relations between the endogenous variables make impossible the determination of a general, definite, solution. These models can be solved only with the assistance of computer simulations; and the solution assumes the form of time-paths, instead of equilibrium positions, for the endogenous variables. The solution is obtained by computer after setting numerical values for the parameters of the equations and the initial conditions of the model. These values have to be as realistic as possible in order to assure a robust result in terms of time-paths.

One of the main limitations of the post-Keynesian paradigm in economics, as stressed several years ago by Solow (1979), is the inexistence of a common-unifying framework to analyze the capitalist dynamics. In a certain sense, post Keynesian economics is more a collection of alternative theories about growth, income distribution, inflation, business cycles than a theoretical approach to all these problems. There are several, and not necessarily compatible, theories to deal with the same problems. Take investment theory, for example. The approach of Davidson (1978) and Minsky (1975), although fully compatible with John Maynard Keynes ideas on the subject, is very different from the approach of Kalecki (1954) that stress the importance of the acceleration principle for investment decisions.

The objective of the present article is to present the structure and the first computational simulations of a macro-dynamic simulation model for an open economy with post-keynesian features that serves as a common framework for post-Keynesian economics. The elements of the post-keynesian paradigm incorporated in the model are: (i) The principle of effective demand; (ii) Differentiated saving propensities between capitalist and workers; (iii) Mark-up pricing; (iv) Investment decision based on the “two-price theory” of Minsky; (v) The relevance of capital structure over investment and pricing decision of firms; (vi) Inflation based on distributive conflict between capitalists and workers; (vii) Endogenous money supply; (viii) Endogenous technical progress a la Kaldor (1957). The computational simulations of the model reproduces some important features of capitalist dynamics as the occurrence of irregular, but non-explosive, fluctuations of the growth rate of real output; the stability of the profit rate in the long-term; the maintenance of idle capacity, the occurrence of a single episode of great reduction in the level of real output over the entire simulation period with is in accordance with the rare character of “Great Depressions” in the history of capitalism, the increasing importance of financial wealth for the dynamics of the wealth of capitalists and the irrelevance of real exchange rate for the dynamics of balance of payments.

The present article is structured in five sections, including the introduction. In the second section, we will present the six building blocks of the model at hand. Third section is dedicated to the calibration methodology of the model. Section four presents the main simulation results and section five concludes the paper.

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1 For a critique of Solow (1979), see Carvalho (1992).
2. A Macro-dynamic Simulation Model for an Open Economy

In this section we will present the structure of a macro-dynamic model with a productive and a financial sector in open economy with governmental activities. There are only two inputs available for production: capital and labor (both are homogeneous) and only one good is produced, consumed and invested in this economy.

The solution of the model starts with the definition of the initial values for the endogenous values in time zero as well as the values of the structural parameters of the model. Some parameters, however, do not have constant numerical values, but their values changes through time in a random way according to a uniform probability distribution. This procedure is not only an attempt to represent the uncertainty surrounding the estimation of the “true” parameters of the model, but also a source of exogenous shocks to the system. The dynamical equations will then be used to calculate the values of the endogenous variables from period 1 to period 69, which is supposed to be the end of the simulation span.

The non-linear nature of the dynamic equations of the model at hand in addition to the occurrence of exogenous shocks will produce a time path for endogenous variables that is dependent upon the particular history of the exogenous shocks. Therefore, the “solution” of the model is path-dependent and “history matters”.

We must also emphasize that in the model at hand the time-paths for the endogenous variables are not, in general, determined by attractors or by any kind of equilibrium. If a steady-state for the endogenous variables could, in principle, be calculated; it will be a very particular solution (one in a entire set of non-equilibrium solutions) and a no interesting one, since there are no mechanisms by which the selection of this particular solution can be assured. The general case is a non-equilibrium dynamical path for the endogenous variables.

The model at hand will be structured in seven interconnected modules: (i) module 1 - the components of the effective demand; (II) module 2 – the determination of the level of output and income; (III) module 3 – income the determination of income distribution; (IV) module 4 – inflation and monetary policy (v) module 5 – financial system; (vi) module 6 - foreign sector, and (vii) module 7 – balance sheet of the private sector.

2.1. Module 1: Effective demand.

We initially assume that government consumption expenditure in real terms grows, in each period, by an exogenous rate \( h \). Thus, we can write the following equation:

\[
G_t^C = (1 + h^C) G_{t-1}^C
\]

Where: \( G_t^C \) is the government consumption expenditures in real terms in period \( t \).

Government also makes investment expenditures, mainly for the sake of public infrastructure. We will suppose that this kind of expenditures is pro-cyclical. Then we have: \( G_t^I = h^I Y_{t-1} \). Where:

\[1 > h^I > 0\].

Regarding investment expenditures by the private sector, we will suppose that private investment expenditure is determined by a two-stage process. In the first one, businessmen determines the desired level of capital stock for that period and, given the stock of capital inherited from the past, the level of investment that they want to undertake. The level of desired investment will depend on the state of long-term expectations and the liquidity preference of businessman. In the second stage, businessmen match the desired investment with the financial restriction to invest. This restriction will be determined by the maximum level of banking debt that businessmen are willing to accept. If desired investment is superior

\[2\]

It must stressed that the simple introduction of non-linearities in a economic model is not a sufficient condition to produce bounded fluctuations of real output or the growth rate of real GDP. As shown in Nasica (2000, pp.68-69), a non-linear dynamic model can produce monotonic convergence (or divergence) to a steady-state position depending on the values of the parameters. In a pure mathematical exercise, we can choose the numerical values for the parameters that are required for the existence of sustained fluctuations of growth rate of GDP. However, in a more realistic exercise, under the point of view of economic theory, such degrees of freedom could not exist. So, the introduction of random shocks could be considered an adequate strategy for the modeling of economic dynamics. Besides that, in a deterministic, although non-linear, economic model, path-dependence is almost always the result of differences in initial conditions (Ibid, p.63). In a stochastic non-linear dynamical model, however, path-dependence is the result of the effects of different “histories” of external shocks over the structure of the system.
to the financial restriction, than firms will only be able to invest the amount permitted by their financial restriction. On the other hand, if desired investment is lower than the feasible investment, the firm will be able to invest all that it wants.

The capital stock that is desired by businessmen at each period has two components. The first one is: \( \frac{\sigma Y_{t-1}}{\alpha_0} \), where \( \sigma \) denotes the output-capital ratio and \( \alpha_0 \) is the coefficient of sales projection that is used by entrepreneurs to form their expectations about the future level of production and sales from last period level of production. This is a simple formalization of a conventional behavior of expectations formation where the present situation is considered to be a guide for the future (Possas, 1993). We have to notice that \( \alpha_0 \) is supposed to be a random variable with a uniform probability of distribution defined in the interval \([-0.3; 3.7]\). Fluctuations in the value of \( \alpha_0 \) represent changes in the state of long-term expectations in the Keynesian theory of investment behavior.

The second component of the desired stock of capital is:

\[
\begin{align*}
\frac{\alpha Y_{t-1}}{\sigma} - \alpha_1 \left( \frac{P^D_t}{P^S_t} - 1 \right) ; \quad & \text{if } P^D_t > P^S_t \\
\frac{\alpha_0 Y_{t-1}}{\sigma} ; \quad & \text{c.c.}
\end{align*}
\]

The demand price of capital assets in period \( t \); \( P^D_t \) is the supply price of this equipment in the same period and \( \alpha_1 \) is a positive constant. This component incorporates the investment decision into a general theory of portfolio choice, since by this mechanism investment is capital assets is compared in terms of profitability and liquidity with alternative forms of wealth accumulation. More specifically, businessmen have always the alternative of using money instead of capital assets as a time vehicle to accumulate wealth (cf. Davidson, 2002, p.71). A demand price of capital assets higher then the supply price of capital goods is a signaling for entrepreneurs that investment in capital assets is superior than hoarding money as a strategy for wealth accumulation.

Desired investment and desired capital stock can be expressed by:

\[
I^D_t = K^D_t - K_{t-1} \quad (3)
\]

\[
K^D_t = \begin{cases} 
\frac{\alpha Y_{t-1}}{\sigma} - \alpha_1 \left( \frac{P^D_t}{P^S_t} - 1 \right) ; \quad & \text{if } P^D_t > P^S_t \\
\frac{\alpha_0 Y_{t-1}}{\sigma} ; \quad & \text{c.c.}
\end{cases} \quad (4)
\]

Assuming again a conventional behavior in expectations formation, one can compute the present value of expected revenues from capital equipment (the demand price of capital equipment). By means of a simple projection of last period profits to the future (Possas, 1993), we arrive at:

\[
P^D_t = \frac{(1-\tau)m_{t-1} P^S_{Y_{t-1}}}{d_t} \quad (5)
\]

Where: \( \tau \) is the tax rate over business profits, \( m_{t-1} \) is the profit share in the income of the period \( t-1 \), \( P_{t-1} \) is the general level of prices in \( t-1 \), \( Y_{t-1} \) is the real income in \( t-1 \) and \( d_t \) is the discount rate applied to expected revenues of capital equipment.

The equipment replacement cost (or the supply price of equipment) is equal to the value of capital stock evaluated at current prices of this equipment. Since output is a homogenous good, the supply price of capital goods is equal to the general level of prices. So, we have:

\[
P^S_t = P_{t-1} K_{t-1} \quad (5a)
\]

The discount rate, applied to expected revenues of capital equipment, depends on two elements: (i) The rate of interest of bank loans (a proxy for opportunity cost of investment projects, \( i_{t-1} \)); and (ii) the borrower’s risk, a weighted mean of the insolvency risk \( (\delta_{t-1}) \) and liquidity risk \( (f_{t-1}) \). So we have:

\[
d_t = i_{t-1} + \theta \left( \frac{L_{t-1}}{P_{t-1} K_{t-1}} \right) + (1-\theta) \left( \frac{i_{t-1} + \gamma L_{t-1}}{m_{t-1} P_{t-1} Y_{t-1}} \right) = i_{t-1} + \theta \delta_{t-1} + (1-\theta) f_{t-1} \quad (6),
\]
Where: $\theta$ is the weight factor of insolvency and liquidity risk (it indicates the degree of managerial aversion to the risk of insolvency vis-à-vis to liquidity risk); $L_t$ is the total amount of debts with the banking sector; $\gamma$ is the amortization coefficient of debts; $\delta_{t-1}$ is the ratio between firms’ total debts and their capital stock; and $f_t$ is a coefficient of financial fragility, given by the ratio between firms’ financial debts and their operational profits.

Once determined the expected investment, firms should match their investment intentions with their financial restriction. The financial restriction to investment expenditures will depend on: (i) the amount of new loans that they can contract with the banking sector, taking in consideration the maximum level of debts that businessmen are willing to accept to have with banks; and (ii) the amount of retained profits that are at hand for the financing of investment decisions. We suppose that government does not tax retained profits. Thus, the financial restriction to investment is the sum of the increase in the level of indebtedness that firms are willing to accept and retained profits.

With effect, the investment, at the $t$ period, that firms can support is determined by:

$$I_t = \min \left\{ I_t^P, F_t \right\} \quad (8)$$

Regarding the consumption expenses, we suppose the existence of different propensities to consume on, respectively, wages and profits (Kaldor, 1956 and Pasinetti, 1962). Specifically, we consider that “workers spend all they get” (their propensity to save is equal to zero). On the other hand, we assume that productive capitalists (in other words, the non-financial company owners) have a propensity to save out of operational profits equal to $s_c$. These capitalists own a stock of foreign assets (sovereign bonds issued by an external government) that is inherited from previous period ($A^*_{t-1}$). Supposing that $i^*_t$ is the interest rate paid over foreign bonds, than productive capitalists receive a foreign income equal to $A^*_{t-1}i^*_t$ measured in foreign exchange and $E_t A^*_{t-1}i^*_t$ measured in domestic currency, where $E_t$ is nominal rate of exchange at period $t$.

Finally, we suppose that financial capitalists (i.e., the banks owners) have a propensity to save over net receipts of financial intermediation equal to $s_f$. Thus, the nominal expenses of consumption in the period $t$ are determined by the following expression:

$$PC_t = w_{t-1}N_{t-1} + (1-s_c)[(1-\tau)(1-\theta)(P_{t-1}Y_{t-1} - w_{t-1}N_{t-1} - (i_{t-1} + \gamma)L_{t-1}) + A^*_{t-1}i^*_tE_t] + (1-s_f)(1-\tau)L_{t-1}i^*_{t-1} \quad (9)$$

The effective demand at the $t$ period, $Z_t$, is determined by

$$Z_t = C_t + I_t + G^c_t + G^f_t + X_t - e_t M_t \quad (10)$$

Where: $X_t$ is the quantum of exports at the $t$ period; $M_t$ is the quantum of imports at the $t$ period; $e_t = \frac{E_t P^*_t}{P^*_t}$ is the real rate of exchange; $P_t$ is the domestic price level; $P^*_t$ is the international price level.

We suppose that the quantum of imports is determined by the following equation:

$$M_t = j \left( \frac{E_t P^*_t}{P^*_t} \right)^\zeta Y_{t-1} \quad (11)$$

In which: $j$ is a positive constant; $\zeta$ is the income-elasticity of demand for imports (positive); $\chi$ is the price-elasticity of demand for imports (negative).
The exports function, in a similar manner, is given by\(^6\):
\[
X_t = \chi \left( \frac{E_{t-1} P_t^*}{P_{t-1}} \right)^{\Omega} Y_{t-1}^* \quad (12)
\]

In which we have: \(x\) as a positive constant; \(\chi\) as the income-elasticity of demand for exports (positive); \(\Omega\) as the price-elasticity of demand for exports (positive); and \(Y_{t-1}^*\) representing the external demand defined in the previous period.

### 2.2. Module 2: Production, Income and Technological Progress.

According to the principle of effective demand, the production level is determined by effective demand for goods and services (Pasinetti, 1997, p.99). This occurs if and only if there is no full utilization of productive capacity (i.e., there is a “idle” capacity). We suppose that firms meet all changes in the demand for their products with changes in the output level until they reach the potential output.

The level of potential output is determined by two restrictions: (i) the manpower available; (ii) the maximum level of capacity utilization.

With regard to the first restriction, there is a minimum value that unemployment rate can reach. This rate \((U_{\text{min}})\) will determine the full-employment level of output \(Y_{t}^{\text{max},f}\) given by:

\[
Y_{t}^{\text{max},f} = \frac{N}{q_t} (1 - U_{\text{min}}) \quad (13),
\]

where \(q_t\) is labor requirement per-unit of output.

The variable \(q_t\) is not constant, but changes through time due to the occurrence of technical progress. Regarding technical progress, we will suppose the existence of dynamical economies of scale, as, for example, “learning by doing”. This means that the rate of change of labor productivity is determined by the rate of change of real output and/or the rate of change in the level of capital stock. Here we adopted a Kaldorian technical progress function (Kaldor, 1957) as the one written above:

\[
q_t = q_{t-1} - \rho_0 \left[ \frac{(1 - \psi)K_{t-1}^i + I_{t-1}^i + G_{t-1}^i}{(1 - \psi)K_{t-2}^i + I_{t-1}^i + G_{t-1}^i} - 1 + \varepsilon \right] q_{t-1} \quad (14)
\]

In expression (14), we can see that any increase in the stock of aggregate capital will reduce the labor requirement per unit of output, i.e. will increase the productivity of labor.

The uncertainty surrounding the process of technical change will be modeled by means of a random variable \((\xi)\), which is assumed to have a uniform probability distribution in interval \([-1,1]\).

Another assumption, just as emphasized by Steindl (1952), is the existence of an upper limit to the level of capacity utilization. So the maximum output compatible with the normal level of capacity utilization is given by:

\[
Y_{t}^{\text{max},c} = u_{\text{max}} \bar{Y}_{t-1} \quad (15)
\]

In equation (15), \(u_{\text{max}}\) is the normal level of productive capacity utilization; \(Y_{t}^{\text{max},c}\) is the level of output compatible with a normal level of capacity utilization and \(\bar{Y}_{t-1}\) is the maximum output at period \(t-1\).

Maximum output \(\bar{Y}_{t-1}\) is given by:

\[
\bar{Y}_{t-1} = \sigma K_{t-1} \quad (15a)
\]

Where: \(\sigma\) is “social productivity of capital”, a technical coefficient that describes the amount of output that can be obtained through the utilization of one unit of capital.

In this context, the level of output compatible with both restrictions will be the smaller value between (13) and (15):

\[
Y_{t}^{\text{max}} = \min \left[ \frac{N}{q_t} (1 - U_{\text{min}}) , u_{\text{max}} \sigma K_{t-1} \right] \quad (16)
\]

If the level of output is smaller than the potential output - determined by equation (16a) - then real output in period \(t\) will be determined by the effective demand of this same period - given by equation (10). However, we have to consider another restriction. Real output cannot increase between periods at any rate. In fact, there is a maximum rate of increase of real output between periods due to the existence of adjustment costs. These costs are related to selecting, contracting and training new workers.

\(^6\) Based on Thirwall (1982, p.221).
Therefore, we will consider the existence of a maximum real GDP growth rate between periods, \( g_{\text{max}} \). In this way, the production level at the \( t \) period will be determined by:

\[
Y_t = \min \left[ Z_t, Y_{t-1}^{\text{max}}, (1 + g_{\text{max}})Y_{t-1} \right]
\]

(16a)

2.3. Module 3: Income distribution

According to a classical-marxist view, in an industrial economy, just as the one supposed by the model at hand, national income should be conceived as the wealth expressed in material terms (products) and created along a certain period. Hence, there are only two kinds of income in an industrial economy: wages and gross profits. The government and the financial sector do not create wealth, because they do not create added value. They just appropriate a share of the profits (constituted by taxes and interests). Taxes and interests do not affect the profits amount. By this perspective, the income evaluated in nominal terms and generated along the period \( t \) is just wages plus gross profits. So, we have:

\[
P_t Y_t = w_t N_t + r_t P_t K_t
\]

(17). Where \( r_t \) is the profit rate and \( w \) is the nominal wage rate.

Profit rate \( r_t \) can be seen as the product between profits share in the income (\( m_t \)), the level of productive capacity utilization (\( u_t \)) and the “social productivity of capital” (\( \sigma \)). Thus, we can re-write the expression (17): \[ m_t = 1 - V_t q_t \] (18). Where: \( V_t \) is the real wage rate.

Equation (18) shows that, given the labor productivity, there is an inverse relationship between real wage and profits share.

2.4. Module 4: Inflation and Monetary Policy

In order to determine the rate of inflation of the economy at hand, we will suppose that: (i) Exports and imports are made only with final goods. So, variations in nominal exchange rate do not have any direct impact upon production costs of firms; (ii) International mobility of labor is zero so that nominal and real wages are determined only in accordance with the conditions prevailing in domestic labor markets; (iii) Domestic firms operate under an oligopolistic market structure, so they have market-power. In other words, firms are price-makers in goods market; (iv) Due to the existence of uncertainty regarding the level of demand for their products (which is the direct result of strategic interaction between price-decision of firms under oligopoly), firms fix the prices of their products by means of a mark-up rate over unit direct costs. Based on this discussion, we have:

\[
(1 + r_t) = \frac{P_t}{P_t - 1} \left[ \left( \frac{1 + z_t^f}{1 + z_{t-1}^f} \right) \frac{w_t}{w_{t-1}} \frac{q_t}{q_{t-1}} \right]
\]

(20)

Where: \( \pi \) is the inflation rate in the period \( t \).

The first step to determine the inflation rate at the \( t \) period \( t \) is to compute the wage inflation (in other words, the rate of variation of the nominal wages between period \( t \) and the period \( t-1 \)). For this regard, we suppose that the nominal wages are the result of a bargaining process among firms and unions. In this negotiation process, unions demand higher wages to cover the inflation losses of the previous period and to increase the real wage rate to a certain target rate. This target rate is determined by the conditions prevailing in the labor market and by productivity growth, since unions desire to incorporate productivity gains in real wages. Greater is the bargaining power of unions, greater will be the target real wage in the determination of nominal wages.

In this sense, we have:

\[
\left( \frac{w_t - w_{t-1}}{w_{t-1}} \right) = \left( \frac{P_{t-1} - P_{t-2}}{P_{t-2}} \right) + \phi (\bar{V}_t - V_{t-1})
\]

(21), where \( \bar{V}_t \) is the target real wage in the period \( t \).

Target-real wage is determined by:

\[
\bar{V}_t = \phi_1 - \phi_2 U_{t-1} + \phi_3 \frac{1}{q}
\]

(21a)

Using (19), (21) and (21a) in (20), we arrive at the following expression:
The equation for interest rate rule is given by:

\[ i_t^* = (1 - \lambda)\pi_{t-1} + \lambda \left[ \beta_0 (\pi_{t-1} - \pi^*) + \beta_1 (g_{r-1} - \eta) + \beta_2 \right] \] (22)

Where: \( i_t^* \) is the short-term interest rate determined by the Central Bank; \( \lambda \) is the interest rate inertia factor; The coefficients \( \beta_0 > 0 \) and \( \beta_1 > 0 \) represents, respectively, the given weight, in interest rate composition, to the divergence of inflation rate of the previous period concerning the “natural” growth rate (\( \eta \)); and the to the divergence of the growth rate of real output (at the previous period) concerning the natural growth rate. \( \beta_2 \) is a constant.


Just as in the case of the productive sector, we suppose the existence of an oligopolistic market structure in the banking sector. Thus, banks are price-makers too. Therefore, they fix the interest rate of their loans to the productive sector. The commercial banks define the interest rate charged by their loans (\( i_t \)) through the application of a mark-up (\( z^b_t \)) over the short-term interest rate fixed by the Central bank (Rousseas, 1986, pp.51-52). Thus, we have the equation: \( i_t = (1 + z^b_t) i_t^* \) (23)

Banking mark-up is not constant through time. It varies between periods according to the economic conjuncture and/or to the banks market power. Just as Aronovich (1994), we suppose that banking mark-up is countercyclical, varying in the opposite direction of productive capacity utilization. In fact, increases in the level of productive capacity utilization are related to sales increase and, by extent, to a reduction of productive sector’s firms default risk. This reduction allows to banks reduce their spreads.

Other assumption in this module: increases in the inflation rate induce the commercial banks to increase the rate of mark-up. The intuition here is a higher inflation forces the Central Bank to increase the short-term interest rate. Such action is designed in order to prevent a divergence between current inflation rate and the target rate of inflation. But it increases the volatility of the short-term interest rate, contributing for an increase of “interest risk” (Ono et alli, 2005). In response to this increase, commercial banks charge higher spreads. Banking mark-up is described just as it follows:

\[ z^b_t = \max \left( z^b_{min}; z^b_t u_{r-1} + z^b_t \pi_{r-1} \right) \] (24)

Once fixed the loans interest rate, commercial banks meet the entire credit demand of the productive sector. That means that there is no kind of credit rationing, just as we see in new-keynesian macroeconomic models. Therefore, the effective volume of credit conceded by commercial banks at the t period is entirely determined by credit demand. Such assumption is in accordance with the hypothesis of endogenous supply of money (Kaldor, 1986 and Moore, 1988). Creation of demand deposits is,

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7 For the compatibility between Inflation Targeting and Post-Keynesian economics see Setterfield (2006).

8 The only restriction to the application of equation (22) as a rule for fixing the short-term interest rate is that nominal interest rates can never be negative. So we can define a roof for the short-term interest rate. We will define this minimum value for short-term interest rate as \( l_{min} \), so the value of short-term interest rate in period t is given by:

\[ i_t = \max \left(l_{min}; (1 - \lambda)\pi_{t-1} + \lambda [\beta_0 (\pi_{t-1} - \pi^*) + \beta_1 (g_{r-1} - \eta)] + \beta_2 \right) \]

9 According to Barbosa (2004), Central Banks operate monetary policy in a manner to smooth interest rate changes over time. This interest rate smoothing generates a certain level of inertia in interest rate determination.
Therefore, determined by expansion of banking credit. So, we have: \( D_t - D_{t-1} = L_t - L_{t-1} \) (25). Where \( D_t \) is demand deposits at period \( t \).

The government's fiscal deficit \((DG_t)\), by hypothesis, is entirely financed by the expansion of the monetary base \((H_t)\) at period \( t \): \( DG_t = H_t - H_{t-1} \) (26).

2.6. Module 6: External sector

The economy considered in the model at hand is a small open-economy with a floating exchange rate regime that operates in a setting of imperfect mobility of capital. With regard to the degree of openness of capital account, we adopt the assumption that capitalists of productive sector are the only agents that are authorized by law to use a part of the distributed profits buy foreign assets. Direct investment is the only way by which the capitals can flow out the domestic economy. There is no interest-rate arbitrage\(^{10} \).

Hence, the prices and the international inflation are determined by the next equations:

\[
P_t^* = (1 + \pi^*) P_{t-1}^* \quad (27) \quad \text{and} \quad \pi^* = \bar{\pi} + \varepsilon_t \quad (28)
\]

In which: \( \varepsilon_t \) is a white noise and \( \bar{\pi} \) is the average inflation of the rest of the world (an exogenous and random variable with constant average and variance).

The growth rate of the world economy, given by \( g_t^* \), is supposed to fluctuate in a random way around a constant level, as we can see in equations (29) and (30):

\[
Y_t^* = (1 + g_t^*) Y_{t-1}^* \quad (29) \quad \text{and} \quad g_t^* = \bar{g} + \eta_t \quad (30)
\]

In which \( \eta_t \) is the term of white noise related to the growth process.

Similarly, we suppose that the international interest rate fluctuates in random manner around a constant value (an exogenous variable): \( i_t^* = \bar{i} + \gamma_t \) (31). Where: \( \bar{i} \) is the average interest rate in international economy and \( \gamma_t \) is a white noise associated with the international interest rate movements.

The current account balance is given by:

\[
STC = E_t P_t^* (X_t - M_t) + E_t i_{t-1}^* A_{t-1}^* \quad (32)
\]

The result of the capital account is determined by:

\[
SKC = -E_t \left( A_t^* - A_{t-1}^* \right) \quad (33)
\]

We assume the existence of a pure floating exchange rate regime. Because of that, the result of the balance-of-payments is necessarily equal to zero, once the monetary authorities do not intervene in the exchange market. So, we can write:

\[
E_t P_t^* (X_t - e_t M_t) + E_t i_{t-1}^* A_{t-1}^* = E_t \left( A_t^* - A_{t-1}^* \right) \quad (34)
\]

Productive capitalists allocate their savings in foreign assets, so that capital account balance can be written as:

\[
E_t \left( A_t^* - A_{t-1}^* \right) = s_t \left( 1 - \tau \right) \left( 1 - \bar{g} \right) \left( P_{t-1} Y_{t-1} - w_{t-1} N_{t-1} - (i_{t-1} + \gamma) L_{t-1} \right) + E_t i_{t-1}^* A_{t-1}^* \quad (35)
\]

Combining the expressions (11), (12) and (35) in (34), with some calculus we have:

\[
E_t \left( P_t^* \left[ \left( E_t P_{t-1}^* \right)^\Omega \left( Y_{t-1}^* \right)^Y - j \left( E_t P_{t-1}^* \right)^\chi \left( Y_{t-1}^* \right)^\xi \right] + i_{t-1}^* A_{t-1}^* \right) = (1 - \tau) \quad (36)
\]

In the equation (36) the nominal rate of exchange (at the period \( t \)) is the variable that adjusts the result of current account balance to the result of capital account in order to maintain the equilibrium in the balance of payments. Solving the equation (36) to \( E_t \), we obtain:

\[
E_t = \left( 1 - \tau \right) \left( 1 - \bar{g} \right) \left( P_{t-1} Y_{t-1} - w_{t-1} N_{t-1} - (i_{t-1} + \gamma) L_{t-1} \right) \quad (37)
\]

2.7 Module 7: Balance Sheet of the Private Sector

---

\(^{10}\) So, the transfer of liquid funds from one monetary center (and currency) to another to take advantage of higher rates of return or interest is prohibited by law.
The private sector of the economy has three social classes: productive capitalists, financial capitalists and workers. Working class’ propensity to save assumed to be equal to zero. Workers have only their own capacity to work; they do not have any other kind of wealth. Productive and financial capitalists own, respectively, productive firms and banks, which existence is totally independent and distinguishable of their owners. In this way, assets and debts of firms and banks are completely separated of the personal wealth of their owners.

Productive firms own only capital stock as asset. It is financed by retained profits (net wealth, \(W^F_t\)) and bank loans (\(L_i\)). Therefore, their balance sheet can be expressed by: \[PK_t = L_i + W^F_t\] (38)

Bank assets are represented by loans to productive firms and by bank reserves. Bank liabilities are composed by net wealth (\(W^B_t\)) and demand deposits (\(D_i\)). In algebraic terms: \[L_i + R_i = D_i + W^B_t\] (39)

The productive capitalist use its savings for the purchase of foreign currency assets. Thus, we have: \[E_iA^* = W^cf_i\] (40). Where \(W^cf_i\) is the wealth of productive capitalists.

Finally, financial capitalists allocate his wealth (\(W^cf_t\)) in two assets: demand deposits and cash (\(M_i\)). So, we can write: \[D_i + M_i = W^cf_t\] (41)

Aggregating all assets and debts, we have:
\[W_t = W^B_t + W^F_t + W^cf_t + W^sp_t = R_i + M_i + PK_t + E_iA^*\] (42)

Where \(W_t\) is the net wealth of private sector.

Monetary base (\(H_t\)) is equal, by definition, to the sum of bank reserves and cash owned by the public. In this way, we have: \[H_t = R_t + M_t\] (43).

As we have seen, only banks and financial capitalists take portfolio decisions. Productive capitalists and firms allocate their wealth entirely in only one asset.

With regard to financial capitalists, we suppose that the amount of cash that they wish to hold (at the \(t\) period) is equal to the stock of money they retained (at the previous period) plus the saved share of net profit (at the previous period). Hence, \[M_t = M_{t-1} + \left((1 - \tau)s_f(i_{t-1}L_{t-1})\right)\] (44)

Finally, with some calculus, from (26), (44) and (43), we have:
\[R_t = (H_{t-1} - M_{t-1}) + \left[DG_t - \left((1 - \tau)s_f(i_{t-1}L_{t-1})\right)\right]\] (45)

3. Calibration of the Model

The model at hand has 34 structural parameters and 45 equations. In order to solve for the dynamical paths of the system we must start by defining the values of the structural parameters. Some of these parameters, however, are random variables generated by a uniform probability distribution. The generation method of these parameters is detailed in Table I below:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha_0)</td>
<td>Coefficient of sales projection (random variable generated from the [-0.3; 3.7] interval)</td>
</tr>
<tr>
<td>(G^{*})</td>
<td>Growth rate of international economy (random variable generated from the [0.005; 0.045] interval).</td>
</tr>
<tr>
<td>(\zeta)</td>
<td>Stochastic term of productivity growth (random variable generated from the [-1;1] interval)</td>
</tr>
<tr>
<td>(i^{*})</td>
<td>International nominal interest rate (random variable generated from the [0.01; 0.12] interval).</td>
</tr>
<tr>
<td>(\Omega)</td>
<td>Price elasticity of demand for exports (random variable generated from the [0.0; 0.8] interval).</td>
</tr>
<tr>
<td>(\chi)</td>
<td>Price elasticity of demand for imports (random variable generated from the [-0.7; -0.55] interval).</td>
</tr>
<tr>
<td>(\nu)</td>
<td>Income elasticity of demand for exports (random variable generated from the [0.85; 0.92] interval).</td>
</tr>
<tr>
<td>(\xi)</td>
<td>Income elasticity of demand for imports (random variable generated from the [0.95; 1.02] interval).</td>
</tr>
</tbody>
</table>

For the remaining parameters we attributed numerical values that can be found in real world economies whatever was possible to do it. More specifically, we considered the growth rate of labor force equal to 1.8% per period, the inflation target equal to 5.0% per period, propensity to save out of distributed profits and out of interest income equal to 0.2, profits retention coefficient equal to 0.75, the output-capital ratio equal to 0.5 (that is equivalent to a capital-output ratio of 2.0); the maximum growth rate of real output as equal to 7% per period, the minimum unemployment rate equal to 2,5% of the labor force, the maximum level of capacity utilization equal to 0.9 and the growth rate of government
expenditures in consumption equal to 1% per period. All these values can be found in the historical record of capitalist economies in the last 30 years. The other parameters, however, are free parameters in the sense that we do not have a priori estimate about their numerical values. This means that we are free to change the numerical values of these parameters in order to search for a “good simulation”, which means a dynamical path for the endogenous variables of the system that reproduce some stylized facts about long-run dynamics of capitalist economies.

These considerations allow us to propose the following methodology for computer simulation of our model: (1) Choose an initial set of values for the parameters and initial conditions of the system. For those parameters that have reliable estimates about their numerical values, use then to calibrate the model; (2) Run the model in the computer in order to obtain the time paths for the endogenous variables; (3) Check if the simulated paths replicate some general properties or “stylized facts” of the dynamics of capitalist economies; (4) If the simulated paths do not replicate the “stylized facts” of capitalist dynamics, than choose a new set of values for parameters and initial conditions, holding constant only those values for which there are reliable estimates about then.

The stylized facts that we want to reproduce with the model at hand are: (a) Real output has a positive trend growth in the long run, but growth is not a smooth process. Growth occurs by means of irregular, although non-explosive, fluctuations along the trend (cf. Blanchard e Fisher, 1989, p.1); (b) The real rate of return on capital shows no tendency to fall in the long-term (cf. Kaldor, 1957); (c) Secular growth in the real wage rate, as a consequence of productivity growth; (d) Existence of idle productive capacity in the long-run (cf. Kalecki, 1954); (e) Huge falls of the level of economic activity – a “great depression” – are relatively rare phenomenon in the history of capitalism. In other words, depressions only occur one or two times in a century (cf. Leijonhufvud, 1996); (f) There is a long-run tendency for the share of financial wealth in total wealth of the productive sector to increase over time; (f) The level of real exchange rate appears to have no systematic influence over the situation of the balance of payments, since the behavior of exports and imports are determined by non-price competition in the long-run (cf. McCombie and Roberts, 2002, p.92).

Based on the simulation methodology presented so far, we selected the following set of values for the parameters of the system.

<table>
<thead>
<tr>
<th>Table II: Parameters Values Used in Computer Simulation</th>
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<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>α</td>
</tr>
<tr>
<td>hI</td>
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<tr>
<td>hC</td>
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<tr>
<td>α₀</td>
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<tr>
<td>α₁</td>
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<tr>
<td>z₀</td>
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<td>θ</td>
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4. Simulation Outcomes

The model presented in section 2 was run in an EXCEL spreadsheet with the numerical values presented in Table I for a time horizon of 69 periods. Given the path-dependent nature of the model at hand, for each possible realization of the random variables listed in the structural equations, there will be
one different time-path for endogenous variables. However, the main qualitative features of the dynamic behavior do not change between realizations of the random variables. In other words, the qualitative behavior of the dynamic system is robust. In order to make an intelligible presentation of the simulation outcomes, we will aggregate the simulation outcomes in three groups. The first group will be the one related with growth and distribution variables. In this group we will show the dynamics of the following variables: growth rate of real output, unemployment rate, degree of capacity utilization, real wage rate, profit-share and profit rate. The second group will be the one related with the monetary and financial side of the economy. In this group we will show the dynamics of the short-term nominal interest rate, the banking rate of interest, the rate of inflation, the level of firms debts with the commercial banks and the proportion of financial assets in the private sector wealth. Finally, in the third group we will present the dynamics of the endogenous variables relates to the external sector, that is: real exchange rate and net exports as a share of real output.

4.1 The Dynamic Behavior of Growth and Distribution Variables.

The first qualitative feature of the behavior of the system that is interesting for us (and that is independent of the specific realization of the random variables) was the occurrence of growth-cum-cycle path for real output. The observed fluctuations the growth rate of real output are irregular, but non-explosive, as we can see in figure 1.

In figure 1 we can also see the occurrence a major recession from periods 21 to 25, when growth rate of real GDP fall to a level of -4% per period, and a great depression from periods 49 to 50, when growth rate of GDP fall to a level of -10% per period. Along the entire simulation span, growth rate of GDP oscillates in an irregular way, presenting phases of modest growth of more or less 2.3% per period and phases of robust growth of almost 7% per period.

The average growth rate of real output was 2.9% per period along the time span of the simulation. The natural rate of growth for the economy at hand is estimated in 2.6% per period, resulting from a 1.8% per period rate of increase in the labor force plus a 0.8% per period rate of increase in labor productivity. This means that in the model at hand economic growth was not constrained by the natural rate of growth. This result is in accordance with Post-Keynesian models of growth as those developed by Harrod (1939) and Robinson (1962). Another interesting remark is related to the relation between the growth rate of autonomous demand and the effective growth rate of the economy. In the model at hand, the source of autonomous demand is the government expenditures in consumption plus exports demand. Taking a weighed average of the growth rate of government consumption expenditures and the growth rate of exports for the simulation span we arrive at an average growth rate of autonomous demand of only 0.7% per period. This means that the growth of autonomous demand is not the source of long-run
growth of this economy, a result that is at odds with the traditional Kaldorian theory of demand-led growth (cf. Kaldor, 1988).

If long-run growth is not determined neither by natural rate of growth and the growth rate of autonomous demand, what is the source of long-term growth in this economy? The only possible explanation is the one advanced by Kaldor (1954) in a almost forgotten article about the relation between economic growth and cyclical fluctuations: the trend growth is the result of a strong animal spirits of capitalists (cf. Kaldor, 1954, p.67). In the model at hand, this means a high, although unstable, coefficient of sales projection in the investment demand equation. Due the fluctuations in the growth rate of real output, unemployment rate also exhibits ups and downs during the time span of the simulation. As we can see in figure 2, the major increases in the unemployment rate occur precisely in periods of sharp decrease in the rate of economic growth. However, the time-series for unemployment rate shows no tendency to increase over time and the average unemployment rate was 8.4% of the labor force. This is a number that is very close to those observed in OECD countries.

![Figure 2 - Dynamics of Unemployment Rate](image)

Another interesting qualitative feature of the dynamic behavior of the system is the occurrence of excess capacity in the long-run. As we can see in Figure 3, the level of capacity utilization fluctuates along the time span of the simulation – due to fluctuations in effective demand – but exhibits an average level of 23%.

In terms of income distribution, the first interesting qualitative behavior of the system is related to the dynamics of the real wage rate. As we can see in figure 4, the real wage rate increases more than 15% during the time span of the simulation, which means an average growth rate of real wages of 0.23% per period. Since productivity growth was 0.8% per period, we can conclude that the economy at hand shows a tendency for decreasing (increasing) the wage-share (profit-share) in income. This result was shown in figure 5.
A final result concerning income distribution is the stability of the profit-rate in the long-run. As we can see in figure 6, profit rate fluctuates through time, basically as a result of the fluctuations in the level of capacity utilization (as can be observed by means of a comparison between figures 3 and 6). In the long-run, however, the profit rate fluctuates around an average value of 5% per period.
One final remark is important about the behavior of growth and distribution variables. Since the profit rate remains stable over the simulation span, the upward trend observed in profit-share time-series must be followed by a downward trend in the level of capacity utilization time series. This trend can be observed in figure 3. This means that in the economy at hand capacity utilization and profit-share changes in opposite directions. In other words, the accumulation regime is a wage-led type.

**4.2 Dynamic behavior of monetary and financial variables.**

We will turn our attention to the monetary and financial side of the economy. The first interesting result is regarding the behavior of the inflation rate, as we can see in Figure 7. Along the entire simulation span, we can observe only two single episodes of inflation acceleration: from period 1 to 5 and from period 26 to 31. From the rest of the time span, inflation rate is either falling or stable at a zero per cent level. This behavior of the inflation rate is explained by two factors: the increase in productivity of labor at a rate greater than the growth in real wages (while the former is increasing at an average rate of 0.8% per period, the latter increases at an average rate of only 0.23% per period) and the downward trend in the level of capacity utilization which force productive firms to reduce the mark-up rate. The combined effect of these factors is strong enough to countervailing the effect of the gap between the target and effective real wages rate over the rate of inflation (Figure 8).
Monetary policy in the economy at hand is supposed to be operated under a *Inflation Targeting Regime*. This means that *Central Bank* should increase short-term interest rates whenever is necessary to stop any acceleration in the rate of inflation. As we can see in figure 5, Central Bank increases short-term interest rates in two episodes: from period 1 to period five and again from period 26 to period 31, that are precisely the same episodes of inflation acceleration. However, one important thing to notice is the huge increase in short term interest rate that is required to stabilize the rate of inflation. In fact, in the first episode of tight monetary policy, Central Bank raised short-term nominal interest rate to 37.5% per period, and in the second episode, short term nominal interest rate is raised to 24.8% per period.

This raises the following question: why a huge increase in nominal short-term interest rate is required to stabilize the rate of inflation? In order to answer this question we have to analyze the channels by which monetary policy can affect the rate of inflation according to our model. Inflation is determined in the model at hand by equation (20a), according to which inflation will accelerate whenever: i) target real wage is greater than last period real wage; ii) the level of capacity utilization is increasing; iii) the level of firms’s endebtness with the banking sector in increasing and iv) the rate of capital accumulation is decreasing. The problem with a tight monetary policy as an instrument to stabilize the rate of inflation is that it contributed, at the same time, for the factors that are responsible for...
a decrease and the factors that are responsible for an increase the rate of inflation. In fact, an increase in short-term interest rates will increase the cost of capital due to its effect through the discount rate of expected profits. This will reduce the level of desired investment. If the reduction in desired investment is followed by a reduction in the realized investment than two effects will be produced: a) the level of effective demand will be reduced, contributing to a reduction in the level of capacity utilization (that reduces inflation) and b) the rate of capital accumulation will be reduced, resulting in a reduction in the rate of productivity growth that increases the rate of inflation. Besides that, an increase in the short-term interest rate will produce an increase in banking rate of interest (see Figure 10). This increase will accelerate the rate at which the level of firms’s endebtness with the banking sector increases, what contributes to acceleration in the rate of inflation. The aggregation of all these effects results in a weak, if any, capacity of a tight monetary policy to stabilize the rate of inflation. This result is also in accordance with the empirical literature of the effects of interest rates over inflation (cf. Arestis and Saywer, 2006, p. 14).

![Figure 9: Dynamics of Short-Term Interest Rate](image)

![Figure 10: Dynamics of Banking Rate of Interest](image)
In relation to the firm’s level of endebtness with the banking sector, Figure 11 shows that the ratio of firm’s debts to the current value of their capital stock stabilizes at 0.9 in the long-term. This means that productive firms are well succeeded in their attempt to prevent an explosive increase in the level of endebtness.

To conclude the remarks about the dynamics of financial variables, as we can see in Figure 12, there is a long-run tendency for the increase in the share of financial wealth in total wealth. This increase is the result of the fact that productive and financial capitalists use their personal savings only to buy financial assets. Productive assets are bought only by means of investment decision of productive firms. This result is consistent with the ‘stylized facts’ of capitalist long-run dynamics; according to which financial wealth tends to increase in importance in the long-term.

4.3. Dynamic behavior of the external sector

I will now turn our attention to the dynamic behavior of variables related to the external sector of the economy. Regarding the rate exchange rate, we can see in Figure 13, the occurrence of continuous real exchange rate depreciation during the time span of the simulation. The upward trend in the real
exchange rate is caused by the difference between the domestic and international rates of inflation. In our simulation exercise, we supposed that international inflation is at 2.5% per period, but the average rate of domestic inflation was only 0.2% per period. In other words, international inflation is supposed to be 10 times greater the rate of domestic inflation. Besides that we can see the occurrence of isolated episodes of hyper-devaluations of real exchange rate that resulted from a huge, but temporary, devaluation of nominal exchange rate.

Despite the upward trend in the real exchange rate, Figure 14 shows no trend in the behavior of net-exports as a share to real output during the time span of the simulation. In fact, net exports as a share to real output fluctuates around a level of 4%. This result confirms the post-keynesian assertion that the behavior of relative prices does not have a significant impact upon the situation of the balance of payments (cf. McCombie and Roberts, 2002, p.92).

1. Final Remarks.
The central goal of this article was to build a simulation macrodynamic model for an open economy with governmental activities, a flexible exchange rate-regime, inflation targeting, endogenous technical
progress and imperfect capital mobility. This model was though to be a theoretical framework that is capable to integrate real and financial aspects of post-keynesian economics, making a progress in the task of answering Solow’s (1979) critique according to which post-keynesian economics was more a “state of mind” than a coherent challenge to mainstream economics.

Simulated trajectories reflect some general features of the dynamic of capitalists economies, especially the existence of irregular fluctuations of the growth rates of real output. The analysis of the external sector of the economy shows that the behavior of net export surplus as a share of real output is largely independent of real exchange rate. Another important result that was obtained from the model is the increasing share of financial assets in national wealth. In other words, the model provides an economic result that suggests a growing share of financial assets in the capitalists’ aggregate wealth of this economy.

The model presented in this article can be used to evaluate the effects of changes in the policy variables – for example, an increase in the growth rate of government expenditures in consumption or an increase in the target rate of inflation – over the dynamics of the endogenous variables. Such exercise, however, could not be done in this paper for reason of lack of space.

REFERENCES


