Characterizing the Brazilian Term Structure of Interest Rates

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Abstract

This paper studies the Brazilian term structure of interest rates and characterizes how the term premia has changed over time. We employ a Kalman filter approach, which is extended to take into account regime switches and overlapping forecasts errors. Empirical evidence suggests that term premia depends on international global liquidity and domestic factors such as the composition of public debt and inflation volatility. These results provide guidance for the formulation of fiscal and monetary policies.

Resumo

Este artigo estuda a estrutura a termo de juros brasileira e caracteriza como o prêmio pelo termo tem variado no tempo. Utilizamos a abordagem do filtro de Kalman, que é estendida para levar em conta mudanças de regime e erros de previsão superpostos. A evidencia empírica sugere que o prêmio pelo termo depende da liquidez internacional global e de fatores domésticos tais como a composição do déficit público e a volatilidade da inflação. Estes resultados fornecem uma orientação para a formulação das políticas fiscal e monetária.

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1. Introduction

Extensive research on the behavior of the term structure of interest rates has been conducted in the past decades, especially for the US. One of the main lines of research focuses on the short and long-term interest rates relationship (Campbell and Shiller (1991), Hardouvelis (1994), Evans and Lewis (1994), and Rudebusch (1995)).

One of the main theories that has been used to test for the long and short-term interest rates relationship is the expectations model of the term structure. The Expectation Hypothesis (EH) states that long-term interest rates are weighted average of expected future short rates over the lifetime of the long asset plus a risk premium, which is held to be constant.

Empirical evidence is controversial on whether the EH holds in developed countries (Arize et al. (2002), Bekaert et al. (2001), Campbell and Shiller (1991), Cuthberson (1996), Fama and Bliss (1987), Evans and Lewis (1994), Hardouvelis (1994), Jondeau and Ricart (1999), Kugler (1996), Mustia and Décclesiab (2007), Sarno et al. (2007), Sola and Driffill (1994) and Tzavalis and Wickens (1997)). Many papers present evidence against the EH, suggesting that time-varying premia may important in explaining term structure behavior. Furthermore, very little research has been conducted for emerging markets[1]. It is not clear whether the EH holds for emerging markets and what are the reasons that may explain its failure for a variety of countries.

Overall, the results highlight the importance of incorporating time-varying term premia in empirical tests of the EH, and also incorporating nonlinearities such as changes in regime. Besides, an important issue is what explains changes in term premia.

This paper contributes to the literature by examining the EH for the Brazilian term structure of interest rates by employing a time-varying term premium specification. Empirical evidence suggests that term premia depends on international global liquidity and domestic factors such as the composition of public debt and inflation volatility. These results provide guidance for the formulation of fiscal and monetary policies.

The paper is organized as follows. Section 2 summarizes the methodology employed for estimating term premia in the term structure of interest rates. Section 3 describes the data used in the study. Section 4 presents the empirical results. Finally, Section 5 concludes the paper.

2. Brief Literature Review

There is a large body of evidence rejecting the EH for the US. In an influential paper Campbell and Shiller (1991) find that the slope of the term structure almost

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[1] See Arize et al. (2002), Cooray (2003), Ghazali and Low (2002), Konstantinou (2005), Tabak and Andrade (2003), Lima and Issler (2003).

always gives a forecast in the wrong direction for the short-term change in the yield on the longer bond, but gives a forecast in the right direction for long-term changes in short rates[2].

Many explanations have been given on why the EH may not hold for the US and other countries. Ang and Bekaert (2002) present evidence supporting the presence of regime shifts in developed countries. Bekaert et al. (2001) find that anomalies in the term structure of interest rates may be due to a generalized peso problem in which a high interest rate regime occurred less frequently in the sample than was rationally anticipated. The authors show that a model that combines time-variation in term premiums with peso problem effects is largely consistent with data from the US, UK and Germany.

Bekdache (2001) study the effects of the maturity composition of the US federal debt on the term structure of interest rates. The author finds convincing evidence that the maturity profile of debt has a non-trivial impact on the term premia.

Clarida et al. (2006) examine the relationship between interest rates of different maturities for the US, Germany and Japan. Their modeling allows for asymmetric adjustment and regime shifts. The authors present convincing evidence that such non-linear models provide good in-sample fits and have satisfactory out-of-sample forecasting properties. For the US the empirical results presented in Lanne (2003) lend support to the expectations hypothesis at the short end of the maturity spectrum of US Eurodollar deposit rates once a potential regime shift is allowed for. The author also shows that the peso effect was working only in an early period in the sample when the interest rates were at a high level.

Engle et al. (1987) develops an ARCH-M model to show that there is a time varying risk premium in interest rates, which is highly significant. Therefore, the authors explain the failure of the EH of the term structure in the US as a result of such time-varying term premium. Iyer (1997) finds considerable variation in estimated premiums and significant persistence in their volatility over time for the US. Kozicki and Tinsley (2002) show how term premia may depend on the policy rule specification and policy rate uncertainty. They find that more aggressive policy rule involves an economically important increase in term premia.

Gravele and Morley (2005) study the relationship between term premia and economic variables for Canada and find strong evidence of a positive relationship between term premia and interest rate volatility and other macroeconomic variables. The authors find that non-observable variables play an important role in explaining term premia as well as economic fundamentals.

Engsted and Nyholm (2000) study the Danish term structure, using a regime-shift approach, and find that several implications of the EH are consistent with the data, especially in the later part of the sample.

Beyaert and Perez-Castejon (2000) study the Spanish interbank market using regime switching regressions and reject the EH. Ghazali and Low (2002) study the

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[2] This is known as the Campbell-Shiller Paradox.

Malaysian market and find evidence that support the EH. Cooray (2003) finds evidence that spot and forward rates are cointegrated for Sri Lanka, but rejects the hypothesis that forward rates are unbiased predictors of spot rates.

Arize et al. (2002) test the EH using vector error correction modeling and finds evidence supporting the EH with the exception of the UK. The authors study 19 countries, with both developed and emerging markets. Mustia and D'Ecclesiab (2007) show that the EH holds for Italy and European countries, employing vector error correction modeling techniques.

Tabak and Andrade (2003) test for Expectation Hypothesis (EH) for the Brazilian term structure of interest rates and find evidence of its rejection. Furthermore, they show that risk premia in the yield curve is positively related to the volatility of interest rates. Lima and Issler (2003) test the EH using a present value approach and reject partially the EH for Brazil. They find evidence of a common factor for long and short-term interest rates. Brito *et al.* (2004) also test for the EH using a different methodology and reject this hypothesis for the Brazilian economy. They present evidence of overreaction of the long-short term interest rate spread to changes in short-term interest rates.

Overall, empirical results depend on the methodology that is being used, the country that is being studied and the maturity profile of interest rates. Three main issues seem to be relevant from the extant literature that deals with testing the EH: 1). There seems to be relevant and economically significant time-varying term premia; 2). Time-varying term premia may be explained by economic factors and also by non-observable factors; 3) Specific conditions of bonds market in the country under study may explain why one observes such varying behavior across countries.

Our focus in this paper will be in studying whether there is significant time-varying term premia in the Brazilian term structure of interest rates and on testing what specific variables may affect the term premia.

3. Methodology

Our focus is to examine the evolution of the risk premia embedded in the Brazilian term structure of interest rates.

We develop a space-state specification to estimate and model the risk premia. Let $f_{t,j}$ be the forward interest rate in period j with maturity in $t+j$. The forward interest rate may be decomposed in:

$$f_{t,j} = E_t [r_{t+j}] + \tau_{t,j} , \qquad (1)$$

where $E_t [r_{t+j}]$ represents agent's expectations for the spot rate j periods ahead, given information available in instant, and $\tau_{t,j}$, the term premia. When we subtract the realized spot interest rate j periods ahead from both sides of equation (1), we obtain the excess return associated to the forward interest rates ($x_{t+j,j} \equiv f_{t,j} - r_{t+j}$), which can be expressed as:

$$x_{t+j,j} = \tau_{t,j} + u_{t+j}, \tag{2}$$

where $u_{t+j} \equiv E_t [r_{t+j}] - r_{t+j}$ is agent's prediction errors

The term premia is a non-observable variable, which can be estimated via the Kalman filter. However, we have to impose a structure for the term premia. For the time being, we assume that the term premia follows an autoregressive structure:

$$\tau_{t,j} = \tau_{t-1,j} + v_t, \tag{3}$$

where v_t is an error component, independent and identically normally distributed with zero mean and variance σ_v^2. Expression (3) is the case in which the term premium is non-stationary.

The term premium may also be modeled to follow a first-order autoregressive (AR) process:

$$\tau_{t,j} = c + \phi \tau_{t-1,j} + v_t, \tag{4}$$

where $|\phi| < 1$.

We also employ a model in which the intercept is assumed to follow an AR(1) process with a regime switching intercept:

$$\tau_{t,j} = c + \gamma R_{t+j} + \phi \tau_{t-1,j} + v_t, \tag{5}$$

where R_{t+j} is a regime indicator variable with possible values 0 or 1, depending on which interest rate volatility regime prevails in period $t+j$.

Gravelle and Morley (2005) argue that the specification provided in (5) allows testing for the interaction of term premia and interest rate volatility. Tabak and Andrade (2003) show that risk premium, for the Brazilian term structure of interest rates, are time-varying, and positively related to the level of interest rate volatility. Therefore, we would expect that specification (5) hold for the Brazilian term structure of interest rates.

4. Data

All variables were measured at monthly frequencies over the period January 1995 to September 2006 and the data sources were as follows: interest rates for different maturities were provided by the São Paulo Mercantile Exchange (BM&F) at the close, debt composition, international reserves, exchange rate data and foreign capital flows (annex IV) were provided by the Central Bank of Brazil, the risk aversion coefficient

was provided by *Merril Lynch*, and industrial production and inflation were provided by IBGE[3].

In the following Table we present descriptive statistics provided by expression (2) three and six-months excess forward returns. In all cases we strongly reject the normality assumption.

Table 1. Descriptive statistics for Forward Excess Returns.

	3-months	6-months
Mean	0.0076	0.0147
Median	0.0064	0.0114
Maximum	0.1611	0.1672
Minimum	-0.1545	-0.1250
Std. Dev.	0.0404	0.0486
Skewness	-0.63	-0.03
Kurtosis	8.78	4.03
Jarque-Bera	175.04	5.12
Probability	0.00	0.08
Observations	120	115

Standard unit-root test, reported in Table 2, reject the hypothesis of a unit-root for each maturity. Furthermore, the Kwiatkowski-Phillips-Schimdt-Shin test cannot reject the hypothesis of stationarity. Therefore, excess forward returns seem to be stationary.

Table 2. Unit-root Tests.

Forwards	ADF	PP	KPSS	DF-GLS
3-months	-5.35***	-4.19***	0.07	-5.39***
6-months	-3.12	-4.10***	0.08	-3.15**

Notes: unit root tests with intercept and trend. ADF denotes test statistic from the Augmented Dickey-Fuller test, PP denotes the test from the Phillips-Perron test, DF-GLS is the GLS detrended Dickey-Fuller test proposed by Elliott, Rothenberg and Stock (1996), and the KPSS is the Kwiatkowski-Phillips-Schimdt-Shin test statistic. The ADF, DF-GLS and PP test the null hypothesis of a unit root and the KPSS test the null hypothesis of stationarity. The lag order of the ADF and DF-GLS were chosen using the Schwarz criterion, while the PP and KPSS are specified using the Bartlett kernel wit automatic Newey-West bandwith selection. A significance level of 1%, 5% and 10% is indicated by ***,**, and *.

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[3] The Brazilian Institute of Geography and Statistics (Instituto Brasileiro de Geografia e Estatística - IBGE), is the agency responsible for statistical, geographic, cartographic, geodetic and environmental information in Brazil.

5. Empirical Results

To test the expectation hypothesis we estimate four models: with constant, stationary, non-stationary and regime-switching term premia. Table 3 reports log-likelihood values for these specifications for two forward rate horizons (3 and 6 months). Both stationary and non-stationary specifications are considered in order to test whether term premia are non-stationary or mean reverting. In the former case shocks to term premia are permanent, while in the latter shocks are transitory.

The results provide clear evidence of rejection of the expectation hypothesis. We compare the constant specification with the stationary alternative. The implicit log likelihood ratios are 14.86 and 10.62 for the 3 and 6-months forwards, respectively. Therefore, we are able to reject the null hypothesis that the term premia are constant at better than 1% significance level. If we use the non-stationary specification we obtain similar results with log likelihood ratios of 30.78 and 24.92 for the 3 and 6-months forwards, respectively[4].

Table 3. Log-likelihood Values

Forward Horizon*	Term Premia Specifications			
	Constant	Stationary	Non-stationary	Regime-Switching
j=3	-238.27	-230.84	-222.88	-230.84
j=6	-206.41	-201.10	-193.95	-204.13

For the non-stationary model the term premium is assumed to follow a random walk. The stationary and regime-switching models follow a stationary AR(1) and a stationary AR(1) with switching intercept.

 * We study 3 and 6-months forwards for the 3 and 6-months interest rates, respectively.

Another way of checking for the validity of the expectation hypothesis is to display the evolution of the term premia over time. The graphical inspection (Figures 1-4) of the estimated time-varying term premia suggests that the expectation hypothesis should be rejected. In all cases there is significant variation in the term premia over time[5].

<Place Figure 1-4 About Here>

Table 5 presents the results of estimating equation (2) using OLS with a Newey-West correction for serial correlation and heterocesdasticity. Among several estimated

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[4] Since likelihood ratio tests may be biased it is difficult to draw any conclusions regarding the stationarity of the term premia (Dickey and Fuller (1981)).
[5] We are presenting the figures for the complete model with regime switching term premia. However, results for the stationary model display a similar pattern and suggest high variability of term premia reinforcing these results and are not presented here to conserve space. These figures are available upon request from the authors.

models, we opted for the one that presented more significant parameters in the statistical point of view.

It is inferred from Table 5 that the international liquidity (risk aversion) has a fundamental role in the explanation of the risk premia implicit in the domestic interest rate, just as domestic indicators. This way, either inflation reduction in the recent past, or the profile change in the public debt, have contributed to the reduction of the risk premia implicit in the domestic interest rate. Inasmuch, this movement had been stimulated by the high liquidity observed in the international financial markets. Furthermore, exchange rate depreciation also plays a role in explaining term premia for the Brazilian economy.

Table 5. Determinant of the Term Premia for the Term Structure of Interest Rates (Stationary Term Premium Model)

	j = 3	j = 6
Constant	1.92E-05	-0.0040
	(0.0026)	(0.0033)
Risk Aversion $_t$	0.0013	0.0028**
	(0.0008)	(0.0011)
Inflation $_t$	0.0025	0.0072*
	(0.0038)	(0.0026)
Participation of Price Indexed Bonds $_t$	-0.0646	-0.0834***
	(0.0393)	(0.0439)
Exchange Rate Depreciation $_t$	0.1657*	0.2085*
	(0.0210)	(0.0357)
Term Premia $_{t-1}$	0.8247*	0.8999*
	(0.0794)	(0.050)
Adjusted R^2	62.56%	81.28%
F-statistic	24.06	60.07
p-value	0.00	0.00

*, ** and *** stand for rejection of the null hypothesis at the 1, 5 and 10% significance levels, respectively. We study the determinants of the term premia for 3 and 6-months forward horizons for 3 and 6-months interest rates.

6. Final Considerations

This paper tests the EH for the Brazilian term structure of interest rates. Differently from most previous studies we model a term varying term premium employing Kalman filtering techniques. Furthermore, we allow for a regime switching in the term premium, which implies that term premium is correlated to interest rate volatility.

Our empirical results suggest that the EH is rejected for the data as there is an economically and statistically significant time-varying term premium. Furthermore, we are able to explain changes in the term premia with economic fundamentals.

Evidence suggests that term premia depends on international global liquidity and domestic factors such as the composition of public debt and inflation volatility. These results provide guidance for the formulation of fiscal and monetary policies.

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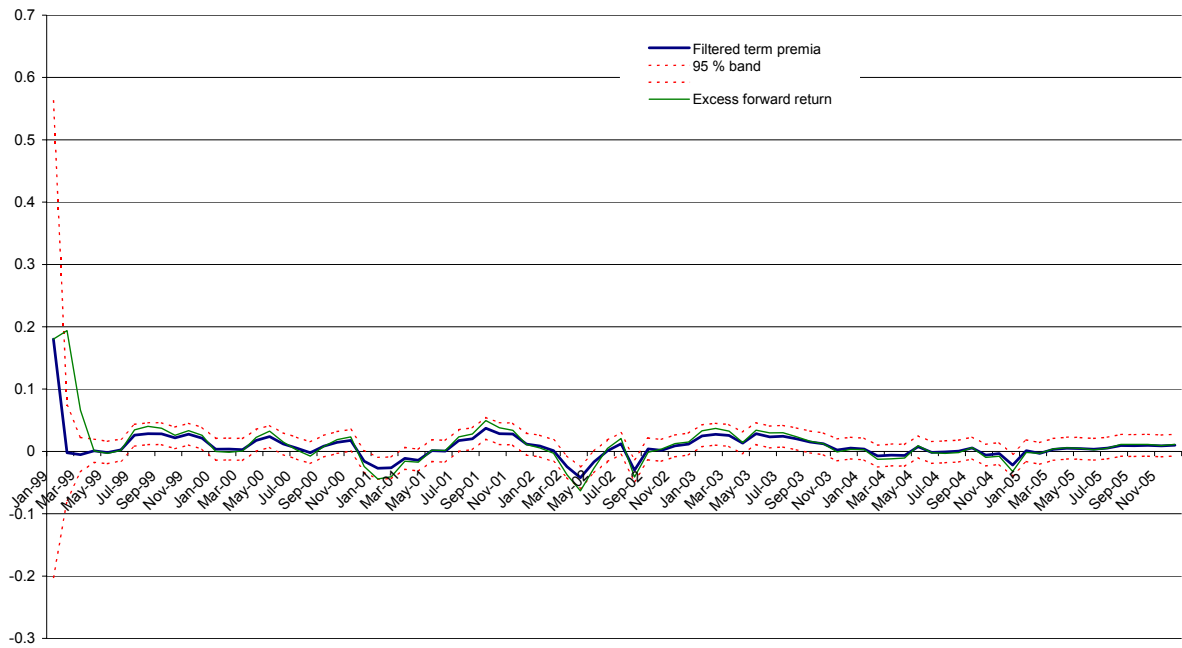
Figure 1. Filtered Term premia for Switching Model with 95% confidence bands (dashed lines) for 3-months forward.
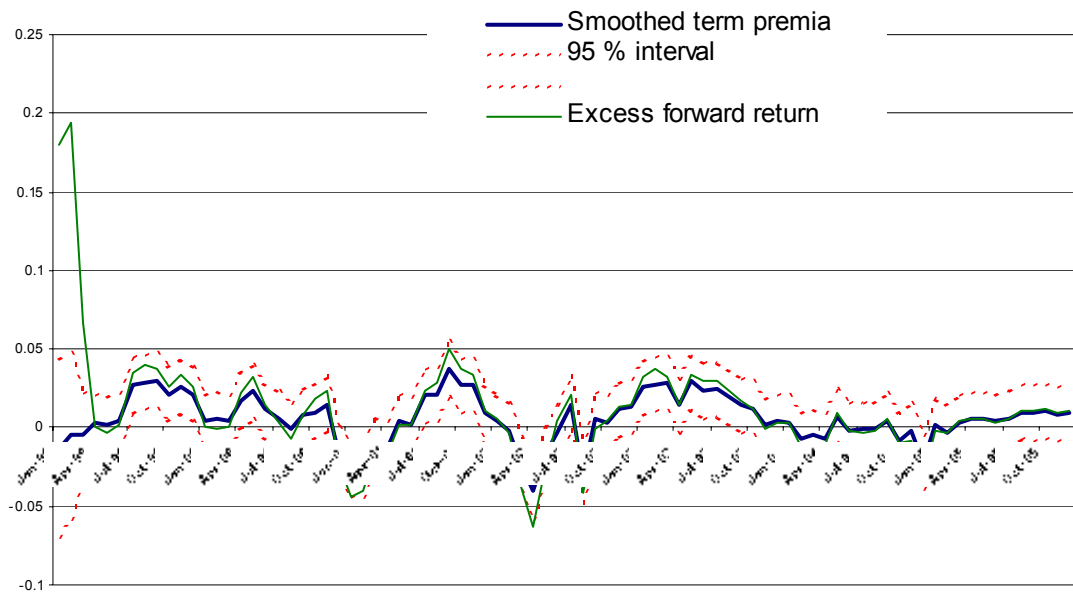
Figure 2. Smoothed Term premia for Switching Model with 95% confidence bands (dashed lines) for 3-months forward.
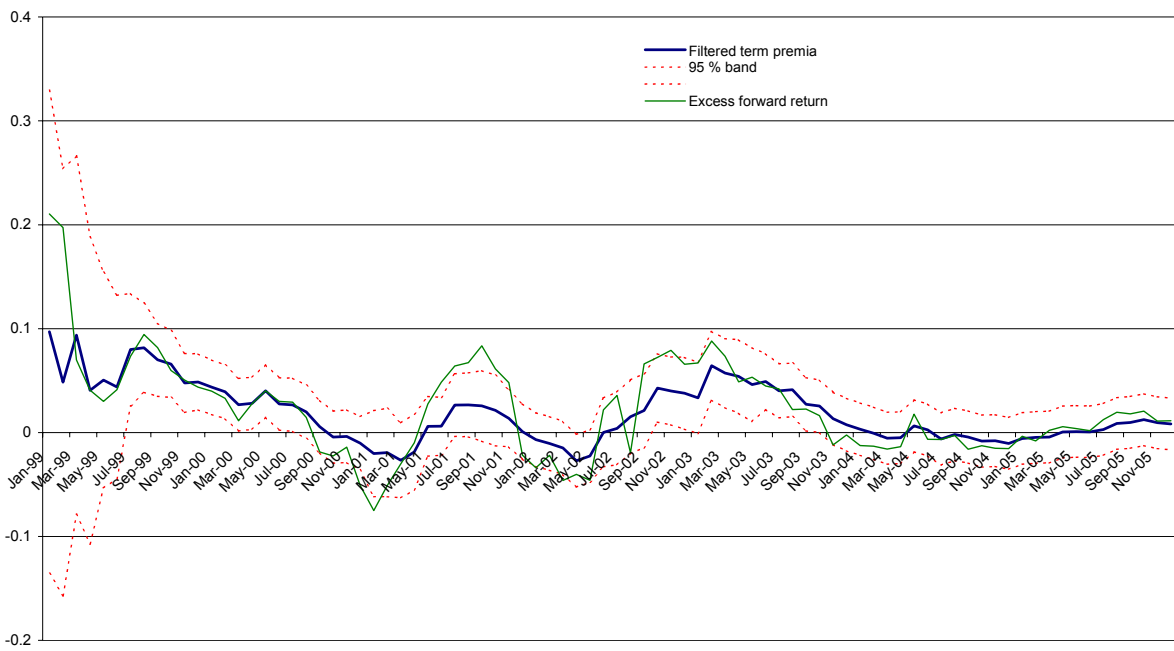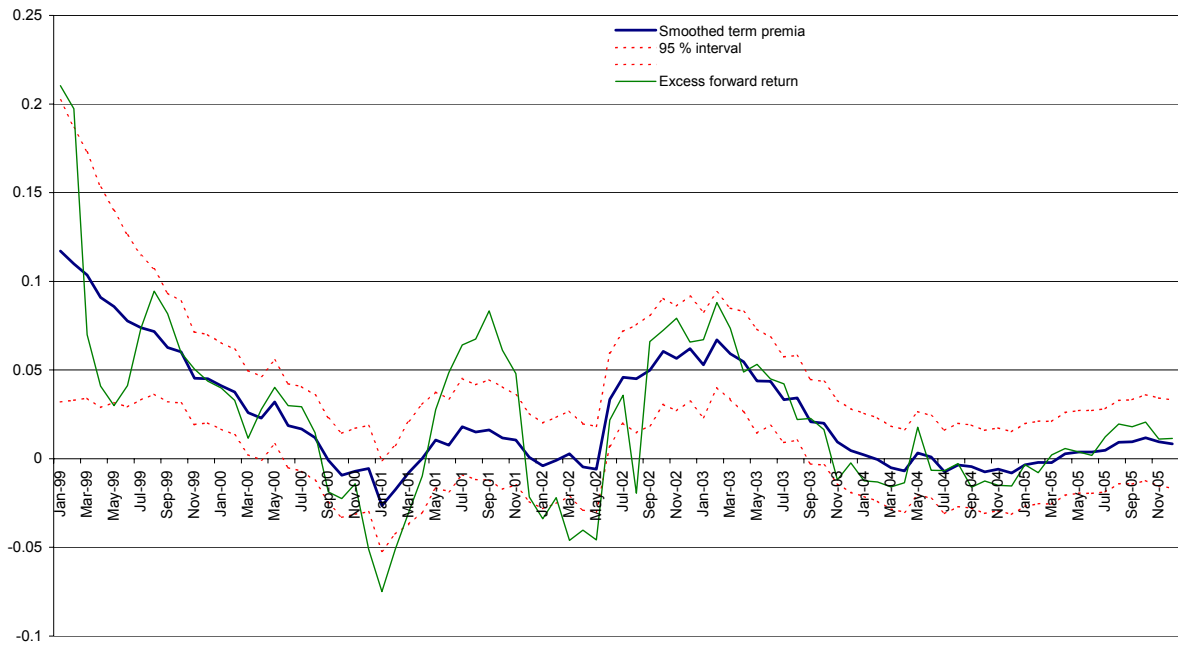
Figure 3. Filtered Term premia for Switching Model with 95% confidence bands (dashed lines) for 6-months forward.

Figure 4. Smoothed Term premia for Switching Model with 95% confidence bands (dashed lines) for 6-months forward.