TECHNOLOGICAL PROGRESS AND AVERAGE
JOB MATCHING QUALITY

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Abstract

Our objective is to study, in a labor market characterized by search frictions, the effects of technological progress on the average quality of the job matches. We will see as results that the effects of technological progress on the labor market depend upon the initial conditions of the economy. If the economy is totally characterized by the presence of low-quality job matches, an increase in the technological progress is accompanied by an increase in the average quality of jobs. In turn, if the economy is totally characterized by the presence of high-quality job matches, an increase in the technological progress rate implies the reverse effect. Finally, if the economy is totally characterized by the presence of very high-quality jobs, an increase in the technological progress rate implies an increase in the average quality of the job matches.

Resumo

Nosso objetivo neste artigo é o de estudar, em um mercado de trabalho caracterizado por fricções, os efeitos de variações na taxa de progresso tecnológico sobre a qualidade média das parcerias produtivas. Veremos, como resultados do modelo, que os efeitos de variações na taxa de progresso tecnológico sobre o mercado de trabalho dependerão das condições iniciais da economia. Se a economia for totalmente caracterizada pela presença de parcerias produtivas de baixa qualidade, então uma variação positiva na taxa de progresso tecnológico virá acompanhada de um aumento na qualidade média dos empregos. Por sua vez, se a economia for totalmente caracterizada pela presença de parcerias produtivas de alta qualidade, uma variação positiva na taxa de progresso tecnológico virá acompanhada de uma redução na qualidade média dos empregos. Finalmente, se a economia for totalmente caracterizada pela presença de parcerias produtivas de elevadíssima qualidade, um aumento na taxa de progresso tecnológico virá acompanhado de um aumento na qualidade média dos empregos. Finally, if the economy is totally characterized by the presence of very high-quality jobs, an increase in the technological progress rate implies an increase in the average quality of the job matches.

Palavras-Chave: Criação e Destruição de Empregos, Progresso Tecnológico.
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1. Introduction

A model for the labor market has to be able to explain several aspects related to job creation and job destruction dynamics. Davis, Haltiwanger and Schuh (1996), for example, found that job creation and destruction flows are typically high, heterogeneous, and asymmetric and that the job destruction process is the one which normally dictates job changes. In other words, according to these authors, a labor market model should explain that, independently of the current economic state, job creation and job destruction flows are normally high, distinct for different sectors and periods, and that the destruction process should be the main responsible for the changes in the job match characteristics.

Apart from these facts, the authors also argued that a labor market model should be able to explain that the best job matches present lower creation and destruction rates and that the break ups are, in a great majority, driven by worker decisions, if the matches are of high quality, and by firms if they are of low quality.

Based on these empirical evidences, several studies were conducted in an attempt to explain some of these facts, determining their consequences over particular aspects of the labor market as, for example, the evolution of the average job quality. Caballero and Hammour (1994), for example, formalized the idea that recessions are periods of high job destruction and low job creation flows, and that if considered together, the result should be an increase in the average match quality. According to these authors during a recession there is a process of elimination of the less productive jobs from the market. The idea was that these periods are characterized by a *cleansing effect* whereby the least efficient jobs would be the first ones to be eliminated, only the most productive jobs remaining in the market. However, although Caballero and Hammour showed the consequences of a recession over the average quality of jobs, their results do not have empirical support, as Bowlus (1995) demonstrated.

Bowlus found, with an empirical analysis of the evolution of job quality in the US, that what is happening is precisely the opposite of what Caballero and Hammour suggested. In other words, Bowlus observed that recessions are periods not only of no improvement, but actual deterioration in the average quality of matches. Based on these findings, Barlevy (2002) considered, in addition to the effect of a recession over job destruction, the effects of recessions over the job creation dynamics. This author argued that considering on-the-job search, we would have the creation of an additional effect, empirically stronger than the one proposed by Caballero and Hammour, which he denominated *sullying effect*.

The idea defended by Barlevy was that recessions were periods characterized by low quality job creation. In this way, if these effects overcome those generated by the destruction dynamics, we would have a reduction, and not an improvement, in the average quality of job matches.

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1. Shimer (2005) argues that the job creation process is the one that drives the job changes.
However, although Barlevy demonstrated the reduction in the average quality of jobs during recessions, the proposed dynamics goes against the stylized fact proposed by Davis, Haltiwanger and Schuh that the job destruction process must be the main responsible for the characteristics of the existing jobs in the market. In order to address this issue, this paper has as its principal goal the study of average job quality, bearing in mind the greater importance of the destruction dynamics in the determination of the quality of matches, and that the job destruction process is determined by workers, if the matches are of high quality, and by the firms, if they are of low quality. The proposed model is more closely related to the creative destruction technological progress model of Mortensen and Pissarides (1998), with the introduction of heterogeneity both in job matching quality and in the information set of the worker and the firm. The idea behind this last aspect is to consider that the information sets of the worker and the firm, which are responsible for the individuals’ evaluation of the job quality, do not coincide. We will see that these simple changes are enough to create heterogeneity in the job destruction process and a greater relative importance of the destruction dynamics in basic job aspects, as argued by Davis, Haltiwanger and Schuh. We will also see that for specific parameters and initial conditions, it is possible to create either the sullying or the cleansing effect proposed by Caballero and Hammour (1994) and Barlevy (2002).

The intuition of the model is that, for the group of low quality matches, an increase in the technological progress rate implies both an increase in the job destruction flow as well as a reduction in the job creation dynamics. Taking these two flows into consideration, we will have a reduction in the average job quality. In turn, for the high-quality matches, both effects will imply not a reduction, but an increase, in the average quality. Thus, and considering both previous scenarios, the effect of higher growth rates over the average job quality will depend on the relative dimension of high versus low job quality intervals. However, although the present model is compatible with the results of Caballero and Hammour (1994) and Barlevy (2002), two aspects will differentiate it from those models. First, in the Barlevy model the sullying effect is the result of a high-low quality job creation, while in our model this effect is obtained via job destruction dynamics. Second, in these models the idea of quality is closely related with the notion of job match productivity, while in our model the concept of quality is associated with the idea of non-monetary job attributes. In other words, according to Caballero and Hammour (1994) and Barlevy (2002), an improvement in the average quality of job matches is equivalent to an improvement in the average match productivity, while in our model an improvement in the average quality implies a qualitative average improvement, not necessarily implying any change in the average productivity. In this way, the heterogeneity proposed by the model is equivalent to differences in match attributes, which create value, both for the worker as well as for the firm, without implying any change in the match productivity.

This chapter is organized in the following way. In the next section we will characterize the cleansing and sullying effects. In the following section we will develop a theoretical model, and in the fourth section we will see the main results obtained concerning the effect of changes in technological progress rate over the average job matching quality. In the final section we will see the main conclusions.
2. Cleansing and Sullying Effects

The technological progress dynamics of creative destruction type implies the existence of a constant process of creation and destruction of job matches due to the hypothesis that only the most-recently jobs have the highest level of technology. In other words, the idea behind the *Schumpeterian* growth models is that new job matches are located on the technological frontier. However, the only way of updating an already existing job match is via the destruction of this relationship and formation of a new one.

With this idea in mind, Caballero and Hammour (1994) studied the effects of lower technological progress rates over the average quality of job matches. They found that there are two flows acting simultaneously over the labor market during periods of low growth. On one side, there is an increase in the job destruction dynamics, as the matches with older technology suffer a rise in the probability of becoming unproductive and being destroyed. However, on the other side, there is a reduction in the job creation process, as it is less advantageous for the firms to create new job matches during periods of lower growth.

The main goal of Caballero and Hammour was to determine which of these effects prevailed during less favorable periods of economic growth. The basic result obtained was that during periods of low growth there is a *cleansing effect* acting over the labor market which created an average improvement in the quality of job matches. In other words, the authors noted that during low-growth periods the job destruction flow exceeds the job creation, creating an average rise in the quality of the ongoing job matches.

The authors also found that it was to be expected that an efficient economy would concentrate all its creation and destruction processes in these periods, since the opportunity cost of job match restructuring would be more beneficial in less-favorable periods. Similar results were also obtained by Mortensen and Pissarides (1994), Gomes, Greenwood and Rebelo (2001), Hall (1991) and Hall (2000).

In another scenario, Mortensen and Pissarides (1994) considered both the effects of aggregate and specific shocks over the dynamics of creation and destruction of jobs. The authors detected a negative correlation between job creation and job destruction processes during periods of recession, and that the destruction flow appeared to have a more volatile dynamic than the job creation flow, a result closely related to those obtained by Caballero and Hammour.

In still another model, Gomes, Greenwood and Rebelo (2001) found that when explicitly introducing a rule of acceptance and rejection of a job match, together with the existence of incomplete markets, in a model that considers aggregate and specific shocks, the result would be an improvement in the average quality of matches, similar to the findings of Caballero and Hammour.

In turn, Hall (1991, 2000) also approached the effect of low growth rates in a creative destruction scenario on job matches, obtaining an increase in the unemployment rate followed by an improvement in the average quality of job matches.

However, although all these authors have modeled that the consequences of a recession over the average quality of job matches would be negative, these results were not empirically supported, as Bowlus (1995), Davis, Haltiwanger and Schuh (1996) and Fernandez (2004) demonstrated. According to these authors, the average quality of matches is not negatively, but rather positively related with economic growth rates. Thus,
we can expect deterioration in the average quality of jobs during periods of recession and an improvement in their quality during good periods. Based on these empirical results, Barlevy (2002) showed that the simple introduction of on-the-job search in a job matching model would be enough to obtain results different from those defended by Caballero and Hammour, although more compatible with the empirical evidence. Barlevy argued that during a recession there were two processes acting simultaneously in the labor market. On one hand, there was an increase in the destruction of less-efficient jobs, which gave way to the *cleansing effect* proposed by Caballero and Hammour. On the other hand, there was a process of low-quality job creation that gave way to an additional effect over the labor market that Barlevy denominated *sullying effect*. In other words, the idea defended by Barlevy was that during a recession there were two effects acting in opposing directions in the labor market. On one side of the market, via destruction dynamics, there was an elimination of the less-efficient job matches, while on the other side, there was a process of creation of less-efficient job matches and more workers were stuck in mediocre matches. In this way, the final result would depend on the magnitude of these two fluxes. If the low-quality job creation dynamics exceed the destruction process we will have a reduction in the average quality of job matches. The model of Gomes, Greenwood and Rebelo’s model argues for a *cleansing effect* during periods of recessions, they maintain that the model is also compatible with the *sullying effect* proposed by Barlevy. According to them, this effect occurs because during recessions the worker obtains reduction in his rent, which in turn implies an increase in the probability that the worker accepts job offers with lower wages, creating job matches with lower quality.

### 3. Theoretical Model

In this section we will present the characteristics of our theoretical model. The modeled scenario is founded on Mortensen and Pissarides (1998) technological progress model with the inclusion of heterogeneity, both in match quality and the information set of the worker and firm in the evaluation of the expected quality of jobs.

**Agents and Preferences.** The economy is formed by a constant population of workers, who live infinitely, and a great number of firms that together give rise to the production activity. Firms and workers are heterogeneous, neutral to risk and maximize the discounted expected present value of rents, subject to the available information. Suppose transferable utility, the existence of a perfect capital market and that the worker and firm discount the future at an exogenous and constant rate $r$. Each firm has only one job position, which can be occupied or not, and employs only one worker at each moment. In turn, each worker can be employed in only one job per period or unemployed.

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**Search and Production.** Before the beginning of production, firms and workers undertake an individual, independent and expensive process of search. Let $P(t)c$ be the search cost for the firms, where $P(t) = e^{g}t$ represents a common factor of growth.

Assume that there is no on-the-job search and that the quantity of job matches formed per period is given by function $m(v, u)$ non-negative, concave, homogeneous of degree one and increasing in its two arguments, where $v$ represents the vacancy rate and $u$ the quantity of unemployed workers in the economy, per period.

The unemployed workers move to an employment situation according to a rate $q(\theta)$, where $\theta$ is the ratio of $v$ and $u$, while a job vacancy becomes occupied at a rate $q(\theta)$.

Before production begins, firms fix their technology at the cost $P(t)K$. Assume that it is irreversible and is in the technological frontier at the moment that the match is formed⑥.

Note that with this technological progress format, since a job match, once created, cannot be updated, its production will remain constant at its initial level, $\tau$.

Suppose that the occupied jobs can be destroyed due to an idiosyncratic shock that follows a Poisson Process with arrival rate $\lambda$, which aims to capture, for example, any negative exogenous events that affect the productivity of a particular job matching.

Observe from the two previous conditions that a particular job match can be destroyed due to an aggregate shock, represented by larger growth rates that affect the obsolescence rate of all existing job matches, or due to a specific shock, which acts individually over a particular job match.

Every occupied match has two components that affect its value for the firm. The first one is given by $P(\tau)x$, and represents the production generated by a job match formed in $\tau$.

The second one is represented by the term $\alpha$ and aims to capture differences in quality of the job matches created at the same moment $\tau$⑦.

Assume that $\alpha$ is time invariant and uniquely determined at the moment that the match is formed, in an exogenous and independent way, from a particular distribution function. Suppose also that it is an experience good, that is, it can only be known after the beginning of production⑧, and that the firm and the worker have different information sets⑨ in the evaluation of the expected value of $\alpha$.

If, apart from the previous conditions, we consider that at any time both firms and workers can terminate the ongoing job match, that is, they can break up the partnership and return to the search process in order to find a new partner, we have that jobs created at each moment will have distinct obsolescence periods, according to their quality and the technological progress rate.

⑥ See Mortensen and Pissarides (1998) and Pissarides (2000) for a different formulation of the technological progress.

⑦ We will consider that, although a higher matching quality does not affect the job productivity, it affects the effective discount rate. In other words, we will consider that the higher the job quality, the less the value of the match will be discounted, as workers and firms prefer a good job in relation to a lower quality one.

⑧ Note that the condition that $\alpha$ is an experience good implies that all meetings will give rise to job matches, since there is no rejection before production starts.

⑨ The idea of this condition is to consider that workers and firms do not valuate the expected value of $\alpha$ in the same way.
Note that the idea behind the previous condition on $\alpha$ is that before the formation of a job match two workers (firms) are seen in an identical way by the firms (workers) and only after the production starts does the quality of the job match start to be revealed. Suppose, in order to facilitate the presentation of the main results of the model, that $\alpha$ is defined in the interval:

$$\left(0, \frac{r}{1-\lambda}\right).$$

Wages. If a productive match is destroyed by the worker, by the firm or even in an exogenous way, both the worker and the firm have to pay the costs related to the return to the search process. In this way, a productive match generates a surplus that has to be distributed among the two parties. Let us suppose that this division is determined from the Generalized Nash Bargain Process between the firm and the worker, where $\beta$ represents the workers’ bargaining power$^{10}$.

Equilibrium. Let $V(t)$ represent the present discounted value for a firm of the expected gains from a job vacancy created at $t$, $J(\tau,t)$ the present discounted value for a firm of the expected gains associated with a job created at $\tau$, with quality $\alpha$ and occupied in period $t$, $U(t)$ the present discounted value for the worker of the expected gains associated with the unemployment, and $W(\tau,t)$ the present discounted value for the worker of the expected gains associated with a job created at $\tau$, with quality $\alpha$, in period $t$. Thus we have:

$$rV(t) = -P(t)e + q(\theta)[(1 + \alpha')J(t,t) - V(t) - P(t)K] + V(t);$$

$$rJ(\tau,t) = P(\tau)x - w(\tau,t) + \alpha J(\tau,t) - 1\alphaJ(\tau,t) - V(t)] + J(\tau,t);$$

$$rU(t) = P(t)b + q(\theta)[(1 + \alpha')W(t,t) - U(t)] + U(t);$$

$$rW(\tau,t) = w(\tau,t) + \alpha W(\tau,t) - \lambda[(1 + \alpha)W(\tau,t) - U(t)] + W(\tau,t);$$

where $\alpha'$ represents the expected quality of the job match for the firm$^{11}$; $w(\tau,t)$ the worker’s wage rate, at moment $t$, in a match formed at $\tau$; $P(t)b$ is the job opportunity cost for the worker, while the last terms represent, respectively, time variations in $V(t)$, $J(\tau,t)$, $U(t)$ and $W(\tau,t)$.

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$^{10}$ See Mortensen and Pissarides (1999b) for alternative formulations of the wage determination.

$^{11}$ Suppose that $\alpha' = \alpha$ and $\alpha' = \alpha - \xi$, where $\xi$ is a small term that represents the deviations in the worker’s expectations in relation to the firm’s expectations.
Note that with these value function specifications the job quality component enters in a proportional manner in \( W(\tau,t) \) and \( J(\tau,t) \). Thus, the term associated with the job quality depends on \( \alpha \) and on the present discounted value of the expected gains associated with an occupied job for the worker and for the firm, \( W(\tau,t) \) and \( J(\tau,t) \).

The main reason behind this formulation is to consider that the effects created by the quality of a match directly affect the workers’ and firms’ valuations of the job matching. In other words, we are considering that the higher the job matching quality, the less it will be discounted, since it is preferred to a lower quality match.

Also note, from expression (2), that when a vacancy is occupied it will generate a gain given by the difference between the value of a vacancy and the value of an occupied job, plus the gains associated with the expected quality of the match to be formed.

In turn, we can verify from expression (4) that once the worker is employed, he will have a gain given by the difference between the value of employment and the value of unemployment, plus the gains associated with the expected quality of the match to be formed.

Continuing the equilibrium characterization, we will assume that \( r + \lambda - \alpha (1 - \lambda) > g \) and \( b < x \), in a way that the effective discount rate is positive and there are strictly positive advantages in the formation of a job match, as results from job search theory.

Considering the usual hypothesis of free entry, we have from (2), that

\[
(6) \quad (1 + \alpha)^e J(t,t) = \left[ \frac{P(t) c}{q(t)} + P(t) K \right],
\]

where we can see that the expected value of a new job match is equal to the sum of the cost of occupying a vacancy, expressed in terms of the rate at which this vacancy is occupied, and the cost to create this new vacancy.

Now, as the surplus generated by the matching is divided according to the Generalized Nash Bargaining Solution, \( w(\tau,t) \) satisfies:

\[
(7) \quad \beta [J(t,t) - V(t)] = (1 - \beta) [W(t,t) - U(t)].
\]

Using expressions (3) - (7), and assuming \( \alpha^e_w = \alpha^e_w = \alpha^e \), the wage rate at moment \( t \), in a match formed at \( \tau \), is given by:

\[
(8) \quad w(\tau,t) = \beta P(\tau) x + (1 - \beta) P(t) \delta(\theta(t)) \omega(\theta(t)),
\]

where \( \delta(\theta(t)) \) and \( \omega(\theta(t)) \) are given by\(^{12}\):

\[^{12}\text{Suppose, to simplify the presentation of the basic results of the model, that the terms defined in expression (9) are always positive.}\]
We can observe from the previous expressions that the wage rate is formed by two terms. The first is associated with the worker’s productivity, and the second is related to his outside options.

We can also observe from the previous expression that the greater the technological progress rate, the greater will be the worker’s wage rate, due to the growth verified in the term related to his outside option, and that the greater $\alpha$ is, the less will be the wage growth, for a given technological progress rate.

Note also that the greater $\alpha$ and $\theta$, the greater will be the wage growth, for a given technological progress rate. The reason for the first effect is that as $\alpha$ increases, the greater will be the expected quality obtained in a new job match, giving way to an increase in the worker’s wage rate in his current job. In turn, the greater $\theta$ is, the greater will be the ratio of vacant jobs to unemployed workers, which implies greater wage rates, due to the congestion effect.

Another interesting aspect that we observe is that the worker’s outside option increases at the technological progress rate, while the wage rate increases at a lower rate. As a consequence, there will be a moment $T^W$ at which the worker will terminate his current match, as his outside option becomes greater than his wage.

In turn, as the worker’s wage grows at a rate proportional to the technological progress rate, while the productivity of the job remains constant, there will be a moment $T^F$ where the worker’s productivity is lower than his wage, implying the destruction of the job match by the firm.

Thus, $T^W$ and $T^F$ represent the moments at which the worker and the firm destroy the job match, respectively.

Using expressions (3) to (5), together with equation (8) and conditions $\alpha^r = \alpha^e$ and $\alpha^w = \alpha^e - \xi$, we have that $T^F$ and $T^W$ are determined by:

$$J(\tau, t) = \max_{\tau^F} \left\{ \int_t^{\tau^F} \left[ (1-\beta) P(\tau) x - (1-\beta) P(s) \delta(\theta(s)) \omega(\theta(s)) \right] e^{-(r+\delta)(t-s)} ds \right\}$$

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13 The idea behind the congestion effect is that as $\theta$ grows, the probability of a firm occupying a vacancy decreases, while the probability on an unemployed worker finding a vacancy increases.

14 The term $\delta_o$ is identical to $\delta$, except that the denominator is $r - \theta q(\theta)(\alpha^e - \xi)$. 
\[
\tau = t
\]

We can observe from the previous expressions that \( T^F \) and \( T^W \) coincide, respectively, with the moment where the value of an occupied job is zero for the firm, and the moment where the value of the employment equals the value of unemployment for the worker.

Considering \( \tau = t \) in the previous expressions, the first order conditions of the previous optimization problems are, respectively:\(^{15}\):

\[
\begin{align*}
(14) & \quad (1 - \beta) x - (1 - \beta) \omega e^{\kappa r} = 0, \\
(15) & \quad \beta x - \omega e^{\kappa r} \left[ \varphi \delta - (1 - \beta) \delta \right] = 0,
\end{align*}
\]

where \( \delta \), \( \omega \) and \( \varphi \) are given by expressions (9), (10) and (13) respectively.

Note, from the two previous expressions, that the optimal moments \( T^W \) and \( T^F \) diminish with higher values of \( \theta \). The reason for this behavior is that the larger \( \theta \) is, the higher will be the worker’s wage, implying a reduction in \( T^F \). In turn, the greater \( \theta \) is, the higher will be the worker’s outside option, which implies a lower \( T^W \).

We can also observe that \( T^W \) and \( T^F \) decrease with higher technological progress rates and \( T^W \) increases with higher positive values of \( \xi \).

Finally, we can observe from these expressions that \( T^W \) diminishes with greater \( \alpha \), while \( T^F \) increases. The reason for this behavior is that the higher \( \alpha \) is, the lower will be wage rate growth, if compared with the increase seen in the worker’s outside option, which will make the obsolescence of the job match faster for the worker. On the contrary, greater \( \alpha \) makes the obsolescence of the job match slower for the firm, as we can see in the figures below.

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\(^{15}\) Note that, in order to have strictly positive \( T^F \) and \( T^W \), \( x > \delta \), \( \omega > 0 \) and \( \beta x > (1 - \beta) \delta > 0 \).
Note from the above figures that the greater $\alpha$ is, the higher will be the present discounted value of the expected gain given by the job match to the worker and to the firm. However, as time goes on, the higher quality job will more quickly lose its value for the worker, and more slowly lose its value for the firm, if compared with a job of low quality.

**Proposition 1** Let $\alpha_i$ be the value of $\alpha$ that makes $T^F = T^W$. Thus, when:

(a) $\alpha \leq \alpha_i$, the job match will exist, at most, until $T^F$;
(b) $\alpha \geq \alpha_i$, the job match will exist, at most, until $T^W$.

**Proof.** We need only to equal $T^F$ and $T^W$ to obtain an expression for $\alpha_i$.

The idea behind proposition 3.1 is the following. If $\alpha$ is given by $\alpha_i$, we will have $T^F = T^W$. However, as $\alpha$ is lower than $\alpha_i$, we will have $T^F < T^W$ and, in this case, the firm will be the agent to destroy the job match. In turn, if $\alpha$ is greater than $\alpha_i$, we will
have the opposite scenario, that is, we will have $T^F > T^W$ and, in this case, the worker will be the agent to destroy the job match. In other words, for different values of $\alpha$ we will have different agents initiating the destruction dynamics. If $\alpha$ is lower than $\alpha_i$ the expression that determines $T^W$ loses its importance in the equilibrium characterization, since the destruction of the job match will be $T^F$, a moment that happens earlier than $T^W$. In turn, if $\alpha$ is above $\alpha_i$ the expression that determines $T^F$ loses its importance in the equilibrium characterization, since the moment of destruction will be $T^W$, a moment that happens earlier than $T^F$, as we can see in the figure below.

**FIGURE 3.3**

*Job Destruction*

$T^F < T^W$

$\alpha < \alpha_i$

$\alpha > \alpha_i$

Firm destroys the Job match

Worker destroys the Job match

Continuing with the equilibrium characterization, when considering $\tau = t$, $J(t,t) = P(t)J$ and the optimum $T^F$ in (11), we have that

(16) \[ J = (1-\beta) \int_0^{T^F} \left[ x - \delta_0 e^{gr} \right] e^{-[r+i-\alpha(1-\delta)]t} \, dt \]

represents the value of a job created at moment $t$ and which will survive, if there is no exogenous destruction, up to moment $T^F$.

Using $J(t,t) = P(t)J$ in (6), we have that (16) and

(17) \[ J = \frac{1}{(1+\alpha^e)} \left[ \frac{c}{q(\theta)} + K \right] \]

determine the job creation dynamics.

These expressions tell us that the expected gains to the firm from a new job match must equal the expected costs associated with the creation of a new vacancy.
Note that, for given $\alpha$, the expressions (14) – (17) determine the equilibrium in the $T–\theta$ space.\footnote{See Pissarides (2000) for details.}

Having characterized the job creation and destruction dynamics, we must now obtain the expression that determines the equilibrium unemployment rate.

The job creation flow, at moment $t$, is given by:

$$(18) \quad JC = \theta q(\theta) u(t),$$

where $u(t)$ represents the unemployment rate, while the job destruction flow, at moment $t$, is given by:

$$(19) \quad JD = \lambda \left[1 - u(t)\right] + e^{-\lambda T} JC; \quad T = T^F, T^W,$$

where the term $\lambda \left[1 - u(t)\right]$ represents the job destruction flow which comes from the exogenous process and $e^{-\lambda T}$ measures the fraction of the job creation flow that survives up to the destruction moments $T^F$ or $T^W$, in accordance with the quality of the job match. Therefore, from the two last expressions, the equilibrium unemployment rate is given by:

$$(20) \quad u = \frac{\lambda}{\lambda + \theta q(\theta) \left[1 - e^{-\lambda T}\right]}, \quad T = T^F, T^W.$$

Observe from this expression that the further $\alpha$ distances itself from $\alpha_i$, above or below, the quicker will be the job match destruction as $T^F$ or $T^W$ diminishes, in accordance with the direction that $\alpha$ moves, leading to an increase in the equilibrium unemployment rate.\footnote{Auer, Efendioglu and Leschke (2004) also found this empirical relationship.} Thus, as the quality of the match distances itself from the quality $\alpha_i$, even in the direction of better qualities, the result will be an increase in the unemployment rate. The basic reason for a higher job destruction rate in better qualities job matches is due to the fact that these types of jobs are less sensitive to what happens in the market. In this way, although a good quality job matching is preferable to a lower quality one, these jobs will be quickly destroyed, since the workers wage rate in these jobs do not follow the average increases in the wage rate.

**Extension.** Now we will see that it is possible to enrich the previous model in order to obtain more interesting results, according to the job match characteristics.

As the model developed up to now does not consider the reaction of an agent when the other part decides to destroy the job match, we will introduce the idea that the results of such an action can be adverse enough that the agent affected by the destruction interferes in the process, avoiding the destruction of the job match.\footnote{Note that we are assuming with this scenario that there is no rigidity in downward wage movements.}
Now, observe that in order to analyze the best response of an agent to the destruction decision of the other, we need to compare the value that this agent receives if such a decision were not taken by the other one.

Let us consider in this section that \( \varepsilon \) is positive. Now, as \( \varepsilon \) is the term that makes \( \varepsilon w \neq \varepsilon f \) and, when considered in the job destruction dynamics, makes \( T^f \neq T^w \), considering it to be positive implies that the only case to analyze is \( T^w < T^w \). In this way, following this condition, at the moment \( T^f \) (at which the firm decides to destroy the job match) the value of the employment is given by:

\[
(22) \quad J(\tau, T^f) = V(T^f) = 0.
\]

**Proposition 2** Suppose that \( T^f < T^w \). Thus, whenever the job match quality is given by \( \alpha_m \), that satisfies

\[
\alpha_m < \frac{r}{1 - \lambda} \frac{P(T^f) \delta \omega}{P(T^f) \delta \omega + (1 - \lambda) W(T^f, T^f)},
\]

then it is more advantageous for the worker to maintain the current job match.

**Proof.** The value of the unemployment option for the worker at moment \( T^f \) is

\[
U(T^f) = \frac{P(T^f) \delta \omega}{r - \alpha(1 - \lambda)}.
\]

Thus, whenever \( W(\tau, T^f) > U(T^f) \) it is advantageous for the worker to maintain the present job match. Manipulating these last two expressions, we arrive at the expression that \( \alpha_m \) must satisfy. Now, if we consider that it is possible to isolate the best jobs within the group of poor-quality job matches, and that \( rW(\tau, T^f) > P(T^f) \delta \omega > 0 \), we have that \( \alpha_m \) is defined in interval (1).

The intuition of the preceding proposition is the following. If, at the moment that the firm decides to destroy the job match, the value of the job for the worker is higher than his outside option, then the worker will react to the job destruction, by accepting lower wages. Now, as the value of the job at moment \( T^f \) is higher than his outside option, we can expect that the worker will accept a wage rate reduction in order to maintain his current job match. Thus, whenever the job match is defined in the previous interval, the job will not be destroyed at \( T^f \).

Note also that the higher \( \varepsilon \) is, the more probable it is that the worker will react to the job destruction. Also note that the worker will preserve the matches with lower \( \alpha \) because they are, at this moment, better than a high quality one – although they do not have good quality, they pay high wages. Therefore, if the job match belongs to the lowest quality subgroup among those of lower quality, it will not be destroyed at \( T^f \), but will last until the moment \( T^w \). In turn, if the
job has quality between $\alpha_m$ and $\alpha_i$, the job will be destroyed at $T^F$, as we can see in the figure below.

4. Technological Progress and Average Job Matching Quality

In this section we will examine the effects of technological progress rate changes over the average quality of the job matches. Assume initially that the economy is completely characterized by job matches defined in the interval of quality $\alpha_m \leq \alpha < \alpha_i$. In this case, we have that $T$ is determined by expression (14) and we can observe that as $g$ grows $T^F$ falls, or in other words, as the technological progress rate grows, the quicker it is that firms will destroy the existing job matches. The reason for this behavior is that the greater $g$ is, the higher will be the wage growth, and as the job has low quality, the quicker it will become obsolete, leading to its destruction.

In turn, regarding the job creation dynamics, as $g$ grows, the less pronounced will be this process, because higher wages will be paid in those new jobs. In this way, we will have as a final result an increase in the job destruction and a reduction in the job creation dynamics. Now, as the destruction occurs from the worst to the best job matches, the worst quality jobs will be the first to be destroyed, leading to an increase in the average quality of the existing matches in the interval $\alpha_m \leq \alpha < \alpha_i$.

Note also that if instead of an increase, there is a decrease in the technological progress rate, the result will be, as before, an increase in the average job match quality. The idea now is that we have a reduction in the job destruction process and an increase in the job creation dynamics. However, the job creation flow would lead to the creation, in an independent manner, of job matches in the previous interval. In this way, we can expect no changes in the job quality coming from the job creation side. Furthermore, as the destruction dynamics follow the same process, that is, the jobs with less quality are the first to be destroyed, we will again have an increase in the average quality of job matches.
Now, consider the job matches defined in the interval \( \alpha < \alpha_M \). In this case we will have \( T \) determined by the expression (15) and as \( g \) grows, \( T^w \) falls, that is, the quicker it is that the worker wishes to destroy the job match.

The reason now is that the wage rate does not respond to every increase in the worker’s outside option, which leads him to destroy his current job match sooner.

In turn, on the job creation side, the results are identical to those previously obtained. Thus, an increase in \( g \) implies an increase in the job destruction flow and a reduction in the job creation dynamics. However, there will be a difference in relation to the earlier results. As the workers destroy the jobs with greater \( \alpha \) followed by the jobs with smaller \( \alpha \), the destruction process will generate opposite results, that is, we will have the job matches being destroyed from the best to the worst ones, leading to a reduction in the average quality of the job matches. Therefore, whenever we consider the jobs defined in the interval \( \alpha < \alpha_M \), the increase in the job destruction will be accompanied by a worsening in the average quality of the matches.

Now observe that if instead of an increase, there were a reduction in the technological progress rate, the result would be a reduction in the average quality of the matches, according to the previous argument. Thus, as before, we would have the job destruction process determining the average quality of the matches, although to a lesser degree.

Finally, for the jobs with quality defined in the interval \( \alpha \geq \alpha_I \), the destruction moment is again given by the expression (15) and the effect of changes in the technological progress over the average match quality are identical to those obtained previously for the jobs defined in the interval \( \alpha < \alpha_M \).

Joining all the previous results, we can see that it is possible to obtain both an improvement and a worsening in the aggregate quality of the matches. The final result will depend on both the distribution of the job matches and the parameters related with the determination of \( \alpha_M \) and \( \alpha_I \).

5. Conclusions

The goal of this chapter was to analyze the behavior of the average quality of job matches, bearing in mind the empirical facts proposed by the literature. Earlier studies defended distinct arguments as to the effect of periods of low growth over the average quality. On one hand, it was argued that these periods were characterized by a cleansing effect, which implied an increase in the average quality of the matches. On the other hand, it was argued that these periods were not characterized by a cleansing effect, but rather by a sullying effect. That is, these periods were moments of reduction in the quality of the matches.

We saw that it is possible to obtain both the cleansing as well the sullying effect, depending on the distribution of the job matches and on the parameters related with \( \alpha_M \) and \( \alpha_I \). The idea advanced was that variations in the technological progress rate create distinct dynamics in the job destruction process, depending on the quality of the job match.

We demonstrated that two simple modifications in the technological progress model of creative destruction type of Mortensen and Pissarides (in order to allow for the existence
of stochastic job matches and differences in perceptions about the average quality of a new job match by the involved agents) was enough to explain the behavior of the average quality of jobs, taking into account the facts that the destruction process must be the main one responsible for this process and that, depending on the job quality, we will have different agents driving the destruction dynamics.

6. References


