The Optimality of the Friedman Rule When Some Distorting Taxes Are Exogenous*†

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Abstract: The Friedman rule is a feature of second-best policies in several monetary models. However, if the Ramsey planner does not have access to a complete set of tax instruments, then that policy prescription often fails to be optimal. In this paper we investigate the relation between the optimality of the Friedman rule and existing tax instruments. The availability of consumption taxes constitutes a sufficient condition for the optimality of the Friedman rule. Moreover, there are cases in which these taxes are not available and that policy prescription is still optimal.

Keywords: Friedman rule, optimal monetary policy, exogenous taxes.
JEL classification: E31, E52, E63, H21.

Resumo: A regra de Friedman é uma das características das políticas do tipo segundo-melhor em diversos modelos monetários. Contudo, se o planejador de Ramsey não tiver acesso a um conjunto completo de instrumentos de taxação do tipo "distorting", então tal prescrição de política freqüentemente não é ótima. Estuda-se neste ensaio a relação entre a otimalidade da regra de Friedman e os instrumentos de taxação disponíveis. Uma condição suficiente para que aquela regra seja ótima é a disponibilidade de impostos sobre o consumo. Adicionalmente, há situações nas quais esses impostos não estão disponíveis e a regra de Friedman ainda assim é ótima.

Palavras-chave: Regra de Friedman, política monetária ótima, taxação exógena.
Classificação JEL: E31, E52, E63, H21.

Área 3 - Macroeconomia, Economia Monetária e Finanças

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1 Introduction

Friedman [11] suggested that a government should set the nominal interest rate equal to zero to lead the economy to an efficient outcome. He argued that only such a policy would maximize the consumer surplus associated with money demand. That policy prescription became known in the literature as the Friedman rule.

Phelps [15] pointed out that Friedman's argument was a partial equilibrium one and implicitly relied on the availability of lump-sum taxes. He then claimed that in a general equilibrium context without lump-sum taxation, the Friedman rule was unlikely to be optimal. The reason for this would be the fact that a second-best policy would generally encompass the use of all available distorting taxes, including inflation. However, Kimbrough [12] later showed that Friedman's prescription could be optimal in a shopping-time model even if all sources of tax revenues were distorting.

After Kimbrough's paper, several essays addressed the optimality of the Friedman rule. A main lesson that emerges from Chari, Christiano and Kehoe [4] and [5]; Correia and Teles [7] and [8]; and De Fiore and Teles [10] is that the optimality of the Friedman rule does not critically rely on any specific type of monetary friction. For standard preferences and technologies, that policy prescription is optimal in cash-in-advance, money-in-the-utility-function and shopping-time economies.

The findings mentioned in the above paragraph make a strong case for the Friedman rule. However, those findings depend on the implicit hypothesis that the planner has access to a sufficiently large set of distorting tax instruments.

Bhattacharya, Haslag and Russell [1] investigated the optimality of the Friedman rule in overlapping generation models. According to them, that policy prescription usually fails to be optimal in that class of models because the inflation tax is an instrument to implement intergenerational transfers. They found that if the government has access to an alternative instrument to carry out these type of transfers, then the Friedman rule is optimal.

Carlstrom and Fuerst [2] studied the problem of selecting the optimal monetary policy in a small open economy. The monetary friction in their model was of the cash-in-advance type. The Friedman rule failed to be optimal in that model. According to them, that failure was due to the existence of an exogenous international interest rate that impacted the behavior of domestic agents and constrained the choices of the Ramsey planner in such way that it was optimal to deviate from the Friedman rule.

Cavalcanti and Villamil [3] analyzed the optimality of zero nominal interest rates in an economy with an informal (untaxed) sector. They introduced money in their model by means of a shopping-time constraint. They concluded that in such a context the Friedman rule failed to be optimal. The reason for this was that the only possible way the government could raise revenue from the informal sector was through inflation.

Nicolini [14] studied the optimal monetary policy in an economy with an informal (untaxed) sector. The monetary friction was of the cash-in-advance type. He found that the Friedman rule was not optimal for the same reason that Cavalcanti and Villamil [3] did.

Schmitt-Grohé and Uribe [17] investigated the optimality of the Friedman rule in a one-sector small open economy with cash and credit goods. As far as we know, they were the first to show that zero nominal interest rates could be optimal in an open-economy setup. In their model, Friedman's prescription would be optimal if the Ramsey planner were able to use only consumption or both consumption and labor income taxes. If the consumption tax were not available, the Friedman rule would not be optimal because inflation acted as a partial substitute for that absent tax.

Sticky prices and imperfect competition may also affect the optimality of the Friedman rule. Schmitt-Grohé and Uribe [18] and [19] studied the properties of the optimal monetary policy under those types of frictions. It turned out that the presence of either one would prevent Friedman's prescription from being optimal.

A way to clarify the contradictions concerning the optimality of the Friedman rule is to take a deeper look at the set of distorting taxes available to the planner. As an example, consider the cash-credit model in Chari, Christiano and Kehoe [4] and [5]. In that single sector, single input and constant return-to-scale economy, there are only two possible distorting taxes: on consumption τ^c and on labor income τ^l. If the planner can select τ^c or τ^l, then the Friedman rule is optimal.

Consider now a situation in which there are three possible generic distorting taxes τ_1, τ_2 and τ_3. There are seven cases to be looked into. Each of these cases is a non-empty subset of $\{\tau_1, \tau_2, \tau_3\}$. One can interpret the non optimality of the Friedman rule in Carlstrom and Fuerst [2], Cavalcanti and Villamil [3], Nicolini [14] and Schmitt-Grohé and Uribe [17] and [19] as its non optimality for a particular subset of all conceivable distorting taxes that could possibly be implemented in their economies.

Chari and Kehoe [6] defined an economy's tax system as *complete* if the number of tax rates the Ramsey planner can select is equal to the number of existing wedges and *incomplete* if the number of tax instruments is smaller than the number of wedges. For the purposes of this paper, we associate the expression *second-best* with policies chosen by a Ramsey planner that has access to a complete tax system. *Higher-order-best* policies constitute an optimal choice under incomplete tax systems. We say that a policy is *Ramsey efficient* if it is a second or higher-order best policy.

Studying the properties of optimal policies when a benevolent government cannot choose some distorting taxes is a relevant exercise. For instance, any actual economy has some informality and household production. It is not a trivial matter to tax these types of activities. The federative nature of many nations may matter too. A central bank and a federal fiscal authority may jointly design a macroeconomic policy and at the same time fail to induce city and state administrations to set their tax rates at optimal levels. International treaties constitute another constraining factor when it comes to the selection of tax rates.

The above discussion naturally leads us to inquire about the optimality of the Friedman rule when the Ramsey planner is not able to choose all of several conceivable distorting taxes. This is the problem we study in this paper. We investigate whether zero nominal interest rates constitute a feature of second and higher-order best policies.

The task we have in mind does not allow us to restrict ourselves to a standard one-sector monetary model. Therefore, in most of this paper we consider a two-sector (tradable and non-tradable) small open economy deterministic version of the cash-credit model of Lucas and Stokey [13]. Consumers face a cash-in-advance constraint on a fraction of their purchases of non-tradables. There is distorting taxation on labor income, on consumption of each type of good and on foreign interest income. Government consumption is exogenous.

Of course, not only open economies have many sectors, goods and taxes. We chose to consider an open economy for two reasons. First, actual economies are open economies. Second, there already are some papers that study the optimality of the Friedman rule in open economies, such as Carlstrom and Fuerst [2] and Schmitt-Grohé and Uribe [17], and we want to relate our findings to theirs.

Our main conclusions are as follows. If the Ramsey planner can select the tax rate on consumption, then the Friedman rule is optimal. The availability of other taxes is irrelevant for this result. Additionally, we also show that there are circumstances in which consumption taxes are not available and the Friedman rule is still optimal.

We wish to emphasize the relevance of the aforementioned findings. It is well known that the Friedman rule is a feature of second-best policies in many monetary models. However, the most common result found in the literature when the Ramsey planner does not have access to a complete tax system is that zero nominal interest rates are not optimal. Our results show that the non-optimality of the Friedman rule critically depends on the assumptions concerning the availability of tax instruments.

Chari and Kehoe [6] emphasized that usually there is more than one tax system that decentralizes a given allocation. We use this type of indeterminacy in this paper. Those authors also pointed out that as the Ramsey planner loses tax instruments, more constraints have to be added to the Ramsey problem. We also use this fact to characterize the Ramsey efficient policies. Our novelty consists in showing that a particular feature (i.e., the Friedman rule) of the optimal macroeconomic policy survives the introduction of several constraints in the Ramsey problem.

The finding that zero nominal interest rates may be a feature of higher-order-best policies is not specific to the cash-credit two-sector small open economy considered in this paper. To each result, we provide an intuitive explanation that can be carried over to other monetary models. Nevertheless, in the Appendix we present a two-sector shopping-time economy and show that the Friedman rule is a feature of a higher-order-best policy in that model too.

This paper is organized as follows. Section 2 describes the cash-credit two-sector small open economy we study in most of our analysis. Section 3 characterizes the set of competitive equilibria and the Pareto efficient allocation and policies of that economy. Section 4 discusses its Ramsey efficient policies. Section 5 presents our concluding remarks. The Appendix briefly discusses Ramsey efficient policies in a two-sector shopping-time monetary economy.

2 The economy

Consider a small country populated by a single infinitely lived household and a government. The household is composed of a shopper and a worker, who are each endowed with one unit of time.

The country produces a non-tradable good. The household and the government consume that good. The respective amounts they consume are denoted by c^N and g^N. The country also produces a tradable good. The household and the government consume it; the respective notations are c^T and g^T. The country can export or import this latter good; x denotes the amount exported. A negative value for x means that the country is importing that good. For future reference, define $S = \{T, N\}$.

Markets operate in a particular way. At a first stage of each date t, a spot market for goods and labor services operates. At a second stage, after the market for goods and labor service closes, a securities and currency market operates.

A domestic currency M circulates in this economy. Agents trade two types of securities: a claim B that pays $(1 + i)$ units of M and a claim B^F that pays $(1 + i^F)$ units of some foreign currency. Both claims mature after one period. Foreigners do not sell or buy claims to the domestic currency.

The worker cannot sell her services outside the country. The shopper faces a cash-in-advance constraint. A fraction c_1^N of the her purchases of c^N must be paid for with the domestic currency, while for the remainder c_2^N there is no such constraint. All other transactions are liquidated during the securities and currency trading session. The date t price, in terms of the foreign currency, of the tradable good is exogenous and equal to one.

Let y^s and l^s denote, respectively, the output of good $s \in S$ and the amount of labor allocated to its production. The inequality $0 \leq y^s \leq (l^s)^{\alpha^s}$ describes the technology. The share parameter satisfies $\alpha^s \in (0, 1]$.

A single competitive firm produces each good. Let l_t denote the amount of labor the household supplies at date t. Other variables indexed by t have analogous meanings. Feasibility requires

$$l_t^T + l_t^N = l_t \leq 1,\ c_{1t}^N + c_{2t}^N + g_t^N = (l_t^N)^{\alpha^N},\ c_t^T + g_t^T + x_t = (l_t^T)^{\alpha^T} \ . \tag{1}$$

A finite set $G^N \times G^T \times I^F$ contains the exogenous sequence $\{g_t^N, g_t^T, i_t^F\}_{t=0}^{\infty}$. Additionally, $G^N \subset \mathbb{R}_+$, $G^T \subset \mathbb{R}_+$ and $I^F \subset \mathbb{R}_{++}$.

The government finances $\{g_t^N, g_t^T\}_{t=0}^{\infty}$ by issuing and withdrawing the domestic currency; by issuing and redeeming B; by purchasing and selling B^F; and by taxing profits, labor income, consumption and interest income on foreign assets. Its budget constraint is

$$\begin{aligned} p_t^N g_t^N + E_t g_t^T + E_t B_{Gt+1}^F + (1 + i_t) B_t + M_t = E_t(1 + i_t^F) B_{Gt}^F + M_{t+1} + \\ B_{t+1} + \tau_t^l w_t l_t + \tau_t^T E_t c_t^T + \tau_t^N p_t^N (c_{1t}^N + c_{2t}^N) + E_t \tau_t^F i_t^F B_{Ht}^F + \sum_{s \in S} \delta_t^s \psi_t^s \ , \end{aligned} \tag{2}$$

where E_t is the nominal exchange rate; p_t^N and w_t are the respective date t monetary prices (in terms of the domestic currency) of the non-tradable good and labor services; B_{Gt+1}^F stands for the foreign assets held by the government at the end of date t, while B_{Ht}^F is people's foreign assets at the beginning of the same period; M_{t+1} and B_{t+1} are the amounts of domestic currency and public debt held by the household at the end of date t; ψ_t^T and ψ_t^N are the date t profits; τ_t^l, τ_t^T and τ_t^N are tax rates on labor income and consumption;

τ_t^F is the tax rate on household's foreign assets income; and δ_t^T and δ_t^N are the tax rates on profits. A negative value for B_{Gt+1}^F means that the government is borrowing abroad, while a negative value for B_{t+1} means that the government is lending to the household. At $t=0$ the government holds an initial amount B_{G0}^F of foreign assets. The boundedness constraint $\left|B_{G,t+1}^F\right| \leq A < \infty$ prevents Ponzi schemes.

The function $u : \mathbb{R}_+^3 \times [0,1] \to \mathbb{R} \cup \{-\infty\}$, $u = u(c_1^N, c_2^N, c^T, l)$, is the household period utility function. It satisfies standard properties. As usual, intertemporal preferences are described by

$$\sum_{t=0}^{\infty} \beta^t u(c_{1t}^N, c_{2t}^N, c_t^T, l_t) \ , \tag{3}$$

where $\beta \in (0,1)$. The household's date t budget is

$$\begin{aligned} &(1+\tau_t^N)p_t^N(c_{1t}^N + c_{2t}^N) + (1+\tau_t^T)E_t c_t^T + M_{t+1} + B_{t+1} + E_t B_{Ht+1}^F \leq \\ &(1-\tau_t^l)w_t l_t + M_t + (1+i_t)B_t + E_t[1+(1-\tau_t^F)i_t^F]B_{Ht}^F + \sum_{s\in S}(1-\delta_t^s)\psi_t^s \ . \end{aligned} \tag{4}$$

The constraint $\left|B_{t+1}/p_{t+1}^N\right|, \left|B_{H,t+1}^F\right| \leq A$ prevents Ponzi games. The household faces the cash-in-advance constraint

$$(1+\tau_t^N)p_t^N c_{1t}^N \leq M_t \ . \tag{5}$$

Given initial cash and bond holdings (M_0, B_0, B_{H0}^F), the household chooses a sequence $\{c_t^T, c_{1t}^N, c_{2t}^N, l_t, M_{t+1}, B_{t+1}, B_{Ht+1}^F\}_{t=0}^{\infty}$ to maximize (3) subject to the constraints (4), (5) and $l_t \leq 1$.

At each date t, the firm that produces the non-tradable good chooses l_t^N to maximize $\psi_t^N = p_t^N(l_t^N)^{\alpha^N} - w_t l_t^N$. In a similar fashion, the other firm chooses l_t^T to maximize $\psi_t^T = E_t(l_t^T)^{\alpha^T} - w_t l_t^T$.

3 Competitive equilibrium

We denote the date t price vector $(p_t^N, E_t, w_t, i_{t+1})$ by φ_t and the sequence $\{\varphi_t\}_{t=0}^{\infty}$ by φ. The date t vector of tax rates $(\tau_t^N, \tau_t^T, \tau_t^l, \tau_t^F, \delta_t^N, \delta_t^T)$ is denoted by ξ_t and $\xi = \{\xi_t\}_{t=0}^{\infty}$ is a *tax policy*. Date t allocations $(l_t^N, l_t^T, x_t, c_{1t}^N, c_{2t}^N, c_t^T, l_t)$ and end-of-period asset holdings $(M_{t+1}, B_{t+1}, B_{Ht+1}^F, B_{Gt+1}^F)$ are denoted, respectively, by χ_t and ζ_{t+1}. Additionally, $\chi = \{\chi_t\}_{t=0}^{\infty}$ and $\zeta = \{\zeta_{t+1}\}_{t=0}^{\infty}$.

Definition 1 *A* competitive equilibrium *for a tax policy ξ is an object (φ, χ, ζ) satisfying: (i) given φ and ξ, (χ, ζ) provides a solution for the household problem; (ii) $w_t = p_t^N \alpha^N (l_t^N)^{\alpha^N - 1} = E_t \alpha^T (l_t^T)^{\alpha^T - 1}$; (iii) (1) and (2) hold. An array $(\xi, \varphi, \chi, \zeta)$ is* attainable *if (φ, χ, ζ) is a competitive equilibrium for ξ.*

Adding the identities $\psi_t^N + w_t l_t^N = p_t^N(\ c_{1t}^N + c_{2t}^N + g_t^N)$ and $\psi_t^T + w_t l_t^T = E_t(c_t^T + x_t + g_t^T)$ to (2), and (4) holding with equality, one obtains the balance of payments

$$x_t + (1+i_t^F)(B_{Gt}^F + B_{Ht}^F) - B_{Gt+1}^F - B_{Ht+1}^F = 0 \ . \tag{6}$$

So, it is not necessary to specify this condition when defining competitive equilibrium.

Next we characterize the set of competitive equilibrium allocations in terms of a few constraints. To simplify the notation, $u(t)$, $u_1(t)$, $u_2(t)$, $u_T(t)$, and $u_l(t)$ will denote, respectively, the value of u and its partial derivatives $\partial u/\partial c_1^N$, $\partial u/\partial c_2^N$, $\partial u/\partial c^T$, and $\partial u/\partial l$ evaluated at the point $(c_{1t}^N, c_{2t}^N, c_t^T, l_t)$. We define $W(t)$ according to

$$W(t) = u_1(t)c_{1t}^N + u_2(t)c_{2t}^N + u_T(t)c_t^T + u_l(t)l_t \ ,$$

while $W_1(t)$, $W_2(t)$, $W_T(t)$, and $W_l(t)$ denote the partial derivatives of $W(t)$.

There are seven conditions with obvious economic meaning that must hold in any competitive equilibrium. A trivial condition is (1). The second requirement is the Euler equation

$$\beta \frac{u_T(t+1)}{u_T(t)} = \frac{1+\tau_{t+1}^T}{1+\tau_t^T} \frac{1}{1+(1-\tau_{t+1}^F)i_{t+1}^F} \ . \tag{7}$$

However, for future convenience, we write this constraint as

$$u_T(t) = \beta^{-t} \frac{1+\tau_t^T}{1+\tau_0^T} \frac{u_T(0)}{\prod_{r=1}^t [1+(1-\tau_r^F)i_r^F]} \ , \tag{8}$$

where the empty product $\prod_{r=1}^0$ is defined to be equal to one. The third constraint is that the household's marginal rate of substitution between tradables and non-tradables must be consistent with the tax rates and the marginal rate of transformation between those types of goods, i.e.,

$$\frac{u_T(t)}{u_2(t)} = \frac{1+\tau_t^T}{1+\tau_t^N} \frac{\alpha^N (l_t^T)^{1-\alpha^T}}{\alpha^T (l_t^N)^{1-\alpha^N}} \ . \tag{9}$$

The fourth,

$$-\frac{u_l(t)}{u_2(t)} = \frac{1-\tau_t^l}{1+\tau_t^N} \frac{\alpha^N}{(l_t^N)^{1-\alpha^N}} \ , \tag{10}$$

is an implementability constraint for real wages. The fifth is

$$\begin{aligned} & u_1(0)c_{1,0}^N + u_2(0) \left[\frac{(1+i_0)B_0}{(1+\tau_0^N)p_0^N} + \frac{M_0}{(1+\tau_0^N)p_0^N} - c_{1,0}^N \right] + \\ u_T(0) \frac{[1+(1-\tau_0^F)i_0^F]B_{H0}^F}{1+\tau_0^T} &= \sum_{t=0}^{\infty} \beta^t \left[W(t) + u_l(t) \sum_{s \in S} \frac{1-\delta_t^s}{1-\tau_t^l} \frac{1-\alpha^s}{\alpha^s} l_t^s \right] \ , \end{aligned} \tag{11}$$

which consolidates all date t budget constraints of the household. The sixth is a balance-of-payment constraint

$$-\sum_{t=0}^{\infty} \frac{x_t}{\prod_{r=1}^t (1+i_r^F)} = (1+i_0^F)(B_{H0}^F + B_{G0}^F) \ . \tag{12}$$

The last one, which ensures that the nominal interest rate is greater or equal to zero, is

$$u_2(t) \leq u_1(t) \ . \tag{13}$$

The above constraints are not enough to characterize a competitive equilibrium. The inequality

$$(1+\tau_0^N)p_0^N c_{1,0}^N \le M_0 \tag{14}$$

is needed to ensure that the date zero cash-in-advance constraint holds. Finally, an implementability constraint for a transversality condition is

$$\lim_{t\to\infty} \beta^t u_1(t) c_{1t}^N = 0 \ . \tag{15}$$

In a longer version of this essay we established the result that follows.

Proposition 2 *Let $M_0 > 0$. A bounded sequence χ and a price $p_0^N > 0$ satisfy (1) and (8)-(15) if and only if they are components of an attainable array $(\xi, \varphi, \chi, \zeta)$.*

The government budget constraint is a linear combination of the household's budget constraint, resource constraints and balance of payments. For this reason the above set of conditions does not include an implementability constraint for equation (2).

Recall that the Friedman rule specifies that, for all t, $i_{t+1} = 0$. From people's first-order conditions, it is easy to conclude that equality holds if and only if $u_2(t+1) = u_1(t+1)$.

To help us analyze the second and higher-order best policies, we will briefly discuss the properties of policies that lead to Pareto efficient outcomes. In this economy, an allocation is Pareto efficient if and only if it maximizes (3) subject to (1) and (12). Under standard assumptions on u, there is a unique Pareto efficient allocation. Conditions (1) and (12) plus

$$u_1(t) = u_2(t) \ ; \tag{16}$$

$$\frac{u_T(t)}{u_2(t)} = \frac{\alpha^N (l_t^T)^{1-\alpha^T}}{\alpha^T (l_t^N)^{1-\alpha^N}} \ ; \tag{17}$$

$$\beta \frac{u_T(t+1)}{u_T(t)} = \frac{1}{1+i_{t+1}^F} \ ; \tag{18}$$

and

$$-\frac{u_l(t)}{u_2(t)} = \frac{\alpha^N}{(l_t^N)^{1-\alpha^N}} \tag{19}$$

fully characterize a Pareto efficient allocation.

Straightforward calculations show that necessary and sufficient conditions that characterize a Pareto efficient policy are the Friedman rule,

$$\tau_t^N = \tau_t^T = -\tau_t^l \ , \tag{20}$$

and

$$\frac{1}{1+i_{t+1}^F} = \frac{1+\tau_{t+1}^T}{1+\tau_t^T} \frac{1}{1+(1-\tau_{t+1}^F) i_{t+1}^F} . \tag{21}$$

The Friedman rule ensures the absence of a wedge between the consumption of cash and credit non-tradables. The equalities in (20) constitute a standard uniform taxation requirement. Condition (21) requires that no wedge exists between the prevailing international interest rate and the rate at which the household can borrow and lend abroad.

4 Ramsey efficiency

Following the tradition that Ramsey [16] started, in this section we study the problem of selecting a best attainable allocation. However, we carry out this task under different assumptions about the ability of the Ramsey planner to select the tax rates.

A standard procedure to compute the Ramsey allocation is to maximize the lifetime utility (3) subject to some implementability constraints. Regardless of which taxes the government can select, conditions (1), (11) and (12)-(15) must constrain the allocation being chosen. The remaining constraints to be considered depend on which taxes are exogenous and which are not. For instance, if both $\{\tau_t^T\}_{t=0}^{\infty}$ and $\{\tau_t^F\}_{t=0}^{\infty}$ are exogenous, then (8) must constrain the Ramsey planner's choice. On the other hand, if the planner picks at least one of these taxes, then that condition can be disregarded.

The reasoning presented in the above paragraph is used throughout this section. We assume that some taxes are exogenous. Then, we identify the constraints the Ramsey planner faces and solve a standard maximization problem. In the last step, we check whether or not a solution to that problem satisfies the Friedman rule.

We next introduce three assumptions on preferences. These assumptions are nested, in the sense that the last one implies the second and the second implies the first.

Let the acronym NH stand for *non-tradable homotheticity and non-tradable separability.*[1] The assumption that follows is a natural extension of the typical assumption of one-sector cash-credit models to our two-sector economy.

Assumption NH *There are functions* $H : \mathbb{R}_+^2 \to \mathbb{R}_+$*, homogeneous of degree* $k > 0$*, and* $F : \mathbb{R}_+^2 \times [0,1] \to \mathbb{R} \cup \{-\infty\}$ *that satisfy*

$$u(c_1^N, c_2^N, c^T, l) = F(H(c_1^N, c_2^N), c^T, l) \ .$$

Observe that the assumption on preferences found in Chari, Christiano and Kehoe [5] places two conditions on those goods whose marginal rate of substitution would be distorted by a positive nominal interest rate: (i) they are aggregated in homothetic fashion; and (ii) that aggregation has to be separable from the remaining arguments of the utility function. Assumption NH requires exactly this.

We next present some examples of period utility functions and discuss whether each of them respects NH. In what follows, θ_1, θ_2, θ^T, θ^l and σ are positive parameters. The period utility functions

$$u = \frac{\left[(c_1^N)^{\theta_1}(c_2^N)^{\theta_2}(c^T)^{\theta^T}(1-l)^{\theta^l}\right]^{1-\sigma} - 1}{1-\sigma} \ ; \tag{22}$$

$$u = \frac{\left\{\left[\theta_1(c_1^N)^{\nu} + \theta_2(c_2^N)^{\nu} + \theta^T(c^T)^{\nu}\right]^{\frac{1-\theta^l}{\nu}}(1-l)^{\theta^l}\right\}^{1-\sigma} - 1}{1-\sigma}, \ \theta^l < 1; \tag{23}$$

[1]NHNS could be a more appropriate shortcut. However, the other hypotheses we consider also involve non-tradable separability. Since the letters N and S would also show up in all the other acronyms, we simply dropped those two letters.

$$u = \frac{\left[(c_1^N)^{\theta_1}(c_2^N)^{\theta_2}(c^T)^{\theta^T}\right]^{1-\sigma} - 1}{1-\sigma} + f(l) \tag{24}$$

and

$$u = \left[(c_1^N)^{\theta_1}(c_2^N)^{\theta_2}\right]^{\frac{1}{\theta_1+\theta_2}} + (c^T)^{\theta^T} + f(l)\ ,\ \theta^T < 1 \tag{25}$$

respect Assumption NH, while

$$u = (c_1^N c^T)^{\frac{\theta_2}{2}} + (c_2^N)^{\theta_2} + f(l)\ ,\ \theta_2 < 1 \tag{26}$$

does not (although it homogeneously aggregates the consumption goods). Most of the quantitative macroeconomic papers adopt period preferences as in (22), (23) and (24), the last one being used mostly in open economy models.

Function (25) respects NH, but it does not aggregate all consumption goods in a homogeneous fashion. If we require, in addition to NH, that all three consumption goods satisfy a homogeneity condition, it is possible to obtain some additional results.

Assumption TNH *There are functions $H : \mathbb{R}_+^2 \to \mathbb{R}_+$, homogeneous of degree $k > 0$, $\tilde{H} : \mathbb{R}_+^2 \to \mathbb{R}_+$, homogeneous of degree $\tilde{k} > 0$, and $\tilde{F} : \mathbb{R}_+^2 \times [0,1] \to \mathbb{R} \cup \{-\infty\}$ that satisfy*

$$u(c_1^N, c_2^N, c^T, l) = \tilde{F}\left(\tilde{H}\left(\left[H(c_1^N, c_2^N)\right]^{\frac{1}{k}}, c^T\right), l\right)\ .$$

The acronym TNH stands for *tradable and non-tradable homotheticity and non-tradable separability.* Functions (22), (23) and (24) respect assumption TNH, while (25) and (26) do not.

The next assumption imposes a stronger separability on labor.

Assumption LATNH *There are functions $H : \mathbb{R}_+^2 \to \mathbb{R}_+$, homogeneous of degree $k > 0$, $\tilde{H} : \mathbb{R}_+^2 \to \mathbb{R}_+$, homogeneous of degree $\tilde{k} > 0$, $f : [0,1] \to \mathbb{R} \cup \{-\infty\}$ and a positive number σ that satisfy*

$$u(c_1^N, c_2^N, c^T, l) = \frac{\left\{\tilde{H}\left(\left[H(c_1^N, c_2^N)\right]^{\frac{1}{k}}, c^T\right)\right\}^{1-\sigma} - 1}{1-\sigma} + f(l)\ .$$

The acronym LATNH stands for *labor additivity, tradable and non-tradable homotheticity, and non-tradable separability.* Functions (22), (23), (25) and (26) do not respect LATNH, while (24) does.

In a longer version of this essay we established the result that follows.

Lemma 3 *If u satisfies NH, then $u_{1T}/u_1 = u_{2T}/u_2$, $u_{1l}/u_1 = u_{2l}/u_2$ and $W_1/u_1 = W_2/u_2$. If u satisfies TNH, then the previous three equalities hold, $u_{2l}/u_2 = u_{Tl}/u_T$ and $W_2/u_2 = W_T/u_T$. If u satisfies LATNH, then the previous five equalities hold and W_T/u_T does not depend on (c_1^N, c_2^N, c^T, l).*

Let us outline the structure of the remainder of this section. We first will show that the Friedman rule is optimal whenever the Ramsey planner can select $\{\tau_t^N\}_{t=0}^{\infty}$. Then we will show that there are tax systems in which the Ramsey planner cannot select $\{\tau_t^N\}_{t=0}^{\infty}$ and the Friedman rule is still optimal.

Proposition 4 *Assume that u satisfies NH. If the Ramsey planner can select $\{\tau_t^N\}_{t=0}^{\infty}$, then the optimal policy specifies $i_{t+1} = 0$ for all t.*

Proof. This proof builds on the arguments of Chari, Christiano and Kehoe [5]. Combine (9) and (10) to obtain

$$(1-\tau_t^l)\alpha^T u_T(t) = -(1+\tau_t^T)(l_t^T)^{1-\alpha^T} u_l(t) \ . \tag{27}$$

Note that given some allocation χ, the Ramsey planner can pick $\{\tau_t^N\}_{t=0}^{\infty}$ so that (9) is satisfied.

For the time being, assume that all taxes but $\{\tau_t^N\}_{t=0}^{\infty}$ are exogenous. Consider the problem of maximizing (3) subject to (1), (8), (27), (11), (12), (14) and (15). If it were not for (13), a solution to this problem would be a best competitive equilibrium allocation. So, if a solution respects that constraint, such a solution will yield the highest attainable utility level.

Let Γ, $\beta^t\gamma_t^N$, $\beta^t\gamma_t^F$ and $\beta^t\eta_t$ be Lagrange multipliers for, respectively, (11), the resource constraint of the non-tradable sector, (8) and (27). The first-order condition for c_{jt}^N, $t \geq 1$ and $j \in \{1,2\}$, is

$$\gamma_t^N = u_j(t)\left\{1 + \Gamma\frac{W_j(t)}{u_j(t)} - \left[\gamma_t^F + \eta_t(1-\tau_t^l)\alpha^T\right]\frac{u_{jT}(t)}{u_j(t)} + \right.$$
$$\left.\left[\Gamma\left(\sum_{s\in S}\frac{1-\delta_t^s}{1-\tau_t^l}\frac{1-\alpha^s}{\alpha^s}l_t^s\right) - \eta_t(1+\tau_t^T)(l_t^T)^{1-\alpha^T}\right]\frac{u_{jl}(t)}{u_j(t)}\right\} \ .$$

The equalities in Lemma 3 lead to $u_1(t) = u_2(t)$.

Let us now consider the case in which the Ramsey planner can choose other taxes besides $\{\tau_t^N\}_{t=0}^{\infty}$. If $\{\delta_t^N\}_{t=0}^{\infty}$, $\{\delta_t^T\}_{t=0}^{\infty}$ and $\{\tau_t^l\}_{t=0}^{\infty}$ are still exogenous, it suffices to drop the proper constraints from the Ramsey problem. If $\{\tau_t^l\}_{t=0}^{\infty}$ is endogenous, use (8), (9) and (10) to conclude that

$$\frac{u_l(t)}{1-\tau_t^l} = \frac{-u_T(t)}{1+\tau_t^T}\frac{\alpha^T}{(l_t^T)^{1-\alpha^T}} = \frac{-\frac{\beta^{-t}\alpha^T}{(l_t^T)^{1-\alpha^T}}\frac{u_T(0)}{1+\tau_0^T}}{\prod_{r=1}^{t}[1+(1-\tau_r^F)i_r^F]} \ .$$

Thus, it is possible to rewrite (11) so that it does not depend on $\{\tau_t^l\}_{t=0}^{\infty}$. Then, exactly the same reasoning establishes that that $u_1(t) = u_2(t)$. Finally, if $\{\delta_t^N\}_{t=0}^{\infty}$ or $\{\delta_t^T\}_{t=0}^{\infty}$ is endogenous, the only circumstance in which the Ramsey planner will not be willing to fully tax the respective profits is when the government can balance its budget without using the available distorting taxes. But this amount to saying that $u_1(t) = u_2(t)$. So, there is no loss of generality in assuming the corresponding profit is taxed at a 100% rate. Hence, it is possible to rewrite (11) as

$$u_1(0)c_{1,0}^N + u_2(0)\left[\frac{(1+i_0)B_0}{(1+\tau_0^N)p_0^N} + \frac{M_0}{(1+\tau_0^N)p_0^N} - c_{1,0}^N\right] + \\ u_T(0)\frac{[1+(1-\tau_0^F)i_0^F]B_{H0}^F}{1+\tau_0^T} = \sum_{t=0}^{\infty}\beta^t W(t) \ . \tag{28}$$

Again, the previous reasoning establishes that $u_1(t) = u_2(t)$. □

The intuition for the above result is as follows. A positive nominal interest rate raises revenue for the government and distorts the margin u_1/u_2. On the other hand, a tax on consumption of non-tradables also raises revenue, but it does not distort that margin. So, it is never optimal to use the nominal interest rate to raise revenue. In other words, as Schmitt-Grohé and Uribe [17] pointed out, in a cash-credit model inflation acts as an imperfect substitute for a tax on consumption.

It follows directly from Proposition 4 that $\{\tau_t^N\}_{t=0}^{\infty}$ being endogenous is a sufficient condition for the optimality of the Friedman rule. We will now establish a sequence of results where $\{\tau_t^N\}_{t=0}^{\infty}$ is exogenous and zero nominal interest rates are still optimal.

Proposition 5 *Assume that u satisfies NH. If the Ramsey planner can choose $\{\delta_t^N\}_{t=0}^{\infty}$, $\{\delta_t^T\}_{t=0}^{\infty}$ and at least three of the four sequences $\{\tau_t^N\}_{t=0}^{\infty}$, $\{\tau_t^T\}_{t=0}^{\infty}$, $\{\tau_t^l\}_{t=0}^{\infty}$ and $\{\tau_t^F\}_{t=0}^{\infty}$, then the optimal policy specifies $i_{t+1} = 0$ for all t.*

Proof. Suppose that $\{\tau_t^l\}_{t=0}^{\infty}$ is exogenous and $\{\tau_t^N\}_{t=0}^{\infty}$, $\{\tau_t^T\}_{t=0}^{\infty}$, and $\{\tau_t^F\}_{t=0}^{\infty}$ are endogenous. Thus, given some allocation χ, the Ramsey planner can pick $\{\tau_t^N\}_{t=0}^{\infty}$ so that (10) is satisfied, $\{\tau_t^T\}_{t=0}^{\infty}$ so that (9) holds and $\{\tau_t^F\}_{t=0}^{\infty}$ to satisfy (8). Thus, the Ramsey planner can disregard these constraints. Of course, the same is true regardless of which sequence is exogenous.

Observe that if a best competitive equilibrium allocation can be implemented by policies that satisfy $\delta_t^N < 1$ or $\delta_t^T < 1$ for some t, this amounts to saying that the available lump-sum tax revenues have not been fully used up. Then, the allocation in question must be Pareto efficient and the proposition is established. So, in what follows, we will assume that $\delta_t^N = \delta_t^T = 1$.

Consider the problem of maximizing (3) subject to (1), (28), (12), (14), and (15). Let Γ and $\beta^t\gamma_t^N$ be the previously defined Lagrange multipliers. For $t \geq 1$ and $j \in \{1, 2\}$, the respective first-order condition for c_{jt}^N is

$$\gamma_t^N = u_j(t)\left[1 + \Gamma\frac{W_j(t)}{u_j(t)}\right] . \tag{29}$$

Apply Lemma 3 to conclude that $u_1(t) = u_2(t)$, which implies $i_t = 0$. □

As discussed in the above proof, when just one of the conceivable distorting taxes is exogenous, the Ramsey problem is a standard second-best one. It is well known that the

Friedman rule is a feature of second-best policies in cash-credit models. Chari, Christiano and Kehoe [5] provided the intuition for this result. Given the homogeneity and separability assumption on the consumption of non-tradables, it is optimal to tax these goods at the same rate. But a positive nominal interest rate would imply taxing c_{1t}^N at rate higher than c_{2t}^N. Therefore, the Ramsey planner will be willing to implement the Friedman rule.

It has been argued that in an open economy, the existence of a given international interest rate will impact the behavior of domestic agents in a way that the Friedman rule may fail to be optimal. A possible way to circumvent such a constraint is to tax the income on foreign assets. This possibility is encompassed in Proposition 5. However, the same proposition also shows that even if $\tau_t^F = 0$, the Friedman rule may still be optimal. All that is needed is for the Ramsey planner to have access to a sufficiently large set of alternative tax instruments.

If we add Assumption TNH to the hypothesis of the previous proposition, we will conclude that all consumption goods are taxed in a uniform way. The intuition is the same one provided for Proposition 5.

Proposition 6 *Assume that u satisfies TNH. Under the remaining assumptions of Proposition 5, the optimal policy specifies $i_{t+1} = 0$ and $\tau_{t+1}^N = \tau_{t+1}^T$ for all t.*

Proof. Since TNH implies NH, the optimality of the Friedman rule follows directly from Proposition 5. For the second equality, let $\beta^t \gamma_t^T$ be a Lagrange multiplier for the tradable sector resource constraint. For $t \geq 1$, The first-order condition for c_t^T is

$$\gamma_t^T = u_T(t)\left[1 + \Gamma \frac{W_T(t)}{u_T(t)}\right]. \tag{30}$$

On the other hand, the first-order conditions for l_t^T and l_t^N imply

$$\gamma_t^T \alpha^T (l_t^T)^{\alpha^T - 1} = \gamma_t^N \alpha^N (l_t^N)^{\alpha^N - 1}. \tag{31}$$

Combine (31) with (29) and (30). Then, apply Lemma 3 to obtain (17). Hence, (9) implies $\tau_t^N = \tau_t^T$. □

The assumption that $\{\delta_t^N\}_{t=0}^\infty$ and $\{\delta_t^T\}_{t=0}^\infty$ are endogenous cannot be dispensed with in Proposition 6. The intuition is as follows. For simplicity, assume that $\delta_t^T = 1$ and $\delta_t^N = 0$. A possible way for the Ramsey planner to indirectly tax the profit ψ_t^N is to raise τ_t^N. This is so because the real profit depends positively on the output. Therefore, the planner faces a trade-off. By selecting $\tau_t^N = \tau_t^T$, the margin u_T/u_2 is not distorted and by selecting $\tau_t^N > \tau_t^T$ it will be possible to tax ψ_t^N. A compromise between these conflicting goals may call for some difference in consumption tax rates.

The issue of the endogeneity of $\{\delta_t^N\}_{t=0}^\infty$ and $\{\delta_t^T\}_{t=0}^\infty$ is a little subtler when it comes to the optimality of the Friedman rule. Suppose that taxes on profits are both exogenous and smaller than 100%. If $\{\tau_t^N\}_{t=0}^\infty$ is exogenous, then the Friedman rule can fail to be optimal. The intuition again is simple. If the planner cannot select $\{\tau_t^N\}_{t=0}^\infty$, she may use the a positive nominal interest rate to tax ψ_t^N in an indirect way.

The above discussion becomes more relevant if we recall that the assumption of monopolistic competition is widely used in the literature. In such a context, equilibrium profits can be positive. Hence, the lack of both consumption and profit taxes may render the Friedman rule non-optimal in that class of models.

The next result is a direct consequence of Proposition 6, but it has some interesting implications.

Corollary 7 *Assume that u satisfies TNH. If consumption tax rates are exogenous and satisfy $\tau_t^N = \tau_t^T$ and the Ramsey planner can select $\{\tau_t^l\}_{t=0}^{\infty}$, $\{\tau_t^F\}_{t=0}^{\infty}$, $\{\delta_t^N\}_{t=0}^{\infty}$ and $\{\delta_t^T\}_{t=0}^{\infty}$, then the optimal policy specifies $i_{t+1} = 0$ for all t.*

Proof. Without loss of generality, assume that $\delta_t^N = \delta_t^T = 1$. Given an allocation χ, the Ramsey planner can set τ_t^F and τ_t^l to satisfy (8) and (10). Consider now the problem of maximizing (3) subject to (1), (28), (12), (14) and (15). As in Proposition 6, its solution will satisfy $u_1(t+1) = u_2(t+1)$. Hence, it remains to show that it satisfies (9). But this can be achieved with the same reasoning we adopted in Proposition 6 to show that $\tau_{t+1}^N = \tau_{t+1}^T$. $\square$

The intuition for the above result is as follows. We showed in Proposition 6 that under TNH a second-best policy satisfies $\tau_{t+1}^N = \tau_{t+1}^T$. Hence, we just introduced an optimality condition as a constraint. Since such a constraint will not bind, the features of the optimal policy are not affected.

The hypothesis of Corollary 7 encompasses the case in which $\tau_t^N = \tau_t^T = 0$. Schmitt-Grohé and Uribe [17] concluded, in a one-sector small open economy setup, that if the Ramsey planner does not have access to consumption taxes the Friedman rule is not optimal. Our conclusion is different from theirs because we allowed the Ramsey planner to pick $\{\tau_t^F\}_{t=0}^{\infty}$, $\{\delta_t^N\}_{t=0}^{\infty}$ and $\{\delta_t^T\}_{t=0}^{\infty}$ in an optimal fashion.

Corollary 7 has implications to the optimal monetary policy in the presence of underground economic activity. For instance, consider a situation in which the government cannot observe the consumption of some goods. However, it can observe the income generated in the sectors that produce these goods. We can relate that situation to Corollary 7 by setting $\tau_t^N = \tau_t^T = 0$. In such a context, the Friedman rule is optimal.

We established in Proposition 6 that if TNH holds, a second-best policy induces an allocation that displays, for $t \geq 1$, properties (16) and (17) of a Pareto efficient allocation. Under assumption LATNH, it is possible to show that a second-best allocation satisfies (18). Of course, this last condition is satisfied in a competitive equilibrium if and only if (21) holds.

Proposition 8 *Assume that u satisfies LATNH. Under the remaining assumptions of Proposition 5, the optimal policy satisfies $i_{t+1} = 0$, $\tau_{t+1}^N = \tau_{t+1}^T$ and (21) for all t.*

Proof. Since LATNH implies TNH, the first two equalities follow from Proposition 6. Let γ_t^T and Γ have the previously defined meanings. The first-order conditions for x_t and x_{t+1}

imply $\beta\gamma_{t+1}^T/\gamma_t^T = (1+i_{t+1}^F)^{-1}$. Combine this equality with (30) to obtain

$$\frac{1}{1+i_{t+1}^F} = \beta\frac{u_T(t+1)}{u_T(t)}\frac{\left[1+\Gamma\frac{W_T(t+1)}{u_T(t+1)}\right]}{\left[1+\Gamma\frac{W_T(t)}{u_T(t)}\right]}.$$

This last expression combined with Lemma 3 and equation (7) yields (21). □

The intuition for the last proposition is as follows. Under LATNH, date t and $t+1$ consumption levels are intra and intertemporally separable from labor decisions. Therefore, the traditional uniform consumption taxation result holds at an intertemporal level. As a side comment, if we interpret the foreign bonds as a storage technology for the tradable good, Proposition 8 has an obvious analogy with the results on the taxation of physical capital income in Chari and Kehoe [6].

We can derive from Proposition 8 a result equivalent to Corollary 7. That is, if we assume that $\{\tau_t^N\}_{t=0}^\infty$, $\{\tau_t^T\}_{t=0}^\infty$ and $\{\tau_t^F\}_{t=0}^\infty$ are exogenous and satisfy $\tau_{t+1}^N = \tau_{t+1}^T$ and (21), then the Friedman rule is optimal. Of course, the implications presented after Proposition 6 remains valid in this context in which the Ramsey planner has access to a smaller set of tax instruments.

As Chari and Kehoe [6] did while studying the Friedman rule, it is possible to relate the findings of this section to the classic paper of Diamond and Mirrlees [9] on production efficiency. We can interpret c_{1t}^N and c_{2t}^N as two intermediate inputs for the production of a final good c_t^N. Production efficiency requires taxing c_{1t}^N and c_{2t}^N at the same rate. In our monetary economy, this entails implementing the Friedman rule.

The analogy with Diamond and Mirrlees is useful, but a word of caution is in order. Consider Proposition 4. Suppose that all taxes but $\{\tau_t^N\}_{t=0}^\infty$ are exogenous and equal to zero. Assume that to balance the government budget, the optimal value of τ_t^N is positive at every period. Hence, $\tau_t^N \neq \tau_t^T$ and the consumption goods are not taxed uniformly. If we interpret c_{1t}^N, c_{2t}^N and c_t^T as inputs to produce a final good c_t, we conclude that the intermediate goods are not taxed at the same rate. However, the optimality of the Friedman rule implies that the Ramsey planner picks a common rate for those taxes she can select.

5 Conclusion

The optimality of the Friedman rule is a classic result in monetary economics. However, that conclusion relied on the hypothesis that the Ramsey planner had access to a complete set of distorting tax instruments. Some papers showed that if policymakers were unable to implement some conceivable distorting taxes, then the optimal monetary policy would prescribe positive nominal interest rates.

In this paper we investigated the relation between the optimality of the Friedman rule and the set of distorting tax instruments available to a benevolent planner. We carried out the study in a two-sector (tradable and non-tradable goods) deterministic small open economy framework. We introduced money by assuming that a fraction of people's purchases of non-tradables had to be paid in cash.

We showed that the Friedman rule is optimal whenever the Ramsey planner can select the tax rate on the consumption of non-tradables. Moreover, there exist several situations in which that tax is exogenous and the Friedman rule remains optimal. Hence, the optimality of the Friedman rule may survive even if we severely restrict the ability of the Ramsey planner to select the tax rates.

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Appendix

We have argued that the results on the optimality of the Friedman rule presented in Section 4 can be generalized to other monetary environments. Although we provide intuition for them, we believe that formally obtaining the equivalent of at least one of those results in a distinct monetary model is a valuable exercise.

Ideally, we would consider an economy that is complex enough to allow us to carry out an exercise similar to those of Section 4 and simple enough so that numerical methods are not necessary. To attain these two competing goals, we adopt in this section a simple two-sector closed economy in which money is introduced by means of a shopping-time friction.

The economy produces two goods, 1 and 2. For convenience, define $J = \{1, 2\}$. Both people and government consume these goods. At each date t, resource constraints require, for $j \in J$,

$$c_{jt} + g_{jt} = l_{jt}^{\alpha_j} \tag{32}$$

where c_{jt} and g_{jt} denote, respectively, people's and government's consumption of good j and l_{jt} denotes the amount of time people work in that sector. As before, α_j belong to $(0, 1]$ and the t subscript denotes time.

The government levies taxes on profits, consumption and labor income. These taxes may vary across sectors. The government budget constraint is

$$\sum_{j\in J} p_{jt}g_{jt} + (1+i_{t-1})B_{t-1} + M_{t-1} = B_t + M_t + \sum_{j\in J} \tau^l_{jt}w_{jt}l_{jt} + \tau^c_{jt}p_{jt}c_{jt} + \delta_{jt}\psi_{jt},$$

where p_{jt} is the price of good j; τ^c_{jt} is the tax rate on the consumption of good j; τ^l_{jt} is the tax rate on the the gross nominal wage w_{jt} paid in sector j; δ_{jt} is the tax rate on the profit ψ_{jt} of firm j; M_t and B_t correspond to household end-of-period cash and bond holdings,[2] while i_{t-1} is the nominal interest rate. We assume that the government has no outstanding debt at the beginning of date zero, that is, $(1+i_{-1})B_{-1} + M_{-1} = 0$.

There exists a single household. Its preferences are described by

$$\sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, h_t) \tag{33}$$

where h_t corresponds to leisure. Its budget constraint is

$$\begin{aligned}\sum_{j\in J}(1+\tau^c_{jt})p_{jt}c_{jt} + B_t + M_t \le (1+i_{t-1})B_{t-1} + \\ M_{t-1} + \sum_{j\in J}\left[(1-\tau^l_{jt})w_{jt}l_{jt} + (1-\delta_{jt})\psi_{jt}\right] \ .\end{aligned} \tag{34}$$

Standard boundedness constraints prevent Ponzi schemes. Purchases of good 1 require household time.[3] Let s_t denote the amount of time allocated to the purchase of that good. That variable must satisfy

$$s_t \ge v\left(c_{1t}, \frac{M_t}{(1+\tau^c_{1t})p_{1t}}\right). \tag{35}$$

Denote the ratio $M_t/[(1+\tau^c_{1t})p_{1t}]$ by m_t. The transaction function v is homogeneous of degree $k \ge 0$ and non negative. It satisfies standard monotonicity, differentiability and convexity conditions. Moreover, if $v_m(c_1, m) = 0$, then $v(c_1, m) = 0$, where $v_m = \partial v/\partial m$.

The household maximizes (33) subject to (34), (35) and $h_t + l_{1t} + l_{2t} + s_t \le 1$. On the other hand, each firm j behaves in a competitive fashion. It selects l_{jt} to maximize $\psi_{jt} = p_{jt}l_{jt}^{\alpha_j} - w_{jt}l_{jt}$.

[2] Here we departed from the dating convention adopted in the previous sections of this paper. So far, a stock variable with subscript t would correspond to the beginning of period value of the variable in question. We believe that the previous convention was the natural one for our cash-credit economy. On the other hand, the dating convention adopted in this section is the one usually adopted in shopping-time models. Needless to say, none of our results depend on the chosen notation.

[3] There are many ways one can interpret the monetary friction and other features of the economy we consider in this section. For instance, set all taxes in sector 2 equal to zero and think of that sector as one that produces household goods. Clearly, in such a context the shopping-time constraint should affect only the purchases of c_1.

As before, $u_1(t)$, $u_2(t)$, $u_h(t)$, $v_1(t)$ and $v_m(t)$ denote the partial derivatives of u and v evaluated at the point $(c_{1t}, c_{2t}, h_t, m_t)$. Similarly, $v(t) = v(c_{1t}, m_t)$. The factor $W(t)$ is redefined according to

$$W(t) = u_1(t)c_{1t} + u_2(t)c_{2t} - u_h(t)(1 - h_t) + (1 - k)u_h(t)v(t) \ ,$$

while W_1, W_2, W_h, and W_m denote its partial derivatives.

The definition of competitive equilibrium is standard. Given a tax policy $\left\{(\tau^c_{jt}, \tau^l_{jt}, \delta_{jt})_{j\in J}\right\}_{t=0}^{\infty}$, an object $\left\{(c_{jt}, l_{jt})_{j\in J}, h_t, m_t\right\}_{t=0}^{\infty}$ is attainable if and only if it respects the feasibility conditions in (32) and

$$\frac{u_2(t)}{u_h(t)} = \frac{1+\tau^c_{2t}}{1-\tau^l_{2t}} \frac{l_{2t}^{1-\alpha_2}}{\alpha_2} \ , \tag{36}$$

$$\frac{u_1(t)}{u_h(t)} = v_1(t) + \frac{1+\tau^c_{1t}}{1-\tau^l_{1t}} \frac{l_{1t}^{1-\alpha_1}}{\alpha_1} \ , \tag{37}$$

$$\sum_{t=0}^{\infty} \beta^t \left[W(t) - u_h(t) \sum_{j\in J} \frac{1-\delta_{jt}}{1-\tau^l_{jt}} \frac{1-\alpha_j}{\alpha_j} l_{jt} \right] = 0 \ , \tag{38}$$

$$h_t + l_{1t} + l_{2t} + v(t) = 1 \ . \tag{39}$$

Condition (36) ensures that the marginal rate of substitution between leisure and good 2 is consistent with the prevailing real wage rate and the tax rates in that sector. Condition (37) can be interpreted in a similar way once we remember that the consumption of good 1 must respect (35). Equation (38) consolidates the household budget constraints (34), while (39) simply ensures that the household uses exactly its endowment of time.

Simple manipulations of household first-order conditions show that

$$\frac{1}{1+i_t} = \frac{u_1(t) + u_h(t)\left[v_m(t) - v_1(t)\right]}{u_1(t) - u_h(t)v_1(t)} \ .$$

Therefore, the Friedman rule will hold in a competitive equilibrium if and only if $v_m(t) = 0$ for all t.

A quick inspection of (37) shows that τ^c_{1t} and τ^l_{1t} may distort the margin u_1/u_h. However, even if we had $\tau^c_{1t} = -\tau^l_{1t}$, that margin could be distorted. For that to happen, it would suffice to have $v_1(t) > 0$. On the other hand, $kv(t) = v_1(t)c_{1t} + v_m(t)m_t$. Moreover, $v_m(t) = 0$ implies $v(t) = 0$. Hence, if $v_m(t) = 0$, then $v_1(t) = 0$. So, implementing the Friedman rule entails not using a tax on money services that distorts the marginal rate of substitution between c_1 and h.

In the spirit of Proposition 4, suppose that the Ramsey planner can choose $\{\tau^c_{1t}\}_{t=0}^{\infty}$. If the intuition we have developed is correct, the Friedman Rule has to be optimal. The reason is simple. Since τ^c_{1t} is endogenous, the Ramsey planner has two instruments to impact the margin $u_1(t)/u_h(t)$. The first is τ^c_{1t} itself. The second is i_t, since $v_1(t)$ depends on m_t which in turn depends on i_t. Vis-à-vis τ^c_{1t}, a positive nominal interest rate has the disadvantage of reducing the time the household has available for leisure and production of goods. Hence, the Ramsey policy should call for implementing the Friedman rule whenever $\{\tau^c_{1t}\}_{t=0}^{\infty}$ is endogenous.

We now formalize the argument of the last paragraph. If $\{\tau_{1t}^c\}_{t=0}^\infty$ is endogenous, the Ramsey problem is to select $\{(c_{jt}, l_{jt})_{j\in J}, h_t, m_t\}_{t=0}^\infty$ to maximize (33) subject to (32), (38) and (39). Condition (36) will not restrain the Ramsey planner if $\{\tau_{2t}^c\}_{t=0}^\infty$ or $\{\tau_{2t}^l\}_{t=0}^\infty$ is endogenous. Otherwise, this constraint will show up in the Ramsey problem.

Let Γ and $\beta^t\gamma_t^h$ be Lagrange multipliers for, respectively, (38) and (39). The first-order condition with respect to m_t is

$$\left[(k-1)\Gamma u_h(t) + \gamma_t^h\right] v_m(t) = 0 \ . \tag{40}$$

The above condition also appears in Correia and Teles [7] and De Fiore and Teles [10]. As these authors pointed out, both Lagrange multipliers are positive. So, for $k \geq 1$, $v_m(t) = 0$ and the Friedman rule is optimal.

The case where $k < 1$ requires additional work. Denote the partial derivative of u_h/u_2 with respect to h by D_h. We assume $D_h < 0$ and $u_{1h}c_1 + u_{2h}c_2 \geq 0$. Besides this, for the case in which the Ramsey planner can choose only τ_{1t}^c, we assume that the exogenous taxes τ_{2t}^l and τ_{2t}^c satisfy $(1-\tau_{2t}^l)/(1+\tau_{2t}^c) \leq R_t^*$, where R_t^* denotes the value that ratio would assume in a second-best solution.

Write (36) as $[u_h(t)/u_2(t)](l_{2t}^{1-\alpha_2}/\alpha_2) = (1-\tau_{2t}^l)/(1+\tau_{2t}^c)$ and denote its Lagrange multiplier by $\beta^t\eta_t$. Combining the first-order condition for h_t with (40) we obtain

$$(k-1)\Gamma u_h(t) + \gamma_t^h = u_h(t) - \eta_t D_h(t)\frac{l_{2t}^{1-\alpha_2}}{\alpha_2} + \Gamma\left\{\sum_{j\in J} u_{jh}(t)c_{jt} + \right.$$

$$\left. k u_h(t) - u_{hh}(t)\left[\{1-[h_t + v(t)]\} + kv(t) + \sum_{j\in J}\frac{1-\delta_{jt}}{1-\tau_{jt}^l}\frac{1-\alpha_j}{\alpha_j}l_{jt}\right]\right\} \ . \tag{41}$$

Our assumptions on τ_{2t}^l, τ_{2t}^c and R_t^* ensure that $\eta_t \geq 0$. Thus, the right-hand side of (41) is strictly positive. Hence, $(k-1)\Gamma u_h(t) + \gamma_t^h > 0$ and (40) implies that $v_m(t) = 0$.

As we have just mentioned, the above result is the equivalent of Proposition 4 for this two-sector shopping-time setup. We could also obtain other results similar to those of Section 4.