REPLACEMENT CYCLES, INCOME DISTRIBUTION, AND DYNAMIC PRICE DISCRIMINATION ⋆

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Summary : This paper analyses how income distribution, Intellectual Property Rights and other regulatory policies such as "minimum quality standards" determine pricing strategies in a dynamic context where a monopoly periodically introduces new generations or upgrades of a durable good. This paper differs from Inderst's (2003) or Koh's (2006) in that discrimination through quality and screening take place in a setup where consumers buy several (not a single) versions of the durable good during a lifetime. It differs from Glass (2001) in that here an equilibrium may emerge in which different consumer types replace their durable generations with different frequencies. Some of the model's predictions are shown to be supported by data from the last Brazilian POF (household budget survey).

Resumo: Este artigo analisa como a distribuição de renda, direitos de propriedade intelectual e outras políticas regulatórias como "padrões mínimos de qualidade" determinam estratégias de precificação num contexto dinâmico em que um monopolista periodicamente introduz novas gerações ou melhorias de um bem durável. Este trabalho difere dos de Inderst (2003) ou Koh (2006) em que a discriminação através de qualidade e tempo têm lugar num arcabouço em que os consumidores compram várias (e não apenas uma) gerações do bem durável ao longo da vida. Ele também difere do trabalho de Glass (2001) pois aqui podem emergir equilíbrios nos quais diferentes tipos de consumidores trocam de geração de durável com diferentes freqüências. Algumas das predições do modelo são corroboradas por dados da última POF-IBGE.

Keywords and Phrases: intertemporal price discrimination, durable good monopoly, optimal pricing strategy, minimum quality standards, imitation

Palavras-Chave: discriminação intertemporal de preços, monopólio de bem durável, imitação

JEL classifications: D23 , D42

Área ANPEC: 7 (Microeconomia, etc.)

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1. Introduction

In this paper we shall analyse how income distribution, intellectual property rights (IPRs), and other regulatory policies, such as minimum quality standards, determine pricing strategies in a dynamic context, in which a single firm (referred to as "the monopolist") periodically introduces new generations or upgrades of a durable good.In point of fact, several durable goods are subject to periodic quality improvements, some of them, like cellular phones or digital cameras, at impressive and ever increasing speeds. When consumers are simplistically assumed to be homogeneous, a single firm producing the state-of-the-art product may charge a price sufficiently low so as to eliminate from the market all older generations of the same product[1].

However, when consumers have different income levels or different valuations of quality improvements, multiple quality levels can sell simultaneously in the market. In this circumstance, time and quality are different elements that might be used by a monopolist to discriminate between consumers with different valuations/income levels. Even if the durable good came in a single quality, the monopolist could first charge a high price and sell the durable to "rich" or high-valuation consumers who have more to loose if they delayed their purchase.

Of course this depends on the monopolist being able to commit not to change the price of its single quality product too fast in pursuit of adding up revenue from sales to poor consumers. This is referred to in the literature as "the commitment problem" or "the firm's time inconsistency problem" and is examined by Inderst (2003). In his setup the monopolist makes only one sale of one unit of the durable good to each consumer, that is, the consumer buys the durable good only once in his lifetime. Time may elapse until the whole market is cleared (all consumers have acquired the durable good and the game ends). When the possibility of selling two different qualities at the same time enters the picture, a monopolist without commitment power will typically clear the whole market immediately, selling a top quality good to high-valuation consumers and a lower quality one to low-valuation consumers, thus "committing" not to make a more attractive offer to these latter in the future.

In our setup, quite differently, a monopolist with full commitment ability sells successive generations of the same durable good to each consumer in his (infinite) lifetime. This dramatically changes the relationship between quality discrimination and monopoly power: the prospect that the present durable good generation will be available in the market at a lower price in the future (when poor consumers make their purchases under discrimination) induces rich consumers to wait or displace their whole consumption program, so that the monopolist must charge a lower price if he intends to prevent this waiting behaviour from high income consumers. Thence a trade-off emerges that was not present in Inderst (2003): by not resorting to quality discrimination the monopolist can charge high prices on rich consumers' purchases but poor consumers' waiting time (or "replacement period") will be longer. A long replacement period, besides meaning a smaller overall number of purchases in a consumer's lifetime, also brings along with it a high probability of imitation, in which event the sale of one of the generations by the monopolist is skipped.

This latter trade-off can be more relaxed the older are the generations the monopolist is free to sell (the price to rich consumers under quality discrimination approaches the price to rich consumers under simple time discrimination or "screening"). The limit to how old generations can be put for sale is determined by

[1] Quality improvement has first been modeled assuming that consumers were homogeneous, new products were perfect substitutes for old ones, the elasticity of demand was unitary, and per unit cost was constant. As a result of these assumptions, only one quality level would sell in the market, assuming the leading innovator engage in Bertrand price competition, employing a limit-pricing strategy. These are now textbook models and one of the ways to generate growth models with endogenous technical progress. See Grossman and Helpman (1991).

competition/imitation or by a regulatory policy setting "minimum quality standards" as examined by Glass (2001).

Glass' (2001) setup is one in which the good that is innovated/upgraded is non-durable, and therefore consumers must buy some quantity every period. Successive generations of this non-durable become available over time in a *quality ladder* model fashion, that is, there is a single constant rate of innovation and successful innovators are able to sell the state-of-the-art generation. With two consumers types (with different valuations) the oldest generation that may sell in equilibrium is the immediately pre-state-of-arts. Basically, there are two different kinds of equilibrium, depending on consumers types' weights in population: in a "separation equilibrium" (corresponding to quality discrimination), whenever there is an innovation, the high-valuation consumers switch to the brand-new generation paying a high price and low-valuation consumers switch to what was until now the state-of-the-art generation paying a lower price. In a "pooling equilibrium" both consumers' types pay the same price and only the state-of-arts generation sells. A common feature of both equilibria is that high and low valuation consumers replace their generations at the same rate, which is the model's innovation rate.

In our model, in contrast, income distribution and population parameters may induce equilibria (pricing strategies) in which both rich and poor consumers replace their generations at the same pace or in which rich consumers have a higher replacement frequency. Of course, in relation to Glass (2001) we loose one degree of generality: in our setup the rate of innovation is exogenous, and we normalise time units so that this rate is one innovation per period.

• empirical evidences

In countries such as Brazil, in which income inequality is one of the worst in the world, it is easy to observe quite different qualities of the same product being sold in the market. In point of fact, data from the Brazilian POF (Household Budget Surveys) reveal that some durables are indeed sold at very different prices. Considering the purchases made in 2002-2003, the last column of Table 1 below shows that the coefficient of variation of prices ranges from 0.38 for microcomputers to 0.93 for walkman. If households are split into two groups of the same size: the rich households being those with income greater than the medium income and the poor households being the other half; the figures in parenthesis in the fifth column of Table 1 show the ratio of the share of rich households to the share of poor ones that bought each durable good in the last year of the survey. Since all the figures are bigger than one, POF data uncover the stylized fact that poor consumers tend to have a smaller replacement frequency of durable goods purchases than rich ones. Also, if we compare those figures in parenthesis with the corresponding figures for price variability (last column) we find a negative correlation between price variability and rich/poor consumers' relative replacement frequency. Assuming that third degree price discrimination[2] is not pervasive, we can hold the view that even when they renew their durable goods, poor consumers are getting lower quality or less than state-of-the-art products.

[2] That is when the monopolist can discriminate between two segments of consumers, charging different prices for the same durable generation at the same time.

Durable good	Total number of buyers	"poor households" buyers	% of "poor households" are buyers	% of "rich households" are buyers	Durable's Price mean deviation / mean price
Micro Computer	263	13	0,084	1,614 (19,21)	0,38
DVD	212	11	0,071	1,297 (18,26)	0,23
Walkman	171	71	0,458	0,645 (1,41)	0,93
Color TV	876	294	1,9	3,757 (1,97)	0,72
CD Player	103	28	0,18	0,484 (2,68)	0,61
Microwave oven	127	12	0,077	0,742 (9,63)	0,4

Table 1 - some results from Brazilian POF (Pesquisa de Orçamentos Familiares 2002-2003, IBGE). Durable acquisitions during one year. For details, see APPENDIX I

According to reports on the markets for cellular phones of the world leading firm for this product, the replacement period in Brazil is twice as long as in Europe and fifty percent longer than in the U.S[3]. Brazil's replacement period is also longer than the average of Latin America where countries like Venezuela and Colombia have quite short replacement periods[4]. Mobile phone companies in Brazil sell on average forty five models in the domestic market at the same time and the ratio of the highest to the lowest price might be as large as thirty. Nokia, for example, continues to sell cellular phones with black and white screens to low income consumers in Brazil along with a top-of-the-line model capable of video recording with high resolution. This suggests that income inequality together with monopolist-kind discrimination strategies may play a role in explaining the number of product generations available and consumers' different average replacement frequencies for each durable good in various countries.

• **literature revision :** as usual, there exists a vast related literature. Here we offer only a sketchy map:

– Multiple (two) qualities; heterogeneous consumers (two). Two competitors instead of a single monopolist: Gabszewicz&Thisse (1979)

– Single quality; dynamic analysis; one monopolist: Stokey (1981)

– Product life cycle and income distribution: Horsky (1990)

– Multiple qualities; dynamic analysis; one monopolist: Bagnoli, Saland and Swierzbinski (1995); Takeyama (1997) introduces copying; Inderst (2003)

– Successive product generations or upgrades; single monopolist: Fudenberg&Tirole (1998); Villas-Boas (1999). Fishman&Rob (2000) introduce endogenous growth
– Successive product generations or upgrades; two competing firms; endogenous growth; quality ladder model: Glass (2001)

[3] Bites, Edition 147, 09/29/2005, www.bites.com.br.

[4] Valor Online, 09/15/2006.

– Agents facing a trade-off between buying a new generation of the durable good and keeping the level of non-durable consumption; durable and non-durable as complementary; intertemporal allocation of income (non-durable consumption); monopolist practising Intertemporal Price Discrimination: Koh (2006)

• paper organisation:

In section 2 we describe consumers' behaviour, showing how reservation prices for durable-goods acquisitions and frequencies of generation renewals depend on income. In section 3 we present the 2 basic dynamic pricing strategies: non-discrimination and discrimination. In section 4 we show how the choice between non-discrimination and discrimination by the monopolist depends on income distribution, minimum quality standards and IPRs. Section 5 concludes. In the APPENDIX we present some empirical evidence from the Brazilian POF (household budget survey).

2. Consumers' behaviour

The infinite time functional is:

$$U_0^\infty = \sum_{t=0}^{\infty} \beta^t \cdot u_t \quad , \quad 0 < \beta < 1 \tag{1}$$

The instantaneous utility is:

$$u_t = \ln(1 \cdot q_t) + \ln(y_t - p \cdot z_t) \tag{2}$$

in which

1 is the quality of durable good
q_t is the quality of the durable good available for the consumer at time t
y_t is the quantity of non-durable good available at time t (*proxy* for income
p is the the durable's price measured in units of non-durable
$z_t = 0$ if the consumer doesn't purchase a new generation of durable at time t
$z_t = 1$ if the consumer purchases a new generation of durable at time t

If we assume that purchasing the new generation of durable good is not a big burden on the consumer's budget (income), then we can tolerably approximate (2) as[5]

$$u_t = \ln(1 \cdot q_t) + \ln y_t - z_t \cdot \frac{1}{y_t} \cdot p_t \qquad , \quad p \ = dy_t \tag{3}$$

Setting $q_0 = 1$, we assume $q_t = \lambda^s$, where $\lambda > 1$ and $s \in [1, 2, ..., t]$ is the last period in which the consumer acquired a new generation of durable, which implies, in particular, $z_t = 1 \Rightarrow q_t = \lambda^t$. So the durable good

[5] If we further assume that what is being purchased is not a new durable good (for example, a new computer), but rather an updating service on the existing durable (a software, for example), then we can skip modelling a secondary market for old durable goods. So a typical consumer will not buy a single unit of the durable good but several updatings during his lifetime.

does not depreciate physically, providing a perpetual constant flow of services/utility per period, whose level depends only on the generation it belongs .Let's assume, for simplicity, that income is invariable over time, $y_t = y \ , \ \forall t$.

Consider a consumer who arrives at time $T-1$ with $q_{T-1} = \lambda^{s_{T-1}}$, $s_{T-1} \in [1, T-1]$. So at time T she/he faces the problem of choosing between

$\overline{u_T} \equiv \ln \lambda^T + \ln y - \frac{1}{y} \cdot p$, if he buys the generation for sale at T

and

$\underline{u_T} = \ln \lambda^{s_{T-1}} + \ln y$, if she/he doesn't buy

So

$$\overline{u_T} - \underline{u_T} = (T - s_{T-1}) \cdot \ln \lambda - \frac{1}{y} \cdot p \qquad (4)$$

By (4) we see that, given p and y, the consumer will be willing to buy a new durable generation at time T, that is, $\overline{u_T} > \underline{u_T}$, when the durable's generation she/he possesses at time *T-1* is very obsolete, that is, $(T - s_{T-1})$ is big. On the other hand, given p and $(T - s_{T-1})$, the consumer will be willing to buy a new durable generation at time T when her/his income y is big.

Let's assume for the moment that "rich" consumers choose replacing their durable's generation every period[6], what amounts to an improvement of λ times in the durable's quality every period. The condition for a rich consumer to do so is:

$$\ln \lambda \geq \frac{1}{y_r} \cdot p \ \Rightarrow p \leq y_r \cdot \ln \lambda \qquad (5)$$

,in which y_r is the rich consumer's income. Expression (5) gives an upper bound to p assuming rich consumers change the durable's generation every period, and is therefore obtained from (4) with $s_{T-1} = T-1$.[7]

Let us analyse now the "poor" consumer's problem: inspection of (3) above shows that if it paid-off buying a new generation at time $t = 1$, then it would pay-off changing the durable's generation every period, since the gain from jumping one generation up is constant and equal to $du = \ln \lambda$. So we define the "waiting time" or "replacement period" of the poor consumer as

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[6] Later, this will be proved to result from an optimal pricing strategy.

[7] Our idea of an upper bound to the price charged by the monopolist is essentially the same as in Fishman and Rob (2000): "*If old models in the consumer's possession continue to be functional after a new model appears, the monopolist can only charge for the incremental flow of services the new model provides*." (pg. 3)

$$r_p \equiv \min(2,3,4,...) \text{ such that } r_p \cdot \ln\lambda \geq \frac{1}{y_p} \cdot p \qquad (6)$$

, in which y_p is the poor consumer's income. If time were continuous, or for a small λ, we would have

$$r_p \cdot \ln\lambda = \frac{1}{y_p} \cdot p \qquad (6')$$

In the above equality, the left side is the gain from changing the durable's generation; the right side is the cost in terms of the utility of foregone non-durable consumption. Obviously,

$$(6) \Rightarrow \frac{dr_p}{dy_p} < 0 \text{ and } \frac{dr_p}{dp} > 0$$

, that is, the poorer the consumer is, the longer it takes for her/him to replace her/his durable's generation. And a price reduction leads the poor consumer to increasing her/his replacement frequency.

From (5) and (6') comes

$$r_p \leq \frac{y_r}{y_p} \qquad (7)$$

That is, if the price charged on new generations is such that rich consumers replace their generations every period, then the time poor consumers wait to do so is not greater than the ratio between income levels.

3. Dynamic pricing strategies

3.1 Non-discrimination

Recall we assumed that innovation is a process with memory so that a single firm launches all new generations in the durable good market. The simplest pricing strategy this monopolist can use is "**non-discrimination**", which here means that one generation of the durable good is sold for a single price over time and no two different generations are for sale at the same time. It may be the case that poor consumers buy new generations with a smaller frequency, but whenever they make their purchases they are taking the same product rich consumers take.

If the single price under "non-discrimination" were just sufficient for the poor to buy the new generation at every period

$$\underline{p} = \ln\lambda \cdot y_p \qquad (8)$$

,then by (6') we would have $r_p = 1$. But this is only one possibility. More generally, consider also

$$\overline{p} = \gamma \cdot \ln\lambda \cdot y_p \;,\; \gamma \geq 1 \qquad (9)$$

, then by (6') we have $r_p = \gamma$ for the poor consumer.

For the rich consumer, two possibilities open up from (9):

9.i) $\bar{p} \le \ln\lambda \cdot y_r \Leftrightarrow \gamma \le y \equiv \frac{y_r}{y_p} \Rightarrow$ the rich consumer will still buy a new generation every period.

9.ii) $\bar{p} > \ln\lambda \cdot y_r \Leftrightarrow \gamma > y \equiv \frac{y_r}{y_p} \Leftrightarrow \bar{p} = \rho \cdot \ln\lambda \cdot y_r$, $\rho > 1 \Rightarrow r_r = \rho$, that is, the rich consumer will buy a new generation less than every period, with r_r being the rich consumer's waiting time, analogous to r_p.

However obvious, there can be established the following

Proposition: 9.ii above is never an optimal strategy for the monopolist

Proof: Starting from $\bar{p} = \ln\lambda \cdot y_r$, increasing the price means multiplying it by a factor ρ , but also multiplying r_p and r_r by ρ, so that if a consumer type $j \in (p, r)$ used to buy a new generation $1/r_j$ times every period when $\bar{p} = \ln\lambda \cdot y_r$, now he/she will buy a new generation $1/(\rho \cdot r_j)$ times every period when $\bar{p} = \rho \cdot \ln\lambda \cdot y_r$, $\rho > 1$. When the price is multiplied by ρ, the frequency is divided by ρ, but since the future is discounted by a factor $\beta < 1$ - see expression (1) above - this amounts to reducing the present value of the revenues (profits) stream of the monopolist.

So the price under "non discrimination" will be given by (9.i) and the high income consumer will buy a new generation every period. In order to calculate monopolist's revenues, we still have to consider **population parameters** and the **possibility of imitation**:

Call $n_r \equiv$ number of rich consumers and $n_p \equiv$ number of poor consumers

We already know that $r_p = \gamma$. Assume also that n_p is big enough so that poor consumers' purchases are uniformly distributed over time. In this case, the expected number of poor consumers changing their durable's generation in an one period span of time is n_p / γ. Obviously, the number of rich consumers changing their durable's generation in an one period span of time is n_r.

Now **imitation** is conceived of as a process by which other firms may become able to produce state-of-the-art generations with probability i from the very moment those new generations become available in the market. Imitation is assumed to be a **memoryless process** in the sense that expertise is not cumulative, that is, if I fail to imitate generation t at time t, my probability of imitating generation t at time $t + 1$ is i, the same as my probability of imitating generation $t + 1$ at time $t + 1$. If there are many rival imitative firms engaged in price competition, then the only Nash equilibrium in imitation strategies will be that only the state-of-the-art generation is targeted by imitators. We also assume that information flows freely and instantaneously inside the imitators' community, so that whenever imitation is successful, all imitators become able to produce[8]. Assuming for simplicity that imitation and production costs are zero, Bertrand competition on the part of successful imitators will drive the price to zero, causing the monopolist a total loss.

Our innovative monopolist will realise its sales only if imitation fails, what happens with probability $(1 - i)$ for rich consumers and probability $(1 - i)^{\gamma}$ for poor consumers, so that its expected revenue per period is

$$V = \overline{p} \cdot \left[n_r \cdot (1-i) + \frac{n_p (1-i)^{\gamma}}{\gamma} \right] \;=\; \text{by (9)} \;=$$

$$= \ln \lambda \cdot y_p \cdot \left[\gamma \cdot n_r \cdot (1-i) + n_p (1-i)^{\gamma} \right] \;;\quad 1 \leq \gamma \leq \frac{y_r}{y_p} \equiv y \qquad (10)$$

Expression (10) reveals a trade-off present in pricing decisions when there is no discrimination: on the one hand, reducing γ is the same as reducing the new generation's price, leading to smaller revenues per period from sales to rich consumers while leaving sales to poor consumers unaltered[9] ; on the other hand, reducing γ leads poor consumers to increasing their replacement frequency, $1 / r_p$, in a context where there is imitation and the bigger r_p the more likely imitation will occur, thus increasing the expected value of sales revenues per period. One may regard the other pricing strategy, **discrimination**, as a means of softening this trade-off.

3.2 Discrimination

Consider a poor consumer willing to buy a new generation at time t. Therefore, it must be $\overline{u_t} \geq \underline{u_t}$ and $\overline{u_{t-1}} < \underline{u_{t-1}}$. Put another way, given the definition of r_p,

$(r_p - 1) \cdot \ln \lambda < \frac{1}{y_p} \cdot p$, that is, given p, y_p and λ , a poor consumer wouldn't buy a new generation one period earlier. But at a smaller price he would:

$$(r_{pd} - \delta) \cdot \ln \lambda \geq \frac{1}{y_p} \cdot P_{dp} \qquad (11)$$

where p_{dp} denotes the "low" price of a durable α generations old. This price is chosen by the monopolist so as to make the poor consumer to be willing to switch generations of the durable good every $r_{pd} - \delta$ periods, where r_{pd} is the time lag for the switching by the poor consumer should she/he face the "high" price p_d:

$$r_{pd} \cdot \ln \lambda = \frac{1}{y_p} \cdot p_d \qquad (6'')$$

So the fundamental feature of "dynamic price discrimination" is that the monopolist will induce low income consumers to buy a product that is already α generations old and to anticipate (δ periods) their purchases,

[8] Put another way, imitators don't run independent projects, and i is already the aggregate probability of imitation.

[9] Sales to poor consumers remain constant because if the price is reduced to, say, a half, then the number of poor consumers that in one period will replace their durables'generation is exactly doubled. This follows from the already known fact that $r_p = \gamma$.

shortening the period in between purchases, while high income consumers will keep buying the state-of-the-art product at every period. This implies that more than one generation of the (same) durable good are for sale in the market[10]. Typically, poor consumers pay the "low" price p_{dp} for the old generation, and rich consumers pay the "high" price p_d for the new generation.

It is worth noticing that α, the age (number of periods since introduction in the market) of the old generation product, doesn't enter expression (11) above. This is because in a "steady-state" in which poor consumers always buy durables that are already α generations old[11], the only thing they care about is the size of the quality step they take, that is, $(r_{pd} - \delta) \cdot \ln\lambda$.

That indifference towards α is not true, however, for rich consumers − they face the *ex ante* choice between buying the durable's generation t at time t, paying p_d for that, or buying generation t at time $t + \alpha$ paying only p_{dp} . When the probability of imitation is properly taken into account, this amounts to comparing

$$\underbrace{\ln\lambda - \frac{1}{y_r} \cdot p_d \cdot (1-i)}_{\substack{\text{expected gain from always} \\ \text{buying the state-of-the-art}}} \quad \geq \quad \underbrace{\beta^{\alpha} \cdot \left[\ln\lambda - \frac{1}{y_r} \cdot p_{dp} \cdot (1-i)^{\alpha+1}\right]}_{\substack{\text{expected gain from buying} \\ \text{an old generation every period}}} \qquad (12)$$

Expression (12) is written as the condition for a rich consumer to be willing to buy generation t at time t, paying p_d for that, rather than buying generation t at time $t + \alpha$ paying only p_{dp}. The left side can be viewed as the expected gain from not waiting and the right side as the expected gain from waiting, although the rich consumer is in both cases replacing generations every period. For the inequality ≥ above to hold, p_d cannot be too much bigger than p_{dp} ; for although the gain from waiting is time discounted (by a factor β^{α}), waiting brings about a bigger probability of imitation, in which case the consumer doesn't pay anything for a new generation.[12]If prices are such that (12) above holds with equality, that is rich consumers are indifferent between waiting and not waiting, we can solve it for p_{dp} :

$$p_{dp} = \frac{(1-i)^{-\alpha}}{\beta^{\alpha}} \cdot \left[p_d - \frac{1-\beta^{\alpha}}{1-i} \cdot y_r \cdot \ln\lambda\right] \qquad (13)$$

Now we are ready to solve for the endogenous variables p_{dp}, p_d and $r_{pd} - \delta$: Using (11), (6'') and (13) comes

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[10] Two generations may be for sale at the same time in Glass's (2001) model, but not in Inderst's (2003) or Koh's (2006) because in their setups there exists only one "new generation" and the consumer buys it at most once in a lifetime.

[11] "Old" in the sense of being an old model, not in the sense of second hand products.

[12] On the right side of expression (12), the factor (1 – i) appears raised to the power α **+ 1** (**plus 1**) for the same reason why the expected cost of not waiting on the left side appears multiplied by (1 – i): as before, we assume there existis a probability of instantaneous imitation, so that if I wait, say, 5 periods to by a generation invented 5 periods ago, there are actually 6 instances in which imitation can take place.

$$p_d = \ln\lambda \cdot \left[\frac{1-\beta^\alpha}{1-\beta^\alpha \cdot (1-i)^\alpha} \cdot \frac{1}{1-i} \cdot y_r - \delta \cdot \frac{\beta^\alpha}{(1-i)^{-\alpha} - \beta^\alpha} \cdot y_p \right] \tag{14}$$

$$p_{dp} = \ln\lambda \cdot \left[\frac{1-\beta^\alpha}{1-\beta^\alpha \cdot (1-i)^\alpha} \cdot \frac{1}{1-i} y_r - \delta \cdot \frac{(1-i)^{-\alpha}}{(1-i)^{-\alpha} - \beta^\alpha} \cdot y_p \right] \tag{15}$$

And, by substituting (15) in (11),

$$r_{pd} - \delta = \left[\frac{1-\beta^\alpha}{1-\beta^\alpha \cdot (1-i)^\alpha} \cdot \frac{1}{1-i} \cdot y \ - \delta \cdot \frac{(1-i)^{-\alpha}}{(1-i)^{-\alpha} - \beta^\alpha} \right] \quad ; \ y \equiv \frac{y_r}{y_p} \tag{16}$$

Inspecting (13), we see that as $\alpha \to 0$, $p_{dp} \to p_d$ and thus discrimination disappears. It is worth thinking of α as a choice variable constrained by what Glass (2001) calls "minimum quality standards", when the government forbids sales of too old generations. The limit case is when the government allows only state-of-the-art generations to be sold in the market, in which case $\alpha = 0$ and discrimination is impossible. When the quality standard is some positive integer $\overline{\alpha}$, the choice of α must befall on $\{1, 2, ..., \overline{\alpha}\}$.In order to calculate the monopolist's expected revenue per period under discrimination, an expression analogous to (10) above, we first notice that a sale for rich consumers will be realised with probability $(1- i)$, and for poor consumers with probability $(1-i)^{r_{pd}-\delta}$.[13] Next, remind that while all n_r rich consumers replace their generations in one period, the number of poor consumers to do so is $\frac{n_p}{r_{pd}-\delta}$. So the expected revenue per period is

$$V_d = n_r \cdot (1-i) \cdot p_d + \frac{n_p}{r_{pd} - \delta} \cdot p_{dp} \cdot (1-i)^{r_{pd}-\delta} \tag{17}$$

Using (14), (15) and (16) in (17):

$$\begin{aligned} V_d = \ln\lambda \cdot \{ & n_r \cdot (1-i) \cdot \left[\frac{1-\beta^\alpha}{1-\beta^\alpha \cdot (1-i)^\alpha} \cdot \frac{1}{1-i} \cdot y_r - \delta \cdot \frac{\beta^\alpha}{(1-i)^{-\alpha} - \beta^\alpha} \cdot y_p \right] + \\ & + n_p \cdot y_p (1-i)^{\frac{1-\beta^\alpha}{1-\beta^\alpha \cdot (1-i)^\alpha} \cdot \frac{1}{1-i} \cdot y \ - \delta \cdot \frac{(1-i)^{-\alpha}}{(1-i)^{-\alpha} - \beta^\alpha}} \ \} \end{aligned} \tag{18}$$

[13] Since a successful imitation in any of the $r_{pd} - \delta$ periods the poor consumer waits would frustrate a paid generation renewal.

In expression (18), the monopolist takes as given the parameters and exogenous variables: λ, n_r, n_p, y_r, y_p, β, i and $\overline{\alpha}$. If $\overline{\alpha}$ = 1, then discrimination can only take place with α = 1 and the only choice variable is δ, the number of periods the poor consumer's generation replacement is anticipated. This choice of δ is in turn constrained by:

a lower bound δ = 0 – since δ < 0 would mean that the monopolist charges p_{dp} such that the poor consumer actually postpones her/his generation replacement relative to what she/he would do under p_d, what in turn would imply $p_{dp} > p_d$, an absurd by definition of p_{dp} and p_d.

an upper bound we find by setting $r_{pd} - \delta = 1$ in expression (16) above – after all, since we assume that new generations are launched at a rate 1 per period, the minimum a poor consumer can wait to replace her/his durable generation is the same as a rich consumer do, that is, 1 period. This upper bound is given by

$$\delta = \frac{1-\beta^{\alpha}}{1-i} \cdot y + \beta^{\alpha} \cdot (1-i)^{\alpha} - 1 \tag{19}$$

4. Income distribution and pricing strategies

We here address two related questions: 1) Given parameters and exogenous variables' values, which value of γ maximises V, the monopolist's expected revenue per period under non-discrimination, and which pair of values (δ, α) maximise V_d, the monopolist's expected revenue per period under discrimination ? 2) Under which circumstances is the maximal V_d > maximal V , so that discrimination is a dominant strategy? In particular, how this depends on population parameters (n_r, n_p), income inequality ($y \equiv y_r / y_p$), Intellectual Property Rights (IPRs, parameterised by i) and time preferences/interest rates (β)?

It is easier to tackle question 2 first, deriving a sufficient condition for maximal V_d > maximal V. To simplify the notation, let us adopt the following normalisation:

$$V' \equiv \frac{1}{\ln \lambda} \cdot \frac{1}{n_r + n_p} \cdot \frac{1}{y_p} \cdot V = s_r \cdot \gamma \cdot (1-i) + s_p \cdot (1-i)^{\gamma} \quad ; \qquad \text{where } s_r \equiv \frac{n_r}{n_r + n_p} \ , \ s_p \equiv \frac{n_p}{n_r + n_p} \tag{20}$$

and

$$V_d' \equiv \frac{1}{\ln \lambda} \cdot \frac{1}{n_r + n_p} \cdot \frac{1}{y_p} \cdot V_d = s_r \cdot \left[\frac{1-\beta^{\alpha}}{1-\beta^{\alpha} \cdot (1-i)^{\alpha}} \cdot y - \delta \cdot \frac{(1-i) \cdot \beta^{\alpha}}{(1-i)^{-\alpha} - \beta^{\alpha}} \right] + s_p \cdot (1-i)^{\frac{1-\beta^{\alpha}}{1-\beta^{\alpha} \cdot (1-i)^{\alpha}} \cdot \frac{1}{1-i} \cdot y - \delta \cdot \frac{(1-i)^{-\alpha}}{(1-i)^{-\alpha} - \beta^{\alpha}}} \quad ; \text{where, recall, } y \equiv y_r / y_p \tag{21}$$

Now suppose γ* is the value of γ which maximises *V'*. If, in V_d', we set,

$$\frac{1-\beta^{\alpha}}{1-\beta^{\alpha}\cdot(1-i)^{\alpha}}\cdot\frac{1}{1-i}\cdot y\ -\delta\cdot\frac{(1-i)^{-\alpha}}{(1-i)^{-\alpha}-\beta^{\alpha}}=\gamma^{*} \tag{22}$$

then we have

$$V_d' - V' = s_r\cdot(1-i)\cdot\delta > 0 \tag{23}$$

But when does this condition apply? Substitute, in (22), the lower bound $\delta = 0$, to obtain

$$\gamma^{*}=\underbrace{\frac{1-\beta^{\alpha}}{1-\beta^{\alpha}\cdot(1-i)^{\alpha}}}_{0<\ "\ <1}\cdot\frac{1}{1-i}\cdot y \tag{24}$$

If the maximizer γ* is smaller than the value given by (24), we have maximal V_d' > maximal *V'* with a positive value for δ. Put another way, more loosely, when *V'* is maximised with a small γ value, we can be sure that discrimination is a better strategy. Inspecting expression (20) we see that this happens when s_p, the share of poor consumers in population, is large and/or when the probability of imitation *i* is high. The intuition behind this result should be obvious to the reader.[14]

However obvious, this result is already different from what we find in Glass (2001): there she calls a "pooling equilibrium" a situation where both types of consumers pay the same price for a new (state-of-the-art) generation, that is, the same as "non-discrimination" here. Well, in our model, except for the trivial cases when α = 0 or s_r = 0, we will never observe a pooling equilibrium with $\underline{p}=\ln\lambda\cdot y_p$, that is, γ = 1 in expression (9) above; because we know that under these circumstances discrimination would be a better strategy. On the contrary, in Glass (2001), owing to the classical *quality ladder* model's feature that the firms which produce the state-of-the-art and the pre-state-of-the-art generations are rival (not a single monopolist), the pooling equilibrium price is always given by the lowest evaluation in the market, that is, the poor consumer's evaluation.

In our model, if non-discrimination (pooling) prevails, then γ^* is surely bigger than 1.[15] This has another important consequence: r_p, the poor consumers' replacement period, will always be > 1, since we know that $r_p = \gamma$. Thus a stylised fact must be observed under non-discrimination: poor consumers will have a smaller replacement frequency than rich consumers. With respect to the replacement frequency, how does discrimination look like? To answer this, we must go beyond the sufficient condition derived in (22) – (24) above.

[14] In PANEL 1 below we plot *V´* as a function of γ for several sets of parameters values.

[15] Actually, γ* will always be maximal (equal to *y*) in those cases, as can be inferred by the shape of *V'* schedule, depicted in PANEL 1 below.

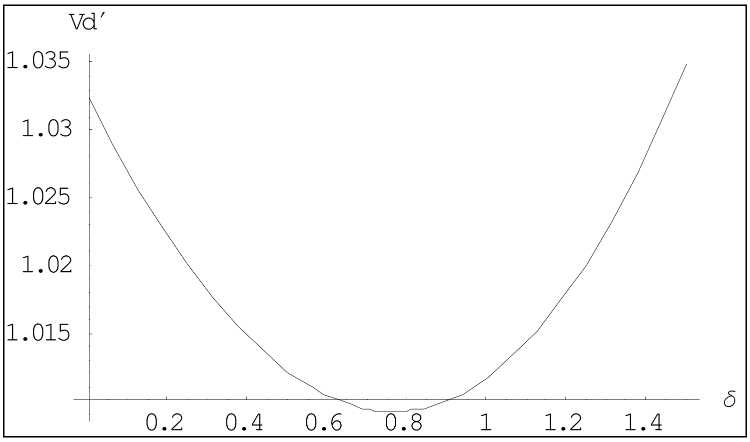
Taking the first derivative of V_d' with respect to δ we get

$$\frac{\partial V_d'}{\partial \delta} = \underbrace{-\frac{s_r \cdot (1-i) \cdot \beta^{\alpha}}{(1-i)^{-\alpha} - \beta^{\alpha}}}_{<0} \; \underbrace{-(1-i)^{-\alpha + \frac{1-\beta^{\alpha}}{1-\beta^{\alpha} \cdot (1-i)^{\alpha}} \cdot \frac{1}{1-i} \cdot y - \delta \cdot \frac{(1-i)^{-\alpha}}{(1-i)^{-\alpha} - \beta^{\alpha}}} \cdot \frac{\ln(1-i) \cdot (1-s_r)}{(1-i)^{-\alpha} - \beta^{\alpha}}}_{>0} \qquad (25)$$

, which has an ambiguous sign. While taking the second derivative we get

$$\frac{\partial^2 V_d'}{\partial \delta^2} = \frac{(1-s_r) \cdot [\ln(1-i)]^2 \cdot (1-i)^{-2\cdot\alpha + \frac{1-\beta^{\alpha}}{1-\beta^{\alpha} \cdot (1-i)^{\alpha}} \cdot \frac{1}{1-i} \cdot y - \delta \cdot \frac{(1-i)^{-\alpha}}{(1-i)^{\alpha} - \beta^{\alpha}}}}{\left[(1-i)^{-\alpha} - \beta^{\alpha}\right]^2} > 0 \qquad (26)$$

So the V_d' schedule looks like the one plotted in GRAPH 1 below[16]:



GRAPH 1 – V_d' as a function of δ

When a function like V_d' is maximised with respect to δ within an interval $[$lower bound to δ , upper bound to $\delta]$, the result is always a corner solution, that is, either $\delta = 0$, or the upper bound $\delta = \left[(1-\beta^{\alpha})/(1-i)\right] \cdot y + \beta^{\alpha} \cdot (1-i)^{\alpha} - 1$.[17] But substituting $\delta = 0$ in expressions (14) and (15), we see that that means $p_d = p_{dp}$, that is, non-discrimination. So the conclusion is: if doing discrimination pays-off, then this is done with δ maximal, what in turn implies $r_{pd} - \delta = 1$, and poor consumers replace generations every period, like rich consumers do. When the monopolist is allowed to sell old enough stuff ($\overline{\alpha} >> 0$), then it maximises its profits by making poor consumers replace their generations

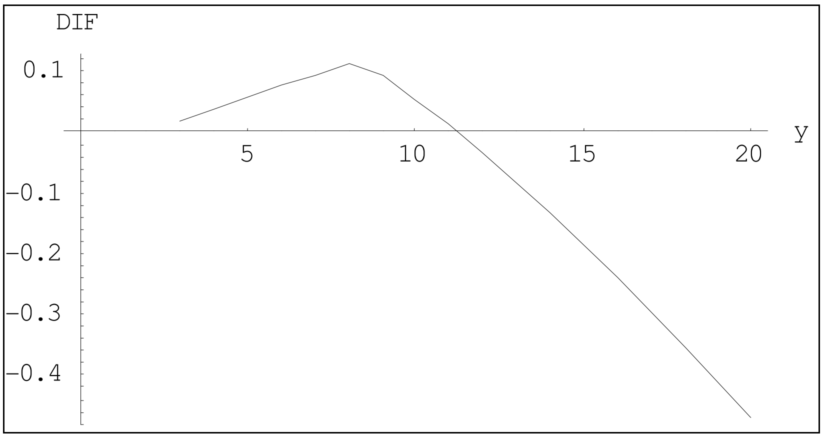
[16] Actually, the graph below is V_d' plotted for $i = 0{,}1$, $\beta = 0{,}8$, $y = 8$, $\alpha = 1$, $s_r = 0{,}1$.

[17] This last value comes from expression (19) above.

with the greatest frequency possible. On the contrary, if $\bar{\alpha}$ is small, so that the upper bound to δ is small, then the monopolist will resort to non-discrimination.[18]

Also, the upper bound to δ is decreasing in the time factor β: When β is small (the interest rate is high) and the future is much discounted, discrimination is likely the best strategy because the monopolist can open a big wedge between a high p_d and a low p_{dp} without fearing that rich consumers will prefer waiting to buy old generations of the durable good.

Finally, the way how **income inequality** (y) affects the monopolist's strategy choice is a bit more complicated. Consider the sufficient condition derived above: By (19) the upper bound to δ is clearly increasing in y, while by (23) the difference $V_d' - V'$ is increasing in δ, so that it might appear that the bigger is y the more it pays-off to discriminate. However, this reasoning holds only as long as the sufficient condition applies, that is, when V' is maximised with a small γ value, which in turn is not true for y too big. Leaving the sufficient condition aside and performing full maximisation of V_d' and V' for a given set of parameters values, we plot below (GRAPH 2) a typical schedule of the difference DIF $\equiv V_d' - V'$ as a function of y:



GRAPH 2 - the difference DIF $\equiv V_d' - V'$ as a function of y
(for parameters values $s_r = 0{,}1$; $i = 0{,}2$; $\beta = 0{,}9$; $\bar{\alpha} = 2$)

---

[18] The reader will notice that in the text we are implicitly assuming first that the upper bound to δ is increasing in α, what, inspecting expression (19) we see it is true for y big enough. Second, and more important, we are assuming the monopolist will always choose $\alpha = \bar{\alpha}$ (maximal allowed) so as to choose the biggest δ and thus maximise Vd'. More formally, what must be considered in choosing α is $\frac{dV_d'}{d\alpha} = \frac{\partial V_d'}{\partial \alpha} + \frac{\partial V_d'}{\partial \delta} \cdot \frac{d\delta}{d\alpha}$. Now we know that for all reasonable parameters values, the positive indirect effect $\frac{\partial V_d'}{\partial \delta} \cdot \frac{d\delta}{d\alpha}$ outweighs the direct, sometimes negative, effect $\frac{\partial V_d'}{\partial \alpha}$ so that $\frac{dV_d'}{d\alpha} > 0$, and therefore our line of argument in the main text is *a fortiriori* true. When $\frac{dV_d'}{d\alpha} > 0$ the simple V_d' schedule plotted above, for a single value of α ($\alpha = \bar{\alpha}$), is already a sure guide to find how V_d' is maximized. More generally, however, what must be considered is an envelope curve; for some δ values are compatible with more than one α value. For each such δ we must choose the α value which maximizes V_d'.

So typically the difference first increases with y (while the optimal non-discrimination price is based on the poor consumers'evaluation, that is, γ close to 1), but eventually it will fall below zero when y gets too big (so that poor consumers don't represent a big share of the market income and the optimal non-discrimination price is based on the rich consumers'evaluation).

This contrasts with Koh's (2006) striking result that, even when consumers and the monopolist have the same rate of discount, (intertemporal price) discrimination is always dominant over a constant price (non-discrimination), while in Stokey (1979) discrimination was always a dominated strategy. Koh's result is due to the realistic assumption that consumers face an intertemporal budget constraint, that is, they can borrow or lend money (non-durable good) over time. So although this is not the main focus of our paper, we are left with explaining a third possibility, namely, that each strategy (discrimination and non-discrimination) may in turn be dominant. An exhaustive demonstration is beside the point here, but we may say that this implication of our model follows from the fact that the monopolist here is constrained by minimum quality standards (recall $\overline{\alpha}$ above), so that it is not entirely free to choose how old are the generations it sells to poor consumers.

5. Conclusions

Recollecting the results we got, we may say that discrimination is likely to occur when income distribution is bad (high s_p and high y) but poor consumers still represent a not negligible share of the economy's income (y cannot be too high); when IPRs are low (high probability of imitation i); and when the future is very much discounted (low β , what can be interpreted as a high interest rate). Another necessary condition for discrimination is that quality standards are absent or not too strict – more generally, we may say that discrimination is likely to be observed in markets where old generations/models sell at the same time that state-of-the-art ones. The pricing strategies (discrimination and non-discrimination) influence the poor consumers' replacement frequency in different ways: under discrimination, poor consumers will have the same replacement frequency as rich consumers, while under non-discrimination generally poor consumers are expected to take much more time to replace their durable good's generation.

To finish, there are two remarks we would like to make:

The first has to do with the impact of income distribution on welfare in our model. To illustrate this point, consider 3 "poor" consumers with identical incomes (as measured in terms of our model's nondurable good), but each living in a different economy, the economies being isolated from one another and each served by a monopolist like the one in our model: the first consumer lives in a place where almost everybody is poor and income inequality is low; the second guy lives in a place where the share of poor consumers is big and income inequality is high; the third lives in a place where the share of poor consumers is small and inequality is big. Then our model predicts that the first guy is better off than the second guy, who is better off than the third guy: the first guy may very well be replacing his durable's generation with a high frequency paying a low price; the second guy will pay a higher price but still enjoy a high replacement frequency; the third guy will pay an even higher price while having a low frequency. [19]

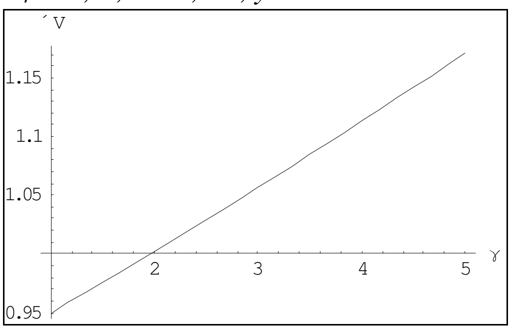
The second remark has to do with the conclusion we reached that discrimination is more likely to be the underlying phenomenon in markets where old generations/models sell at the same time that state-of-the-art ones. Maybe this helps explaining the fact observed in Brazilian POF (Household Budget Survey, see

[19] By a "poor" consumer we should understand someone who is nevertheless able to buy the durable good, so the above illustration does not apply to immiserized populations.

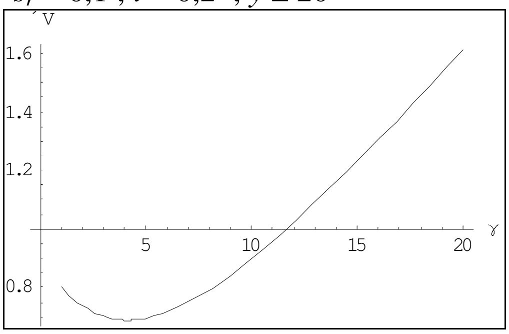
APPENDIX) that while rich consumers replace their computer models with much greater frequency than poorer consumers, both rich and poor consumers seem to be replacing their Walkmans or colour TVs with the same frequency. Indeed, when we inspect what is for sale at those different markets, we see that practically only latest generations of computers are fabricated, while many different vintages/models of Walkmans or TVs are being currently fabricated. Recall that in our model discrimination involves simultaneous fabrication/sales of different generations and that under discrimination both poor and rich consumers are expected to be replacing their generations with the same (maximal) frequency. Prices charged on the same good are expected to differ considerably, what seems to be the case for Walkmans and Colour TVs when we take the price mean deviation divided by the mean price in spot markets. Under non-discrimination, on the contrary, only state-of-the-art durables sell, and poor consumers will be replacing their generations at a smaller pace. There is a single price charged on each good, what grossly seems to be the case for computers and DVDs, which have a much smaller spot price variance.

PANEL 1 - V' as a function of γ for several sets of parameters values

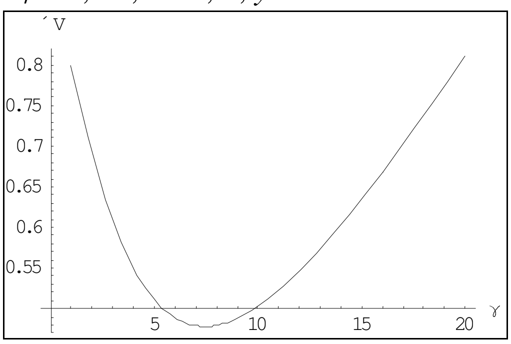
$s_r = 0{,}1$; $i = 0{,}05$; $y \leq 5$



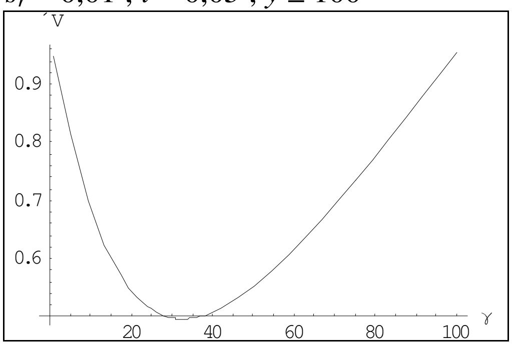
$s_r = 0{,}1$; $i = 0{,}2$; $y \leq 20$



$s_r = 0{,}05$; $i = 0{,}2$; $y \leq 20$



$s_r = 0{,}01$; $i = 0{,}05$; $y \leq 100$



APPENDIX – SOME EMPIRICAL EVIDENCES FROM BRAZILIAN POF

I - price variability and replacement frequency

Total number of households interviewed = 30979 ⇒ medium household = 15490

Income of the medium household (per year) = R$ 1063

Thus we split the universe of households in 2: "rich" households are those whose income > R$ 1063, and "poor" households are those whose income < R$1063

From POF's table 15 (purchases in the last one year, 2002-2003) we extracted the following data:

Durable good	Total number of buyers	"poor households" buyers	% of "poor households" are buyers	% of "rich households" are buyers
Micro Computer	1051	66	0,4261	6,35
DVD	625	47	0,3	3,73
Walkman	269	119	0,768	0,968
Color TV	5560	3058	19,74	16,15
CD Player	412	184	1,18	1,47
Microwave oven	450	81	0,523	2,38

Next we control for the primary market, eliminating second-hand purchases:

Durable good	Total number of buyers	"poor households" buyers	% of "poor households" are buyers	% of "rich households" are buyers
Micro Computer	838	44	0,284	5,126
DVD	612	41	0,2647	3,686
Walkman	238	97	0,626	0,91
Color TV	3915	1820	11,75	13,52
CD Player	328	129	0,833	1,285
Microwave oven	392	57	0,368	2,163

Finally we control for the spot market, eliminating purchases on credit:

Durable good	Total number of buyers	"poor households" buyers	% of "poor households" are buyers	% of "rich households" are buyers	Durable's Price mean deviation / mean price
Micro Computer	263	13	0,084	1,614 (19,21)	0,38
DVD	212	11	0,071	1,297 (18,26)	0,23
Walkman	171	71	0,458	0,645 (1,41)	0,93

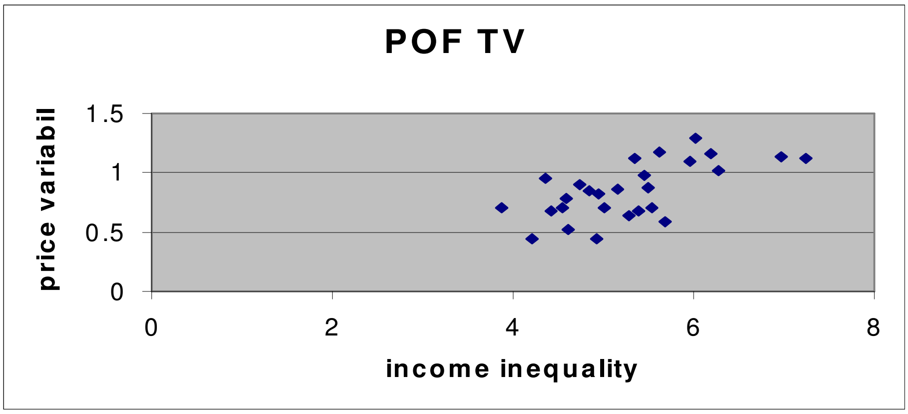
Color TV	876	294	1,9	3,757 (1,97)	0,72
CD Player	103	28	0,18	0,484 (2,68)	0,61
Microwave oven	127	12	0,077	0,742 (9,63)	0,4

In parenthesis is the number of times "% of rich households are buyers" is of "% of poor households are buyers"

The figures inside parenthesis in the fifth column are equal to the figures not in parenthesis in fifth column divided by the figures in the fourth column; they give, thus, relative replacement frequencies of the type "rich consumers' replacement frequency / poor consumers' replacement frequency for durable good i". Comparing these relative frequencies to the price variability indexes in the sixth column, we find that when price variability is low (indicating "non-discrimination", in our model), poor consumers have a much lower replacement rate than rich consumers, as is the case for the durable DVD. The converse is true for durables like Colour TV and Walkman.

II - price variability and income inequality

The second piece of evidence we can extract from POF relates price variability and income inequality at Brazilian States (called UF) level. We use here the same indexes as before: price variability = price mean deviation / mean price and income inequality = mean income of the 50% richest / mean income of the 50% poorest in the population. See for example what we find for the durable good Color TV, by far the most representative in the survey:



Dependent Variable: PRICEVAR
Method: Least Squares
Date: 01/19/07 Time: 12:49
Sample: 1 27
Included observations: 27
PRICEVAR=C(1)+C(2)*INCOMEINEQ

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	-0.139894	0.238363	-0.586893	0.5625
C(2)	0.187350	0.044613	4.199479	0.0003

R-squared	0.413636	Mean dependent var	0.850059
Adjusted R-squared	0.390181	S.D. dependent var	0.234994
S.E. of regression	0.183509	Akaike info criterion	-0.481919
Sum squared resid	0.841889	Schwarz criterion	-0.385931
Log likelihood	8.505908	Durbin-Watson stat	2.038014

Inspecting the plotting and the regression results we find a positive correlation between income and price variability at Brazilian States (UFs) level.

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