What Does It Take to Achieve Equality of Opportunity in Education? An Empirical Investigation Based on Brazilian Data [ℑ]

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Resumo: O trabalho de John Roemer (1998), *Equality of opportunity*, contribuiu decisivamente para a consolidação de uma teoria de justiça que é moderna, conceitualmente clara e facilmente utilizável no desenho de políticas. Inspirado pela teoria de Roemer, este artigo apresenta um exercício simples de simulação de política, combinado a uma análise de microdados. Em um contexto de alocação pura, primeiro analisamos a realocação de gastos educativos que seria necessária para alcançarmos uma situação de igualdade de oportunidades educativas. Usando dados do SAEB-2001, nossos resultados indicam que a implementação de uma política de igualdade de oportunidades entre alunos de diferentes perfis sócio-econômicos requereria que se multiplicasse por 9.2 o gasto médio por aluno praticado atualmente para alunos do perfil sócio-econômico mais desfavorável. Este resultado é obtido, entre outras razões, pela baixa elasticidade do nível de proficiência dos alunos ao nível de gasto por aluno, e pela grande desigualdade educacional brasileira. Em seguida, mostramos que uma redistribuição simultânea de insumos monetários e não-monetários, tais como "qualidade dos colegas" (via de-segregação das escolas) e a "efetividade da escola" (via equalização do acesso às escolas privadas), reduz consideravelmente – em cerca de 40% – a magnitude da redistruibuição financeira necessária. A implementação de uma política de igualdade de oportunidades não traria custos (nem benefícios) em termos de eficiência.

Abstract: Roemer's' 1998 seminal work on equality of opportunity has contributed to the emergence of a theory of justice that is modern, conceptually clear and easy to mobilize in policy design. Inspired by Roemer's theory, this paper is fundamentally a *policy-modeling* exercise coupled to a microdata analysis. In a pure allocation setting, we first analyze the reallocations of educational expenditure required to equalize opportunities (taken to be test scores close to the end of compulsory education). Using Brazilian data, we find that implementing an equal-opportunity policy across pupils of different socio-economic background, by using per-pupil spending as the instrument requires multiplying by 9.2 on average the current level of spending on the lowest achieving pupils. This result is driven by the extremely low elasticity of scores to per-pupil spending. We then show that the simultaneous redistribution of monetary and non-monetary inputs, like peer group quality (ie, de-segregation) and school effectiveness (ie, equalizing access to best-run schools), considerably reduces – by around 40% – the magnitude of financial redistribution needed. Implementing an EOp policy would not come at any particular cost (or benefit) in terms of efficiency.

Key works: Equality of Opportunity, Education, Formula Funding, John Roemer, Quantile Regression.

Palavras-chave: Igualdade de oportunidades, Educação, Financiamento de educação, John Roemer, Regressão Quantílica.

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Introduction

In a nutshell, John Roemer's seminal book on equality of opportunity (1998) defends the view that while some fraction of inequalities of outcome/achievement is determined by morally acceptable factors, another fraction is caused by morally unacceptable factors. Roemer's conception of equity and justice does not rest on gross outcome variables. He prefers instead to choose as relevant attributes *conditional outcome variables*, which somehow take into account the reasons underlying the achievement of a certain outcome. Inequalities caused by morally unacceptable factors (typically circumstances beyond an individual's control like gender, race or socio-economic origin) should give rise to compensations in order to be eliminated. Inequality caused by legitimate factors (effort, autonomous choice etc.), in turn, should not call for compensation. The aim of an equal-opportunity policy is thus to equalize achievements across groups of individuals with similar circumstances. It is not to equalize achievements within these groups.

This simple idea has been discussed and developed in the philosophical literature over the last forty years, and the debate has turned, to a great extent, around where to draw the cut between what fraction of outcome gaps is to be compensated for and what is not. Economists have also contributed to the shaping of conceptions of justice of this kind, but in a different way, usually working at a more abstract level, making use of the mathematic language.

In his celebrated book of 1998, Roemer has, not only translated that widespread conception of justice into a precise mathematical formulation, but he has also provided a simple algorithm ready for policy use. He has labeled his theory with the appealing name of "equality of opportunity" (EOp for short). He has not tried to spell out what is acceptable and what is not. Instead, he has worked with a general and pluralistic demarcation, according to which inequalities due to circumstances – what is out of control of the individual – are considered unacceptable, while inequalities due to choices made by the individual – what is under control of the individual – are acceptable, and that the precise boundary is to be set by each society in the political arena.

The aim of this paper is to apply such a theory to the particular domain of *education policy*. In section 1 we spell the contribution of this paper to the EOp literature and we discuss how Roemer's framework can be applied to education. Section 2 contains a brief presentation of the Brazilian data used. In section 3 we present our empirical strategy. In section 4, we compute the reallocations of educational expenditure required to equalize opportunities. We find that to implement an equal-opportunity policy by using per-pupil spending as the instrument requires multiplying the current level of spending of the lowest achieving type of pupils by at least 9.2. In section 5 we identify ways of reducing financial reallocations needed to achieve equality of opportunity. We show that the simultaneous redistribution of monetary and *non-monetary inputs,* like peer group quality and school effectiveness, considerably reduces the magnitude of financial redistribution required. In section 6, we assess efficiency under alternative allocations. Our results suggest that implementing EOp would not come at any particular cost (or benefit) in terms of efficiency. Section 7 concludes.

1. Equality of opportunity and education policy

Before turning to our subject matter, some words on the originality of this paper and on how it is related to the existing literature. On the one hand, there is a large literature discussing normative issues, mainly at theoretical and conceptual levels, both in welfare economics and in political philosophy. On the other hand, there is another strand of literature which is more policy-oriented, and its main concern is to propose formula funding schemes for the (re)distribution of educational inputs. An important feature of our paper is that it explicitly bridges a link between a particular conception of justice that has been developed within the normative literature – namely, John Roemer's EOp theory – and an application of such theory to the particular domain of education.

There are also some specific features which make of this work an original contribution. First, while there are good reasons to care about income – possibly the "ultimate educational achievement" – such as Betts & Roemer (2004) and Roemer et al. (2003) do, we believe it is also important to focus on intermediate educational achievement. Thus, an original feature of this paper is that we focus on *educational achievements in terms of test scores* as the outcome, and – consequently – on *education policy* as the instrument. A second contribution of this study is our refusal to focus exclusively on financial reallocations of educational resources. Given our knowledge of the education production function, we enlarge the set of policy instruments, investigating how the simultaneous redistribution of monetary and *non-monetary inputs* – like peer group quality and school effectiveness – can reduce the magnitude of financial redistribution needed. Finally, a third important point is that while we follow Roemer's approach in trying to explore some second-best settings (by imposing some restrictions on the extent of the redistribution), we take some distance from Roemer's "compromise solution" to the EOp allocation problem. We do so because of some particular features of the education sector, namely the public-good dimension of educational resources (more on that in section 3.1).

Concretely, how should education policies be specified to equalize opportunity? Although Roemer's framework is relatively easy to mobilize in policy design and can provide guidelines for many real life problems, implementation in the context of education still requires a gradual transposition.

1.1. Circumstances, types and effort

Betts & Roemer (2004) usefully remind us that five keywords constitute the vocabulary of the EOp theory: circumstances, type, effort, objective, and instrument. A *type* is the set of individuals with similar circumstances; the *objective* is the condition for which opportunities are to be equalized; and the *instrument* is the policy intervention used to effect that equalization. The equal-opportunity policy is the value of the instrument which makes it the case that an agent's expected value of the objective is a function only of his effort and not of his circumstances. Thus, in order to equalize opportunities for young people to acquire basic (compulsory) education, the schooling system should be organized in such a way that a pupil's score in math, science or reading be a function only of his *effort* and not of his *circumstances*.

The reasoning starts with the observation that pupils will vary in their propensity of attaining some goal (e.g. get a certain score in math), due to *circumstances* – such as their race, or the socio-economic status of their parents. And the bedrock of the EOp is to consider that they should not be held accountable for these circumstances-related achievement differences.

EOp ethics maintains that differences in the degree to which individuals achieve the goal in question that arise from their differential expenditure of *effort* are, morally speaking, perfectly all right. The partition of causes into circumstances and effort (or autonomous choice) is the central move that distinguishes Roemer's EOp ethic from a strictly outcome-egalitarian conception of educational justice. While the latter vision implicitly holds the individual responsible for nothing, EOp emphasizes that an individual has a claim against society for a low outcome only if he expended sufficiently high effort.

Before moving forward, we must make a comment on the nature of the agents we are dealing with in education. The center of our attention in this paper is scores of pupils whose age is typically 14. Following Roemer (1998), we shall divide these pupils into types assuming that all pupils within a type face the same set of circumstances. All the variation of scores of pupils within a given type shall be assumed to be caused by differential personal *effort*, and given that the amount of effort expended is assumed to be a choice made by the individuals, there shall be no compensation for scores inequalities within each type. A question that might be posed is whether it is reasonable to hold pupils accountable for their effort, given that they are not adults, but kids or teenagers. Can we consider them to be fully able to take autonomous and informed decisions? Can they be held totally accountable for important choices they have to make in their schooling years (e.g., allocation of time between leisure vs. studying)? During a large fraction of their school lives, individuals can not be said to be perfect judges for what is good for themselves, kids are possibly "economically myopic", since they are unable to evaluate all the future benefits that are made available if he or she acquires education in the present time, and they make choices according to other, non-monetary, motivations (Akerlof & Kranton, 2002). To sum up, if we push the argument far enough, we could conclude that circumstances account for virtually all the variability of educational outcomes, that is, that all inequality is unacceptable, which would amount to say that Roemer's theory is not necessary in this case – we could trivially conclude that the policy objective must be one which consists of equalizing pupils' scores.

The objection makes sense. Indeed a great part of inequalities in educational outcomes could reasonably be attributed to circumstances. However, it is also true that a fraction of educational achievements can be credited to the pupil itself, at least to those of a certain minimal age. While it is clear that considering a 5 year-old pupil accountable for his efforts is not reasonable, the claim loses strength when we are talking about a 14-15 year-old youngster, who lives, and is being further prepared to live, in societies where people are, at least partially, held accountable for their acts. For his own benefit, he should be prepared to respond for his acts. Acquiring knowledge and skills depends upon natural and social circumstances (talent, quality of family support etc.), but it also requires personal commitment and effort, and these variables can be considered to be under control of the individual to a certain extent.

1.2. Outcomes and instruments employed in the literature

In principle, any outcome variable is compatible with the EOp agenda when applied to education. A possible candidate is individuals' earnings since they reflect, to a certain extent, his well-being.

Bettts & Roemer (2004) have been trying to assess what would have been the necessary redistribution and/or increase of spending-per-pupil (their *instrument*) in the US in order for the income-EOp objective to be achieved across types of individuals (i.e., race and/or socio-economic groups). They conclude that the extent of the necessary redistribution would be quite substantial, especially when circumstances are defined in terms of race, in which case the per-pupil spending ratio between white and black kids would oscillate between 8 (lower bound) and 80 (upper bound). These results are found because of the extremely low income-elasticities to per-pupil-spending obtained in their regressions. When circumstances are defined simultaneously in terms of parents' education and race, the per-pupil spending ratio between the better-off and the worse-off types would be about 14.

A logical variant consists of using another instrument than per-pupil spending, but at a later stage: income taxation and transfers. Roemer et al. (2003) have tried to asses how well fiscal regimes of eleven industrialized countries perform as far as the income-EOp objective is concerned. They find that fiscal regimes of some countries in Northern Europe do very well in terms of income-EOp objective. They conclude by raising an efficiency issue, namely, on whether redistributive taxation is more or less effective than educational policies as an EOp instrument. They are quite skeptical about education as a means for implementing income-EOp.

1.3. Outcome: our choice

In this paper, in turn, neither do we take income to be the relevant outcome variable, nor do we focus on post-schooling instruments (ie, tax-and-transfers). Rather, we focus on *educational achievements in terms of test scores* (score-EOp) as the outcome, and – consequently – on *education policy* as the instrument. Such a choice requires some justification. While there are good reasons to care about earnings – what could be called the "ultimate educational achievement" – such as Betts & Roemer (2004) and Roemer et al. (2003) do, we believe it is also important to focus on intermediate educational achievement such as scores in tests of cognitive ability.

First of all, because there is evidence on the existence of positive links between the performance of students in tests and their future earning capacity (Currie & Duncan, 2001). If this is true, by aiming at equality of opportunity for achieving scores, we would be setting the seeds for achieving income-EOp years later.[1]

Secondly, for efficiency or political feasibility reasons, it may be relevant to focus on the distribution of test scores instead of that of income. For a given society to achieve income-EOp, the two papers cited above show it would be necessary, either to massively change the allocation of school resources, or to redistribute income massively (with well-known disincentive effects). If reshaping the distribution of test scores involves less dramatic reallocations of resources, that line of action may be a good policy instrument that would contribute for the achievement of income-EOp in 10- to 15-years time.

The third reason for focusing on pupils' skills is related to the widely recognized importance of educational achievements. At least since the seminal works of Schultz (1963) and Becker (1964) economists recognize that education has an important economic value. It is a means, or resource, for achieving a wide array of personal goals. Educational achievements can be good predictors of the access to college, of future earnings capacity and of the social position an individual holds. But education is also likely to be positively correlated to outcome variables or "advantages" valued by various theories of justice, and not only within the normative framework usually adopted by economists (i.e. welfarism). Being more educated might enhance the probabilities that an individual scores higher in the distribution of primary goods defined by John Rawls (1971), of functionings (achievements) and capabilities (freedom) defined by Sen (1985), but also of other attributes such as health status, for example (Grossman, 2005). Finally, beyond all the doors education opens, it can also be seen as an end in itself, as an attribute of a "good life" (Sen, 1985). That is, being educated may have an intrinsic value, regardless of the effect it may have (and will have) on other objectives.

1.3. Instruments: our choice

Having agreed on taking test scores as the outcome, we now turn to the issues of which aspects of education policy are relevant for achieving score-EOp. Like Betts & Roemer (2004) we focus on *per-pupil spending*. In this paper we also provide estimates of the required changes in the distribution of spending per pupil securing EOp. However, we argue that it is useful to enlarge the scope of instruments that can be used and not limit ourselves to reallocations of monetary resources.

Our aim is to better exploit the results highlighted by the abundant literature on education production function (Hanushek, 1986 and 1997; Belfield, 2000). Of course per-pupil spending and its components (teacher salary, class-size, capital expenditure etc.) will always be central to education policy design. Yet, we believe that the production function literature largely legitimizes integrating non-monetary inputs to the EOp

Several authors (Haveman & Wolfe, 1984; Monk, 1992; Vandenberghe, 2002) have shown that a pupil's achievement could indeed be influenced by variables with no immediate monetary expression: the pupils themselves and their human capital background. Education is one of those services wherein outputs depend partially on the customers as inputs. In addition, the presence of other customers (as inputs) often contributes to the output 'experienced' by each customer individually (Rothschild & White, 1995). Human capital endowment of pupils and their aggregation – the student body composition – apparently condition the productivity of more classical inputs (teacher-pupil ratios, teacher salary, capital, sport and scientific

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[1] Note that Currie & Duncan's finding (positive correlation between scores and earnings) is not incompatible with that of Betts & Roemer (low income-elasticity of per-pupil spending).

facility…). The point here is that peer quality – due to well-known segregation phenomena – can be unequally distributed and contribute to (in)equality of opportunity.

Several case studies (Monk, 1992), but also nation-wide empirical research (Hanushek, 1986, 1997) and international studies (Vandenberghe & Robin, 2004) also highlight the critical role played by intra-organizational attributes. The technological relation between inputs and outputs is conditional on the presence of organizational assets. These cannot be directly related to the amount of monetary resources made available by the public authority. There is some evidence that in many countries pupils attending privately-run schools benefit from a higher level of organizational effectiveness than those enrolled in public schools. The point, again, is that school effectiveness can be unequally distributed among pupils and contribute to (in)equality of opportunity.

2. Data

2.1. The SAEB dataset

The data we use come from the 2001 wave of SAEB (Basic Education Assessment System), a survey on pupils' achievement carried out by INEP, a research bureau subordinated to the Brazilian Ministry of Education. While the SAEB is not suitable for international comparisons, its objectives and statistical design, and the procedures employed in the application of the test, have been inspired by, and do not differ very much from, well-known cross-country assessments of pupils' performance, such as PISA, TIMSS/PIRLS, and LLECE[2].

SAEB consists of countrywide tests that evaluate pupils' cognitive abilities in Portuguese[3] and Mathematics. Test score information is coupled with data on relevant features of pupils and their family, as well as teachers', principals' and schools' characteristics. The global database consists of repeated cross-sections (not panels) of representative samples of schools and students. Firstly, schools are randomly chosen to take part in the SAEB. Secondly, one or two classes inside each school are randomly selected. All students of a given selected class have to pass the SAEB exam, but only in one of the subjects.

SAEB focuses on the evaluation of pupils at three key stages of their formal education: 4th and 8th year of primary school, and 3rd year of secondary school. Schooling is mandatory in Brazil for children up to 14 years, regardless of the grade they are attending. The 8th grade sample constitutes a good approximation for the end of compulsory schooling, since most of its students are in fact around 14 years old[4]. Moreover, 8th grade pupils are less likely to have dropped out than 3rd grade of secondary school pupils. Finally, the 8th grade datasets have fewer missing data in key questions (e.g., mother's education) as compared to the 4th grade. For these reasons, we focus exclusively on the 8th grade sample.

Pupils' test scores correspond to subject-specific scales elaborated by INEP staff together with teachers, researchers, and national and international survey experts. Possible scores range from 0 to 500, and are supposed to evaluate skills and abilities of students. The SAEB scale is continuous and hierarchical, which means that a pupil who achieves a certain score – say, 400 in the Portuguese test – has all the literacy skills held by students who scored, say, 150, 300 or 380, plus some additional skills. For example, he might be able to understand and interpret more complex texts than his peers who scored

[2] The INEP website (http://www.inep.gov.br) contains useful information concerning evaluation of students in Brazil, most of which in Portuguese, in the section "Avaliação e Exames". INEP (2002) provides specific information about the 2001 wave of SAEB. INEP (2006) provides information in English about the SAEB exam.
[3] Portuguese is the official language in Brazil and it is the native language of nearly all 180 million Brazilians.
[4] In the final samples used in this study, a majority of 8th grade pupils (71%) were 15 or less by the time they did the SAEB exams. However, the range is actually quite wide – maximum pupil age in the sample is 19 – especially because of grade repetition and irregular school attendance.

lower. Because of the invariance of the scale, pupils' scores are comparable across years and across grades. Scores are not comparable across subjects, but the distributions of scores do not look very different in Portuguese and in Mathematics[5].

2.2. Choice of variables

This data set contains information about teachers' gross monthly wages expressed in "salários mínimos" (*SM*), an index frequently used in Brazilian administrative data[6], as well as the number of pupils in the classroom where the test was implemented (i.e., the teacher-to-pupils ratio). For each pupil, we computed a variable dividing teacher's wage by the number of pupils in the classroom. This variable, hereafter labeled x, gives a reasonable proxy of per-pupil spending at the classroom level; expressed in units of *SM* per pupil.

We are well aware that in an educational production process the input set is inherently multi-dimensional, even if we restrict ourselves to resources which can be directly expressed in monetary units. Besides teacher spending, non-teacher spending (school infra-structure, equipment, material etc.) might have some impact on student performance. However, in this respect we were limited by our data, since it was not possible to build an acceptable variable expressing non-teacher spending.[7] Moreover, even if it were possible to employ a non-teacher spending variable, we believe it would be largely correlated to teacher spending.

Ideally, a *type* should be defined as a set of individuals facing the same circumstances. In practice, however, it is impossible to define types so perfectly, and so we have to turn to some proxy which allows us to define types as sets of individuals facing similar circumstances. The SAEB data set contains a series of socio-economic variables, one of them being the highest degree obtained by the pupil's mother. We assume here that such variable is a good proxy for pupils' circumstances, since it is known to be highly correlated with a number of past, current and future advantages an individual faces. So the highest degree obtained by the pupil's mother is the variable we choose to define pupil's type (t). It should be noted that such definition of type is quite modest in terms of EOp objectives, because, by assuming mother's degree to express circumstances, we implicitly hold pupils accountable for many other important characteristics such as father's education, parents' income, pupils' genetic endowment of talent, his or her geographical location, and so on.

We also use the highest degree obtained by the pupil's mother to compute a proxy of the quality of the peers from which the pupil might benefit/suffer. The variable *PEER* is thus computed as a simple average of the highest degree obtained by classmates' mothers.

Finally, SAEB tells us about the public vs. private nature of the school attended. A public school is a school managed directly by a public authority (the state or the municipality). A private school is a school managed directly by a non-government organization (e.g. a church, business or any other private institution). In brief, the underlying classification is not that of the origin of financial resources, but the legal status of the board. When that variable is used as a dummy (*PRIV*) in a regression, it can help us quantify the importance of the school effectiveness as an input, and assess its potential role in achieving EOp. We make the assumption here that private schools are more efficient than public schools. Descriptive statistics are reported in Table 1.

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[5] SAEB scales have been built in such a way that the mean and the dispersion were identical across subjects, for the 8th grade, in the 1997 wave. Averages were set to 250, and standard deviations were set to 50.

[6] Literally it means "minimum wage", but actually more than defining the actual value of Brazilian wages, it is used as an economic index. In October 2001, when SAEB tests took place, one unit of *SM* was worth 68 US dollars.

[7] All we dispose of are self-reported, subjective, replies to categorical questions on the "general quality" of equipment, buildings and so on (from "very bad" to "very good"). Moreover, no possible translation of these questions into monetary units was possible.

Table 1 – Descriptive statistics

Mother's highest degree (which defines *types*)	Observations	Pct.	Average score in Portuguese test (*S*)	Average per-pupil spending (*X*)	Average index of peer quality (*PEER*)	Probability of attending a private school (*PRIV*)
All	45,030	100.00	246.45	0.153	3.25	0.36
Not studied 1	3,054	6.78	212.92	0.121	2.54	0.06
Primary education 2	11,652	25.88	226.57	0.131	2.67	0.08
Upper primary 3	9,950	22.10	236.11	0.143	3.00	0.20
High school 4	11,741	26.07	258.65	0.158	3.55	0.52
Higher education 5	8,633	19.17	280.47	0.200	4.15	0.82

We notice that the average score (*S*) varies considerably from type to type. The amounts of resources, both monetary (*X*) and non-monetary (*PEER* and *PRIV*), which are available for each type are also very different. The higher the education level of pupils' mothers, the higher are the level of pupils' educational inputs and output.

3. The empirical strategy

In the first chapters of Roemer (1998), the EOp objective function is defined as a continuous maximin across-types. However, in applications to specific problems, such as the one regarding education finance in the US (Roemer & Betts, 2004)[8], the EOp objective function which is actually employed is a discrete version of the original one. We believe the latter approach is appropriate for our purposes, since while the discrete version is computationally less demanding, it is not in conflict with the original spelling out of the EOp theory.

3.1. The EOp algorithm

Following the strategy of Betts & Roemer (2004), but defining the outcome variable as Portuguese test scores (*s*) at the age of 14, we first compute the reallocation of *spending per pupil* (*x*) that would be necessary to equalize opportunities. We consider reallocations of spending per pupil across types of pupils, given a *fixed educational budget per pupil* (*r*).

Similar circumstances are used to partition student data into types (*t*). In this paper, we define only five types using information on *the highest education degree obtained by the pupil's mother*. The idea of effort, in Roemer's framework, is captured by the rank of the student in the within-type conditional distribution of effort. In statistical terms, this rank (and thus the level of effort) can be adequately captured by the quantile, *q*, of the type-specific distribution of score.

Let the education production function connecting resources to (the natural log of) scores be of the form:

$$ln\, s^t_i = \alpha^{tq} + \beta^{tq} \cdot x^t_i + Z^t_i \cdot \gamma^{tq} + \varepsilon^t_i \qquad [1]$$

where:

- i: indexes the observation (i=1, …,n),
- t: indexes pupil's type (t=1,…,T),
- q: indexes within-type score quantile to which the pupil belongs (q=1,…,Q),
- s : score in Portuguese test,
- x : per-pupil spending,
- Z : a vector of control variables,
- ε : error term,
- α, β, γ: coefficients.

[8] A preliminary version of Roemer & Betts (2004) had already been presented in chapter 11 of Roemer (1998).

Then, the core of the EOp allocation problem consists of identifying, for each quantile, *q,* the vector $X^q = (x^{t=1;q}, x^{t=2;q}, \ldots, x^{t=T;q})$ that equalizes (expected) scores across types, subject to the following budget constraint[9]:

$$r = \sum_t (p^t . x^t) \qquad [2]$$

where:

r: the average per-pupil spending

p^t: the share of type t pupils in total population

Finally, it is worth emphasizing that the EOp algorithm as we exposed it so far amounts to maximizing several objectives simultaneously (one per quantile). To come closer to a solution that is practically feasible, some second-best approach must be taken. Roemer suggests a "compromise" which consists of *taking an average*. For example, suppose, as we do in this paper, we work by quartiles of score in each type. Roemer would then first compute, for each quartile the investment policy that equalizes (expected) scores, across the various types, which would give 3 different policies. And then he would declare the EOp policy to be the average of these 3 policies. By adopting such a compromise, Roemer does not assume the instrument can be perfectly allocated according to the quantile, but he assumes it can be perfectly allocated according to the type.

In this paper, we compute Roemer's compromise (reported in Tables 3-5). However, we also report the allocations of the instrument that would secure score-EOp *for each type and quantile* (also in Tables 3-5). While Roemer's compromise may look quite convincing in other contexts, we believe it is less so in the education domain, given the inescapable *public-good dimension* of educational resources in the education production process. Since it is a difficult task to perfectly reallocate the instrument according to type-quantile, we emphasize here the scores distributions which would be the consequence of an ideal, or theoretical, reallocation of the educational resources (ie, ignoring implementation obstacles). We believe it is a worthwhile exercise since it will reveal the *minimal input-reallocation requirements* if an EOp outcome is to be achieved.

3.2. Quantile regressions

Roemer (1998) emphasizes that, as the distribution of effort of a type is a characteristic of the type and not of any individual, it is a circumstance for a particular individual. For example, if an individual's effort is low in *absolute* terms because he belongs to a type whose mean effort is low, this individual should not be held accountable for that. He claims that we should turn our attention to *relative* levels of effort within given types, and that relative effort is best captured by the rank of an individual in the effort distribution of his type, or at least by the quantile to which he belongs in such distribution.

With regards to an empirical application of Roemer's theory to education, we have to bear in mind that the impact of school spending on score for a given type of pupil may vary with the pupil's effort (i.e., the quantile to which he belongs) in the score distribution. Quantile regressions (Koenker & Bassett, 1978), estimated separately for each type of pupils, constitute the most appropriate technique for the application of the EOp theory. The set of coefficients obtained from quantile regressions performed for each type of pupils allows for non-linearities in the relation between score and spending per pupil.

We have estimated equation [1] three times (q=0.25, 0.5, 075) for each of our five types of pupils (t=1,…,5). Vector Z includes current level of peer quality (*PEER*) and private school dummy (*PRIV*).

[9] From this formulation of the problem, it should be clear that we work here in a pure allocation setting. We ignore both potential changes over time, and incentive effects.

Table 2 – Estimates of impact of per-pupil-spending (*x*) on scores (*s*), by type and quantile.

Dependent variable: ln score	Types 1	2	3	4	5
	Panel A: q = 0.25				
X	0.134**	0.170***	0.147***	0.135***	0.120***
PEER	0.036	0.041***	0.047***	0.054***	0.037**
PRIV	0.082***	0.081***	0.099***	0.094***	0.110***
Constant	4.993***	5.045***	5.003***	5.052***	5.021***
	Panel B: q = 0.50				
X	0.097	0.125***	0.143***	0.144***	0.109***
PEER	0.065***	0.045***	0.054***	0.058***	0.049***
PRIV	0.084***	0.073***	0.080***	0.073***	0.084***
Constant	5.135***	5.208***	5.199***	5.250***	5.235***
	Panel C: q = 0.75				
X	0.067	0.119***	0.145***	0.136***	0.088***
PEER	0.038**	0.024***	0.043***	0.051***	0.046***
PRIV	0.070***	0.076***	0.071***	0.058***	0.055***
Constant	5.299***	5.325***	5.340***	5.408***	5.453***

Statistically significant at the: (*) 10% level; (**) 5% level; (***) 1% level.

We can observe in Table 2 that all coefficients are positive, and that almost all of them are statistically significant. Coefficients vary both across quantiles (for a given type) and across types (for a given quantile).

Of course it would be possible to enlarge the set of regressors included in *Z*. However, we must bear in mind that the main objective of this paper is not to assess the contribution of each input to the performance of students.[10] We are rather interested in a policy-modeling exercise, that is, in revealing the reallocations of per-pupil-spending (*x*) which would be necessary in order to achieve score-EOp. Adding regressors to our parsimonious model would most probably reduce the magnitude of the estimated coefficients (βs) and thus increase the necessary reallocations of per-pupil-spending. Our results provide then useful lower bounds for EOp reallocations.

3.3. From quantile regression coefficients to the EOp allocation

Using the set of estimated coefficients and rewriting equation [1] as:

$$ln\ s^t\,(q, x^t) = A^{tq} + b^{tq}\cdot x^t \qquad [3]$$

with:

$$A^{tq} = a^q + c^{tq}PEER^t + d^{tq}PRIV^t \qquad [4]$$

we get 5 linear functions of x^t providing the expected score for each type of pupil. Exploiting the idea that EOp basically means equalizing expected score across types, and using the budget constraint, we develop a system of *T+1* equations, which is resolved incrementally:

- X^{Tq} as a function of *r* and $X^{T-1;\,q},\ldots,X^{1;q}$
- $X^{T-1;q}$ as a function of *r* and $X^{T-2;q},\ldots,X^{1;q}$
- ...
- $X^{1;q}$ as a function of *r* and the set of known (*p*) or estimated parameters *(A(α, c, d), b)*

[10] Those interested in this topic should refer to the abundant literature on education production function (e.g., Pritchett and Filmer (1999), or Vignoles et alli. (2000).)

4. Per-pupil spending and score-EOp: results

4.1. Current situation and two benchmark policies

In Table 3, panel A, we report the current level of per-pupil spending for each type, and the current distribution of scores for each type and quartile. In Table 3 we can also observe the allocation of inputs and distributions of scores obtained as a result of two simulations of redistributions of inputs: (i) Panel B: assuming an input-egalitarian allocation of per-pupil spending, that is, such that $x_i=r$, for all i, and (ii) Panel C: the score-EOp distribution. These are our benchmark policies[11].

Table 3 – Current situation and two benchmark policies

Quantile (q)	Type 1	Type 2	Type 3	Type 4	Type 5
Frequencies (%)	*6.78*	*25.88*	*22.10*	*26.07*	*19.17*
PANEL A: CURRENT	**Current allocation of per pupil spending (X_C)**				
All	0.121	0.131	0.143	0.158	0.200
	Current distribution of scores (S_C)				
q = 0.25	184.69	197.53	206.18	229.38	252.84
q = 0.50	213.88	226.75	237.50	260.31	283.13
q = 0.75	241.67	255.86	265.87	288.26	310.44
PANEL B: EQUAL-RESOURCE	**Equal resource policy (X_{ER})**				
All	0.150	0.150	0.150	0.150	0.150
	Distribution of scores under equal resource policy (S_{ER})				
q = 0.25	185.50	198.28	206.50	229.24	251.44
q = 0.50	214.56	227.38	237.86	260.14	281.69
q = 0.75	242.20	256.55	266.27	288.08	309.16
PANEL C: EOP1	**Allocation of X necessary to achieve score-EOp (X_{EOP1})**				
q = 0.25	1.376	0.727	0.540	-0.200	-1.017
q = 0.50	1.593	0.813	0.415	-0.209	-1.053
q = 0.75	2.071	0.749	0.388	-0.175	-1.151
Average	*1.680*	*0.763*	*0.448*	*-0.195*	*-1.074*
1=current national average (i.e., r=0.15)	***11.201***	*5.087*	*2.984*	*-1.300*	***-7.157***
	Distribution of scores under EOp (S_{EOP1})				
q = 0.25	218.58	218.58	218.58	218.58	218.58
q = 0.50	246.88	246.88	246.88	246.88	246.88
q = 0.75	275.45	275.45	275.45	275.45	275.45

The distribution of scores that results from the input-egalitarian allocation of the education input (S_{ER}) is not extremely different from the current distribution of scores (S_C). In fact, for any pair type-quantile, the changes would be less than 1%, in absolute value. For example, for $t=1$, $q=0.25$, while current average score is 184.69, under an egalitarian allocation of spending per pupil, average score would increase to 185.50, a positive variation of 0.44%. For $t=5$, $q=0.75$, there would be a negative oscillation of 0.41%,

[11] The "compromise solution" suggested by Roemer amounts to averaging, within types, the value of the instrument, that is, the values reported in the line "average". However, it should be noted that the scores reported in panel C (S_{EOP1}) are not in line with Romer's solution. Rather, what we report there are scores which would be obtained if the instrument could be perfectly allocated to pupils of each particular type and quantile (cf. section 3.1).

with average scores decreasing from 310.44 to 309.16. So, moving from current allocation of inputs into an input-egalitarian allocation of inputs would involve a certain amount of redistribution of inputs (especially from type 5 individuals, which would have to face a 25% decrease in their resources), with quite modest impacts on scores. More importantly, such a policy would not be in line with EOp ethics, since it would not provide sufficient compensation for types whose circumstances are not favorable.

In contrast, when we compare the current distribution of scores (S_C) with the distribution of scores under EOp allocation (S_{EOPI}), variations are substantial. For the same types and quantiles mentioned in the previous paragraph, the variations of average scores would be, respectively, of 18.35% and of –11.27%.

If we focus on the values of X securing score-EOp (X_{EOPI}), we immediately note the dramatic and immense reallocation that is required by the EOp agenda. If the aim of the social planner is to equalize expected scores across types following Roemer's compromise solution (allocating within-types average), it will be necessary to multiply the current level of spending of the lowest achieving type of pupils by a factor 11.20, while the level of spending on the highest achiever should actually be multiplied (on average) by a factor –7.16 (thus become negative). Combining information presented on Tables 1 and 3, we can observe that a considerable fraction of the pupils' population would have to make efforts in order for (S_{EOPI}) to be achieved. Ex ante well-off pupils (types 4 and 5), those who would face a decrease in their relative input allocation, represent around 45% of the pupils' population, while the ex ante worse-off groups (types 1, 2 and 3), those who would benefit from the policy shift, represent around 55%.

In contrast to Roemer's compromise, if we turn to the "ideal" EOp allocation, that is, perfectly allocating the resources according to types and quantiles (cf. Section 3.1), then for certain pairs type-quantile, the required reallocations would be even greater. For example, current level of spending for t=5, q=0.75 should be multiplied by a factor –7.67, while the level of spending for t=1, q=0.75 would have to be increased 13.81 times[12].

The reader should take good note of the fact that these results are based on a *conservative interpretation* of the score to spending elasticities (i.e. β). We have indeed assumed that these coefficients capture the sensitivity of scores to the average (or cumulated) per pupil spending since entrance in the education system. But the measure of spending we used (x) is purely cross-sectional and 8th-grade-specific. Should our β thus be interpreted as the effect of current (and not average or cumulated) spending on score? If so, EOp could be achieved gradually, and not on one shot. Table 3 suggests for example that raising type 1's score to the EOp target means spending 1.680 instead of the current 0.121. Assuming constant elasticity (i.e., βs are of similar magnitude across grades) and constant return to scale (i.e., repeated small increments of x produce similar score improvements as a single big one), the same type-1 score could be achieved over a period of 9 years (1 pre-primary grade + 8 primary school grades) and require an increment of 0.173 per year[13]. This could appear as a redistribution of much lower magnitude and possibly be perceived as more politically acceptable.

Yet, from a purely econometric point of view, we believe it makes more sense to stick to our initial and rather conservative interpretation where the βs capture the relationship between score and average (or cumulated) spending. It is well know, from the production function literature, that coefficients estimated using cross-sectional data are generally *upward biased*. The intuitive reason, put into the context of our data, is that 8th grade per-pupil spending is most likely highly correlated with per-pupil spending during all preceding grades. Hence, our assumption that x could be nothing more than a proxy for average or cumulated spending[14].

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[12] (-1.151/0.15)=-7.67 and (2.071/0.15) = 13.81.
[13] (1.680-0.121)/9 = 0.173.
[14] We tend to apply the same reasoning to cross-sectional peer quality (*PEER)* and private school attendance likelihood (*PRIV)*. The values they take for 8th grade are likely to be proxies for average values since the beginning of schooling.

4.2. Imposing a non-negativity constraint

As we can notice from the results in Table 3, equalizing within-quantile scores will virtually never be possible without the absurd implication of imposing negative values of the educational input to some types (that is, pupils belonging to high-performing types would have to be "taxed", transferring resources to their low-performing peers). To avoid such cases, we can impose an additional constraint to our program, to make sure that no type-quantile will be allocated a negative value of the educational resource, that is:

$$x^{tq} \geq 0 \qquad [5]$$

Table 4 contains the results of this simulation, both in terms of the required allocation of resources (X_{EOP2}) and in terms of the resulting distribution of scores (S_{EOP2}).

Table 4 – Reallocation required for achieving score-EOp, respecting non-negativity constraint

Quantile (q)	Type 1	Type 2	Type 3	Type 4	Type 5
Frequencies (%)	*6.78*	*25.88*	*22.10*	*26.07*	*19.17*
Current allocation of per pupil spending (X_C)					
All	0.121	0.131	0.143	0.158	0.200
Allocation of X necessary to achieve score-EOp, respecting non-negativity constraint (X_{EOP2})					
q = 0.25	0.855	0.315	0.065	0.000	0.000
q = 0.50	1.453	0.704	0.000	0.000	0.000
q = 0.75	1.819	0.608	0.000	0.000	0.000
Average	1.376	0.542	0.022	0.000	0.000
1=current national average (i.e., r=0.15)	***9.171***	*3.613*	*0.144*	*0.000*	*0.000*
Distribution of scores under EOp, respecting non-negativity constraint (S_{EOP2})					
q = 0.25	203.81	203.81	203.81	224.55	246.86
q = 0.50	243.53	243.53	232.70	254.44	277.00
q = 0.75	270.84	270.84	260.42	282.11	304.98

An interesting result of this exercise is to be found in the distribution of scores (S_{EOP2}). While there are large gains for some type-quantile pairs (e.g., with respect to the current situation, the average score increases by 13.86% for $t=1$, $q=0.50$), there are not important losses for any pair type-quantile (e.g., with respect to the current situation, the average score decreases by 2.37% for $t=5$, $q=0.25$).

But in this case, roughly 2/3 of the population (types 3, 4 and 5) would be penalized in order to make sure that roughly 1/3 of the pupils (types 1 and 2) achieve average scores that would be in line with this weaker version of score-EOp allocation. And, more importantly, even in this case, per-pupil spending on the low achieving type should be multiplied by the still large factor of 9.17 on average (or up to 12.13 times for t=5, q=0.25).

5. Per-pupil spending, *non-monetary inputs* and score-EOp: results

Following the intuition we exposed in section 2, we now turn to the case where the EOp algorithm is applied after some reallocation of non-monetary inputs has taken place. Algebraically, this means that we redefine A in equation [4] to become.

$$A^* = a^q + c^q PEER^* + d^q PRIV^* \qquad [6]$$

where:

$PEER^*$ is the national average of the peer quality endowment,

$PRIV^*$ is the national average attendance at private schools.

Using parameters (A^*, β) we then identify the EOp solution, following the same logic as the one exposed in section 3.3.

Table 5 – Reallocation required for achieving score-EOp, respecting non-negativity constraint, and with ex-ante redistribution of both monetary and non-monetary inputs

Quantile (q)	Type 1	Type 2	Type 3	Type 4	Type 5
Frequencies	*6.78*	*25.88*	*22.10*	*26.07*	*19.17*
Current allocation of per pupil spending (X_C)					
All	0.121	0.131	0.143	0.158	0.200
Allocation of X necessary to achieve score-EOp, respecting non-negativity constraint, and with redistribution of both monetary and non-monetary inputs (X_{EOP3})					
q = 0.25	0.702	0.258	0.177	0.000	0.000
q = 0.50	0.671	0.295	0.144	0.000	0.000
q = 0.75	1.089	0.290	0.186	0.000	0.000
Average	0.821	0.281	0.169	0.000	0.000
1=current national average (i.e., r=0.15)	**5.473**	1.873	1.129	0.000	0.000
Distribution of scores under EOp, respecting non-negativity constraint, and with redistribution of both monetary and non-monetary inputs (S_{EOP3})					
q = 0.25	213.97	213.97	213.97	216.35	219.82
q = 0.50	244.43	244.43	244.43	246.57	251.14
q = 0.75	274.20	274.20	274.20	274.90	284.16

Table 5 shows that the *simultaneous redistribution of monetary and non-monetary inputs* considerably reduces the magnitude of financial redistribution required. We show that per-pupil spending on the low achieving type should now be multiplied only by a factor 5.47, quite lower than 9.17, the value showed in Table 4. Here once again it is the type-quartile pair t=1, q=0.75 which requires more redistribution: 7.26 times than the current allocation.

In this case, the burden of the policy would be borne by 45% of the population (types 4 and 5). The variations of scores would range from –15.02% ($t=5$, $q=0.25$) to +13.68% ($t=1$, $q=0.25$).

6. Efficiency issues

As pointed by Betts & Romer (2004), policies aimed at equalizing achievement – be it in a strictly egalitarian or in an EOp perspective – are often criticized for being "inefficient".

If we assume that utility is a strictly increasing function of score, and that the reallocation of educational inputs has some effect on the distribution of scores, then any score-EOp policy would lead to a Pareto-inefficient outcome, since higher types will necessarily have to "pay the price" of the redistribution. Even if the score-elasticities of educational resources for lower types were extremely larger than those for higher types (which we know, from Table 2, it is not necessarily true), higher types would be worse-off after the redistribution has ocurred with respect to their pre-redistribution situation.

If, instead, we assume that utility is a function of score, but we admit that we ignore the form of this function, then nothing can be said for sure about the effects, in terms of Pareto-efficiency, of EOp policies such as the ones discussed in this paper. It is not excluded that investing scarce educational resources in lower-types pupils could increase their welfare without harming higher-types pupils.

However, let us define efficiency merely as the "size of the cake", that is, as the average score. In this case, it is indeed possible that when one reallocates per-pupil spending (*X*) the average score (*AS*) diminishes. That would typically occur if relative elasticities (i.e., β) were substantially larger for types 4 or 5 from which resources are being removed. Thus in order to check whether the policy interventions discussed here harm or not efficiency (understood as the "average score") we calculate an efficiency indicator (*F*), simply defined as the ratio between the average score predicted to be obtained after a given policy intervention, and the current average score.

$$F = (AS_k / AS_C) * 100 \qquad [7]$$

where:

F = efficiency indicator
AS = average score
c = current
k = *ER*, *EOp1*, *EOp2*, *EOp3* (ie, the policy interventions simulated in this paper).

Our calculations are based on the assumptions that type frequencies computed from our dataset adequately represent the Brazilian reality. Results are reported in Table 6.

Table 6 – Average scores and efficiency indicator (F)

Scenario	Average Score	Efficiency indicator (F)
C: Current	246.45	-
ER: Equal resource	247.10	100,26%
EOp1: reallocation of monetary resources only, without lower bound	246.97	100,21%
EOp2: reallocation of monetary resources only, with lower bound (x_i=0)	248.63	100,88%
EOp3: reallocation of both monetary and non-monetary resources, with lower bound (x_i=0)	246.09	99,85%

A close look at the results shows that the policies are not efficient-equivalent. The policy that simultaneously redistributes monetary and non-monetary resources (EOp3) could also lead to a slight *decrease* of the average score. By contrast, in the case in which the non-negativity constraint is observed, but in which only monetary resources are reallocated (EOp2), we would obtain a slight *increase* of the average score. In any case, F values all are very close to 100%, which suggests that implementing EOp would not come at any particular cost (or benefit) in terms of efficiency.

7. Conclusions in a policy perspective

We applied Roemer's EOp theory to education policy using Brazilian data, calculating the reallocations of educational expenditure required to equalize score-EOp for pupils of different socio-economic background.

Implementing score-EOp by using only per-pupil spending as the instrument requires multiplying by 9.2 (on average) the current level of spending on the lowest achieving pupils, a result which is driven by the extremely low elasticity of scores to per-pupil spending. In our view, although the required reallocation is considerable, it should be taken as a *lower bound* for an EOp-based redistribution of per-pupil spending, since in our setting: (i) the definition of a type relies on only one characteristic of pupils (thus the fraction of outcome attributed to "effort" is considerable); (ii) the education production function is parsimonious (ie, few control variables have been included). Widening the definition of type or adding controls in the econometric model would probably lead to more demanding redistribution patterns.

Betts & Roemer (2004) finished their paper suggesting that "money alone will not suffice to equalize educational opportunity", and urging for "finding complementary means of improving outcomes for the disadvantaged". We tried to contribute in this sense, and we showed that the *simultaneous redistribution of monetary and non-monetary inputs* would considerably reduce – by around 40% – the magnitude of the financial redistribution needed. We believe this is an important result of our paper, which should be taken into account by EOp proponents and by policymakers.

Reducing segregation (that is, redistributing peer group quality) and increasing the probability of disadvantaged pupils to attend private schools (best-run schools in Brazil) are goals which could be linked to each other. Ideally, some sort of SES-sensitive formula funding would have to be coupled with an equity-sensitive voucher scheme, in such a way that good public schools (through formula funding), and especially good private schools (through voucher scheme), face strong incentives to enroll disadvantaged kids and to mix them with advantaged kids[15].

However, it is clear that in Brazil or in any other country, a number of obstacles would turn out before, or in the course of, the actual implementation of the policies suggested here. First, even if we are working with lower-bound patterns of redistributions here, the magnitude of reallocations is not negligible, and this could mean that EOp policies would most probably face ex ante *political resistance*. Second, we did not draw comments on the *practical obstacles* that would come about in the implementation of an EOp educational policy. For example, given the public-good dimension of educational resources, it would be challenging to perfectly allocate a given amount of spending to pupils of given types and quantiles. On top of that suppose that, following the last policy we discussed (EOp3), one actually managed to de-segregate the system: then it would be even more difficult to perfectly target the amount of educational resources allocated to each type-quantile.

The risk of political resistance and the fine-tuning difficulties related to the implementation of EOp education policies are important issues that might lead one to be quite skeptical about the feasibility of such policies. We believe, however, that political and practical obstracles are inherent to any kind of (redistributive) policy. In addition to that, we believe this paper's contribution is to be found at a more macro-policy level: what it reveals is essentially the *directions in which policy should go*.

The Brazilian schooling system has its own particularities, but it is similar in many respects, to those of other developing countries (highly unequal distribution of inputs and outcomes; coexistence of private and public schools etc.). To a lesser extent, even schooling systems of developed countries share characteristics of the Brazilian one (nowhere can we find a system where all pupils are allocated the same amount of resources, socio-economic background typically has a strong influence on schooling outcomes etc.). So, we believe our results are not limited to observers who are interested in the Brazilian context, or in developing countries.

In our future research, we plan to extend our investigations to other countries. And we also plan to study implementation issues related to EOp education policy.

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[15] Formula-funding and voucher systems experiences from countries such as Belgium (Vandenberghe, 1996) or Chile (Gradstein et al, 2004) could inspire Brazilian educational authorities.

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Appendix

Roemer's EOp algorithm requires that, for each quantile, we find an allocation $X(t,q) = X^1, \ldots X^T$, such that:

$$Argmax_X [Min_T (A^{tq} + \beta^{tq}.x^t)] \qquad [A1]$$

$$st \sum_t (p^t.x^{tq}) = r \qquad [A2]$$

where variables are defined as in section 3.

Assuming types $t=1, 2$; and assuming type 1 will be the worst-off at the solution, [A1] can be restated as:

$$Max_T (A^{1q} + \beta^{1q} x^1) \qquad [A3]$$

st

$$A^{2q} + \beta^{2q} x^2 \geq A^{1q} + \beta^{1q} x^1 \qquad [A4]$$

$$p^1.x^1 + p^2.x^2 = r \qquad [A5]$$

By rewriting and combining [A4] and [A5], we obtain:

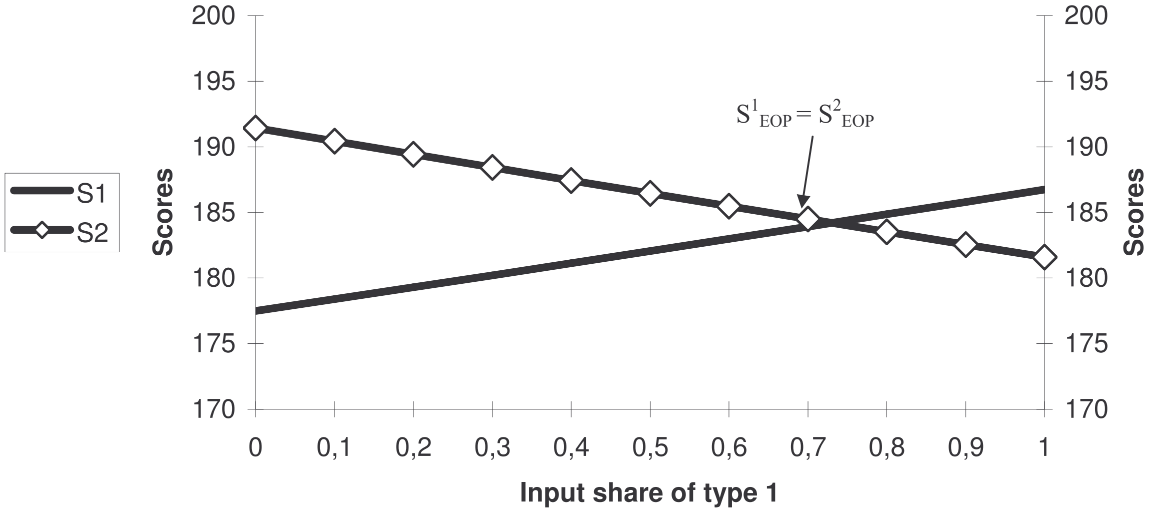
$$x^1 \leq (r.\beta^{2q} + p^2(A^{2q} - A^{1q}))/(p^1.\beta^{2q} + p^2.\beta^{1q}) \qquad [A6]$$

$$x^2 = (r - p^1.x^1)/p^2 \qquad [A7]$$

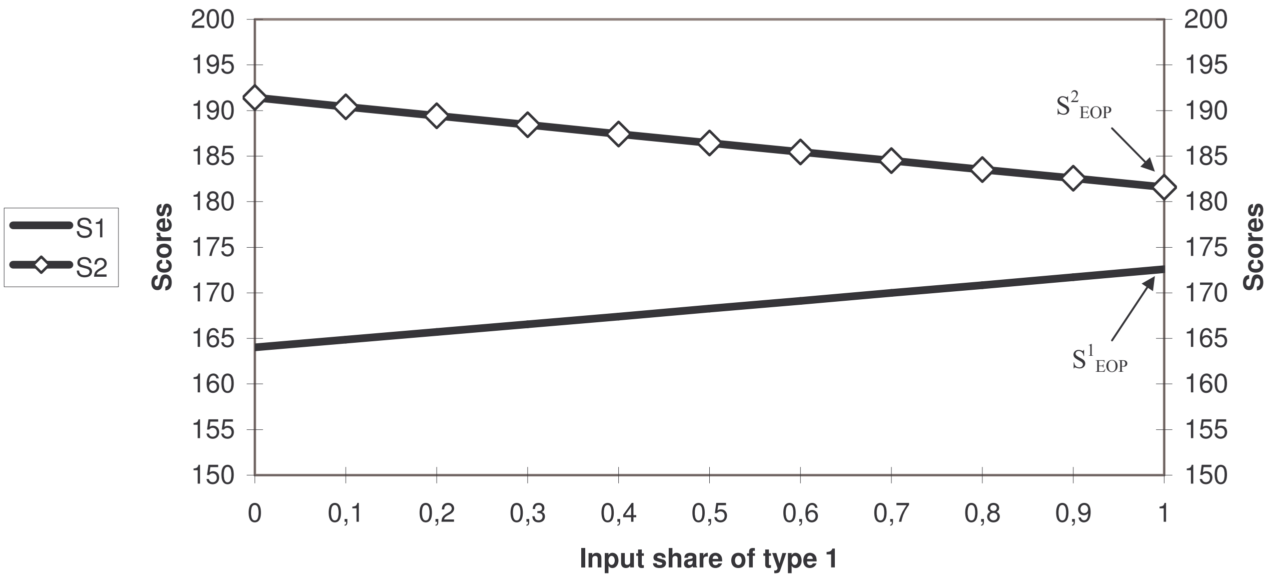
Expression [A6] sets an upper bound for the amount of resources to be allocated to individuals of type 1 at quantile q. As we are interested in maximizing the expected score of the worst-off individuals (type 1) at quantile q, we would like to set X^1 as large as possible, that is, such that [A6] holds with equality. Assuming objective function [A3] is rising with X^1 (i.e., elasticity of score to spending is significantly positive) we have that [A6] at equality and [A7] define a solution candidate.

Generally, we would have to solve the program assuming each type to be the worst-off at the solution, but in the two-type case, this is not necessary. It is in fact redundant, because by combining the expression that defines the upper bound for the resources allocated to individual 2 and the budget constraint, we obtain an optimal allocation which is equivalent to the one provided by [A6] and [A7]. The intuition can be understood in the graphical representation of the "education technologies" below.

Figure 1. EOp solution with 2 types, at a given quantile.
Panel A: "Interior solution".



Panel B: "Corner solution".



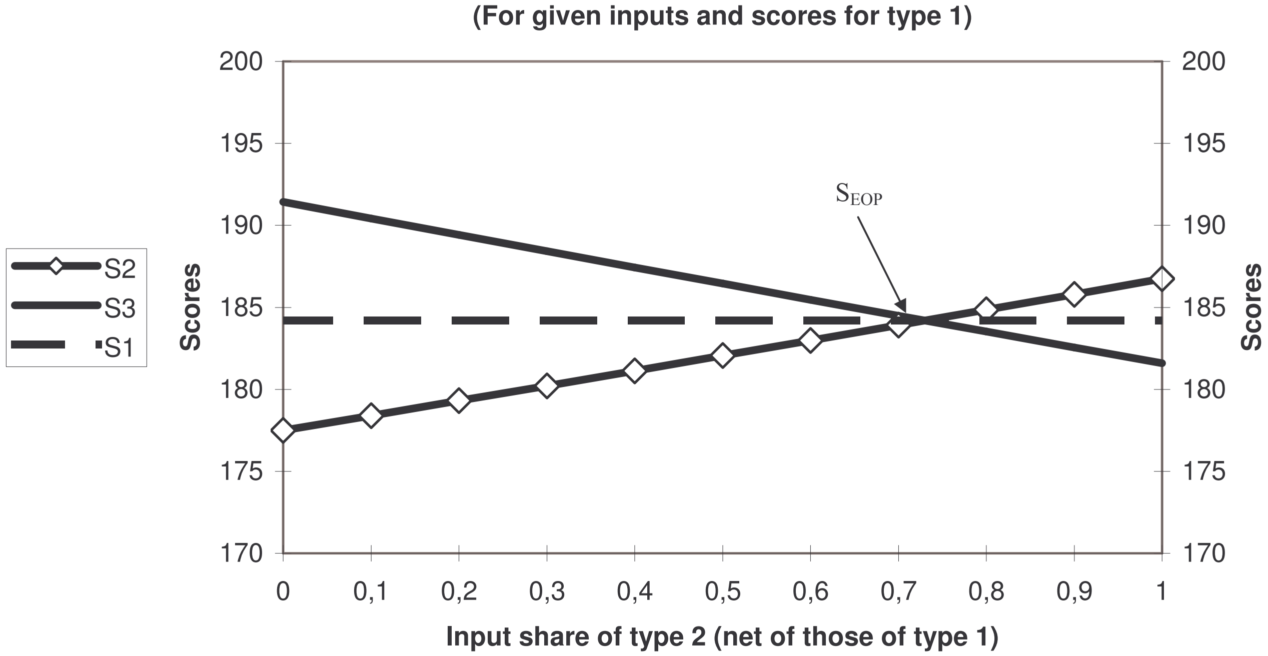
In Panel A of Figure 1, the EOp solution is given by the point where the two lines cross, that is, where $S^1_{EOP} = S^2_{EOP}$, which is exactly the outcome-egalitarian solution. In Panel B, the EOp solution differs from the outcome-egalitarian solution. The latter would require, either the allocation of negative amounts of the educational resource to type-2 individuals (such that their scores are driven down to reach those of type-1 individuals), or the violation of the budget constraint (such that a larger amount of input is allocated to type-1 individuals to improve their scores). If we impose the non-negativity constraint, just as we do in section 4.2, then the EOp solution in Panel B is such that, although type-1 individuals receive 100% of the educational input and type-2 individuals receive nothing at all, still we have that $S^1_{EOP} < S^2_{EOP}$.

To sum up, when technologies are linear, the elasticities are positive (estimated $\beta > 0$), and the differences between the parameters (αs and βs) across types are not spectacular, the optimal distribution of the educational resources are such that the expected scores are equal across types (i.e., outcome-egalitarian solutions obtain, cf. Panel A in Figure 1). However, it is possible that the parameters of the education production function are such that an equal outcome is not feasible, even under a very generous redistribution (cf. Panel B in Figure 1).

If we work with three types, we have to assume each type is the worst-off, and solve the program defined by [A3], [A4], [A5] and an additional constraint that guarantees that a given type is really the worst-off at the solution. For example, if type 1 is assumed to be the worst-off, inequality [A8] would also have to be respected:

$$A^{3q} + \beta^{3q} x^3 \geq A^{1q} + \beta^{1q} x^1 \qquad [A8]$$

Figure 2. EOp solution with 3 types, at a given quantile.



If, for a given quantile and for all pair of types, there were “interior” solutions (Figure 2), then the egalitarian solution would obtain, and there would be no need to solve program [A3]-[A5] and [A8] assuming each type to be the worst-off. If “interior” solutions were not feasible, then the non-negativity constraint would lead to a non-egalitarian solution.