DYNAMIC STRUCTURAL MODELS AND THE HIGH INFLATION PERIOD IN BRAZIL: MODELLING THE MONETARY SYSTEM

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Abstract:

In this paper we develop a linear structural dynamic econometric model for the high inflation period in Brasil. The maing goal is to obtain a parsimonious model that accounts for complex dynamic present in the monetary system during the period describing the relationships among output, inflation rate, interest rates and real money. We start the analysis after the Cruzado plan cast in 1986 following a progressive strategy in deriving the econometric model. The results show that we can identify a long run money demand equation and the model describes parsimoniously and in detail the relationship among the variables despite all the instability present in the second half of the 1980's in Brazil with special attention to the role played by nominal wage inflation in determining the dynamics observed in price inflation.

Key words: VAR, Cointegration, Money Demand, Simultaneous Equations Models

Classificação JEL: E31, E41, C32.

1) Introduction

The empirical analysis of the money demand in Brazil has received some interest in the past given the relative sophistication observed in the monetary system. If we consider Cagan's classical definition of hyperinflation Brazil experienced a very short lived hyperinflation from December 1989 and March 1990, a period which is not even close to the shortest hyperinflations in Europe studied by Cagan (Austria, Greece and Poland) which lasted for 17 months. Nevertheless if we use the definition of high inflation as in Fisher et al (2002) namely those periods where the annual rates crosses 100% and only ends when it stays below 100% for more than one year, then the high inflation period lasted 15 years and 2 months (between April 1980 and May 1995) and the accumulated inflation rate for the period is 20,759,903,275,651% as noticed in Franco (2004).

Remarkably the literature on this subject does not seem to present a detailed analysis of such phenomenon that is comparatively even rarer than hyperinflations.

One strand in the literature has dedicated attention to develop empirical models for the money demand or monetary sector in Brazil rather than testing the Cagan's model adequacy and includes Cardoso (1983), Gerlach and Simone (1985), Calomiris and Domowitz (1989), Fadil and MacDonald (1992). Despite the span of time involved in these works, some of them share the same restriction, namely they do not attempt modelling the period posterior to the Cruzado Plan cast in 1986[2].

A second approach in the empirical literature has been testing Cagan's model adequacy or a variant of it in describing the demand for money in Brazil as in Phylatkis and Taylor (1993) Engsted (1993 a), Rossi (1994) or a variant of it as in Feliz and Welch (1997) and Tourinho (1996).

Whilst most of the evidence in this strand has been favourable to the Cagan model, its relatively simplicity in describing the money demand as a function of expected inflation does not allow a more detailed analysis of the long run relationships present in the sector. The Cagan model was proposed originally to describe short periods of high inflation rates, whereas the history in Brazil has showed a different phenomenon, namely high inflation being questionable its use in modelling empirically the money demand. Furthermore Phylatktis and Taylor (1993) and Engsted (1993 a) concentrated their attention to a period where actually inflation rates were moderate with both samples ending in 1986 when the Brazilian economy experienced the first stabilization plan. Such restriction is present also in other papers that devoted to study the Brazilian case as Juselius (2002), Durevall (1998), Feliz and Welch (1997), all of them imposing 1986 as the ceiling point in the sample length.

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[2] For obvious reasons such "restriction" should not apply to the first two papers.

The findings in Juselius (2002) of a stable liquidity ratio and a long run relationship where prices grow less than proportionally to the expansion of M3, only reinforces the argument that simply testing the adequacy of the Cagan model to the money demand in Brazil and arguing that it adequately describes the data, is a procedure that leaves out subtle economic relationships which could only be explored in a deeper econometric analysis.

The alternative methodology proposed is based on Hendry and Richard (1982), Clements and Mizon (1991), Hendry and Doornik (1994), Hendry and Mizon (1993), Hendry (1995), and is described in detail in Mizon (1995). The core of the analysis is to explore the assumption that valuable information in econometric modelling can come from different sources as economic theory, economic history of the period studied as well as how data is defined and measured. A progressive strategy in the sense that we do not assume the knowledge of the complete economic structure that links the economic variables, or more specifically that the theoretical model coincides with the DGP is followed in selecting the final model. We search for a dynamic structural model (SEM) starting from a general congruent and linear vector auto regression (VAR) which constitutes a basis for inference.

The main objective of this paper is exactly to explore the high inflation period in Brazil by constructing a small econometric model for the monetary sector. The generalized indexation present from prices to wages more specifically was assumed as the main force behind the inflationary spiral that started in 1986 is explored. A theoretical model based on Novaes (1991, 1993) and addressing the indexation from prices to wages is tested on the data through imposing the restrictions implied by the theoretical model on the econometric model.

The paper presents a contribution to the literature not only because it explores in more details the period that cover the stabilization plans in Brazil but also because it uses a methodological approach that allows subtle economic relationships to be drawn from the data. We concentrate our attention to the period that runs from March 1986 until June 1994 when the Real plan imposes the new stable currency. The paper is divided as the following: In section 2 we present the methodology and a brief discussion of its implications in the empirical modelling. Section 3 presents the statistical model used in the analysis and results obtained. Section 4 discusses the role of nominal wage inflation and section 5 concludes. All results were generated using either Pc-Give or Ox.

2) Statistical Model

In this section, we consider the relevant aspects of the statistical model and the modelling strategy. The aim is to show in more technical details how we derive a linear dynamic structural econometric model (SEM) for the Brazilian data, presented in the next section, starting from general linear vector auto regression model (VAR) that is considered then a basis for inference.

The departure point of our analysis is a vector of stochastic variables which have a joint density function given by: $D_z\left(\mathbf{Z}_T^1/\mathbf{Z_0},\Lambda_t^1,\theta\right)$, where the density for the vector $\mathbf{Z_t}$ comprising M variables is conditional to a set of initial values, $\mathbf{Z_0}$ and with θ representing the parameters of interest. The joint density can be then rewritten as a set of sequentially conditional densities through a sequential factorization given by:

$$D_z\left(\mathbf{Z}_\mathrm{T}^\mathbf{1}/\mathbf{Z_0},\theta\right)=\prod_{t=1}^{T}D_z\left(\mathbf{z_t}/\mathbf{Z}_{t-1},\theta\right)\quad(1).$$

Such conditioning process assumes that the joint density is a statistical representation of the economy and indeed that the vector stochastic process $\mathbf{Z_t}$ represents the Data Generation Process (DGP)[3]. It is worthwhile to notice that by mapping the economic mechanism, namely all the agents

[3] Assumes that $\mathbf{Z_t}$ is a $k\times1$ vector. The vector stochastic process $\mathbf{Z_t}$ is formed then by $\mathbf{Z}(\mathbf{t})\equiv\left[Z_1(t),Z_2(t),\dots Z_k(t)\right]$ where each one of the $Z_k(t)$ is a stochastic process, namely, for a given probability space define the function $Z(\cdot,\cdot):S\times\mathrm{T}\to\Re$. The ordered sequence of random variables $\left\{Z(\cdot,t),t\in\mathrm{T}\right\}$ is called then a stochastic random process. In this case for each $t\in\mathrm{T}$ we have a different random variable and for each $s\in S$ we

actions of each single agent in a span of time in the geographic region relevant to the analysis which can be global in open economies into a joint density of a vector of stochastic variables a considerable reduction has taken place since we are assuming that (1) is a statistical representation of all actions in the economy. Notice that despite this the DGP still indeed unmanageably large.

The DGP has this property at this stage because the economy is represented as a system where ultimately all the economic variables represented in the stochastic vector $\mathbf{Z}_t$ are endogenous and determined by interactions among each other. Such representation needs further reductions since usually the sample size of the macroeconomic time series do not allow the estimation of such large econometric models that would represent the DGP, further the DGP is assumed to be unknown since the observed variables which constitute the macroeconomic time series available for modelling are in general aggregations of the original set of variables in $\mathbf{Z_t}$ across time and individuals which implies that some level of marginalization is inevitable in the sense that we do not have their correct representation. This not necessarily means that we can assume that any marginalization is an adequate representation of the DGP. Indeed it is at this point that the approached followed here has its strengthful point. Any final model resulted from the analysis here is subject to test and will provide therefore evidence of the adequacy of the reduction from the DGP that it assumes to be representing and theoretical hypothesis are subject to testing not simply empirical validation.

We assume then that the set of relevant variables $\mathbf{Y_t}$ with $n < M$ is derived by marginalization from equation (1) in such way that we have a partition of $\mathbf{Z}_T^1$ into $\mathbf{Z}_T^1 = \left(\mathbf{W}_T^1, \mathbf{Y}_T^1\right)$ and further marginalization of (1) into:

$$D_z\left(\mathbf{Z}_T^1 / \mathbf{Z}_0, \theta\right) = D_{w/y}\left(\mathbf{W}_T^1 / \mathbf{Y}_T^1, \mathbf{W}_0, \iota_t\right) D_y\left(\mathbf{Y}_T^1 / \mathbf{Y}_0, \Lambda_t^1, \phi\right).$$

This marginalization defines then the Haavelmo distribution as[4]:

$$D_Y\left(\mathbf{Y_T^1} / \mathbf{Y_0}, \Lambda_t^1, \phi\right) = \prod_{t=1}^{T} D_y\left(\mathbf{y_t} / \mathbf{Y}_{t-1}^{t-s}, \mathbf{D_t}, \lambda\right) \quad (2)$$

where Λ represent a set of deterministic variables[5]. The sequence of densities in equation (2) is conditioned on $\mathbf{D_t}$, representing the use of contemporaneous or lagged information only in deterministic terms, on $\mathbf{Y_{t-1}^{t-s}}$ representing a maximum lag length imposed for tractability, and finally on $\lambda = f(\phi)$ meaning that the original set of parameters of interest ϕ might contain many transient parameters.

The Haavelmo distribution plays an important role in the sense that it comprises a set of a priori information, or knowledge about the economic events and empirical observations that defines the relevant variables in equation (2). It is also important to notice that its formulation comprises the assumption that it is possible to learn the parameters of interest, μ say, from ϕ alone in the sense that $\mu = f(\phi)$ only.

Furthermore equation (2) entails also a sequential factorization of $\mathbf{Y_t}$ in such a way that its right hand side generates a mean innovation process given by:

$$\varepsilon_t = \mathbf{y_t} - E\left[\mathbf{y_t} / \mathbf{Y}_{t-1}^{t-s}\right] \quad (3).$$

have a different realization of the process. In this sense the marginalization imposed by (2) and the restrictions imposed in the econometric model that is assumed to represent (2) either in terms of time-heterogeneity (stationarity) or in terms of the memory of the process (VAR(p)) are placed ultimately for tractability.

[4] Such indiscriminate use of the term Haavelmo distribution deserves a more careful definition. According to Clements and Mizon (1991) a well specified statistical model within which is possible to test competing structural hypothesis defines a Haavelmo distribution. Hendry (1995, p. 406) also defines the distribution as given by specifying the variables of interest, their status, their degree of integration, data transformations the history of the process and the sample period.

[5] This set of deterministic variables contains seasonals dummies, step dummies, constant, etc and will be exactly defined for each model used.

Nevertheless a similar sequential factorization of the DGP considering the same lag truncation in (2) generates:

$$D_z\left(\mathbf{Z}_{t-s}^{t-1}/\mathbf{Z}_0,\theta\right)=D_{w/y}\left(\mathbf{W}_{t-s}^{t-1}/\mathbf{Y}_{t-s}^{t-1},\mathbf{Z_0},\mathbf{W}_{t-s}^{t-1},\mathbf{Y}_{t-s}^{t-1},\theta_{a_t}\right)D_y\left(\mathbf{Y}_{t-s}^{t-1}/\mathbf{W}_{t-s}^{t-1},\mathbf{Y}_{t-s}^{t-1},\mathbf{Z_0},\theta_{b_t}\right)$$

$$\eta_t=\mathbf{z_t}-E\left[\mathbf{z_t}/\mathbf{Z}_{t-1}^{t-s}\right] \quad (4).$$

Therefore given the partition of the vector $\mathbf{Z_t}$ into $\mathbf{Z}_T^1=\left(\mathbf{W}_T^1,\mathbf{Y}_T^1\right)$ then the condition to reducing (1) to (2) without loosing information is that indeed: $E\left[\varepsilon_t/\mathbf{Z}_{t-s}^{t-1}\right]=E\left[\varepsilon_t/\mathbf{Y}_{t-1}^{t-s},\mathbf{W}_{t-1}^{t-s}\right]=0$ however we only know that $E\left[\varepsilon_t/\mathbf{Y}_{t-1}^{t-s}\right]=0$. Such condition imply that θ_a is irrelevant in the estimation and that $D_y\left(\mathbf{Y}_{t-s}^{t-1}/\mathbf{W}_{t-s}^{t-1},\mathbf{Y}_{t-s}^{t-1},\mathbf{Z_0},\theta_{b_t}\right)$ can be written as $D_y\left(\mathbf{Y}_{t-s}^{t-1}/\mathbf{Y}_{t-s}^{t-1},\mathbf{Z_0},\theta_{b_t}\right)$ or in other words that $\mathbf{W_t}$ does not Granger cause $\mathbf{Y_t}$ being the former irrelevant in the analysis of the latter.

According to Hendry (1995) it is at the stage of marginalization that most empirical researches eliminate variables which are potentially relevant[6].

In contrast to the approach taken in Phylatkis and Taylor (1993) Engsted (1993) and Rossi (1994) we do follow a progressive strategy, or the LSE methodology, when deriving any structure[7] from the Haavelmo's distribution in (4.2). The progressive strategy in the present context comprises the specification of a general linear dynamic model from which we impose and test restrictions to derive the econometric model, in doing so we expect to relate the empirical model to the actual mechanism that is generating the data rather than to theory only which means then that theory and data form the DGP. The character of progressive comes from the fact that when imposing and testing restrictions we are checking continuously if the proposed model (more restricted) can predict the parameters of the more general model.

In particular we consider that the econometric model of interest imposes a set of restrictions on the statistical system represented by (2) which are delineated from the economic theory. The econometric model can therefore be denoted by:

$$\mathbf{f}_y\left(\mathbf{Y}_T^1/\mathbf{Y}_0;\xi\right)=\prod_{t=1}^{T}\mathbf{f}_y\left(\mathbf{y}_t/\mathbf{Y}_{t-1}^{t-s};\xi\right) \quad (5)$$

where $\mathbf{f}(\cdot)$ represents the postulated sequential joint densities. In general since $\mathbf{f}(\cdot)$ is an econometric model it comprises a practical problem how to choose among the many different econometric models can be postulated to represent the joint densities in $\mathbf{f}(\cdot)$.

In our case we firstly specify a general dynamic model that represents (2) as the Vector Autoregressive Vector (VAR) which assumes the following representation:

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6 Interestingly this discussion is also present in the called *semistructural* VAR approach where according to Canova (1995) the modellers are interested in identifying the behavioural shocks and in predicting the effect of a particular shock on the endogenous variables of the system. Within this approach according to the author there are situations when the observation of current and past values of endogenous variables are not sufficient to achieve the identification of behavioural shocks. Such problem is related to the fact that agents when undertaking their decisions have an information set that is larger than the one available to the econometrician. So potentially there has been some marginalization that threw away relevant variables.

7 The concept of structure is defined in Hendry and Doornik (1994, p 9) as: "... an entity (structural model) which is to be contrasted with a system having derived parameters (reduced form) and even being a synonym for the population parameter". Structure is also defined later in the same page as: "the set of basic invariant attributes of the economic mechanism". Despite the presence of these two definitions they seem to lead to the same concept which is an econometric model as described in the first definition that presents parameters which are invariant (constant across interventions) and constant (time independent). They may also include agent's decision rules as the econometric model used in chapter 2 represents but no assumption of these decision rules being derived from inter-temporal optimization. Within the VAR literature according to Canova (1995, p. 67): "a model is termed as structural if it is possible to give distinct behavioural interpretations to the stochastic disturbances of the model." Our intention in giving these definitions is avoid any confusion with the term structural for the model derived in the next chapter and satisfy at least partially a plea for linguistic stability raised by Sims (1991).

$\mathbf{A}(L)\mathbf{y_t} = \varpi\mathbf{D_t} + \vartheta_t$ (6) where $\vartheta_\mathbf{t} \sim IN(\mathbf{0},\Sigma)$, $\mathbf{A}(L)$ is the matrix polynomial in the lag operator such that: $\mathbf{A}(L) = \sum_{j=0}^{p} \mathbf{A_j} L^j = \mathbf{I}_N + \mathbf{A}^*(L)L$. Since the matrix has order p we have in (6) a p-th order system (VAR) since (6) can be rewritten as the following:

$$
\begin{aligned}
\mathbf{I}_N - \mathbf{A}^*(L)L\mathbf{y_t} &= \varpi\mathbf{D_t} + \vartheta_t \\
\mathbf{y_t} - \mathbf{A}(L)^* \mathbf{y_{t-1}} &= \varpi\mathbf{D_t} + \vartheta_t \qquad (7) \\
\mathbf{y_t} &= \mathbf{A}(L)^* \mathbf{y_{t-1}} + \varpi\mathbf{D_t} + \vartheta_t
\end{aligned}
$$

Finally $\mathbf{D_t}$ is a vector that contains deterministic components as constant, trend, centered seasonal dummies etc. As it stands, the system can be classified as complete and closed. Complete in the sense that the number of equations is equal to the number of variables and closed in the sense that all N variables are modelled despite the marginalization in terms of the deterministic variables.

VAR models have been widely applied in empirical econometrics mostly because they can be seen as the empirical counterpart of theoretical models that assumes rational agents in a framework of inter-temporal optimization, where the relationships to be modelled are of the type: $E\left[\mathbf{A}(L)\zeta_t \middle| I_t\right] = 0$ (8) ignoring deterministic factors. In equation (8) $\zeta_{n\times 1}$ represents a vector of theoretical variables of interest, I_t is the information set and $E\left[\bullet \middle| I_t\right]$ is the conditional expectations operator.

Considering that $\mathbf{y_t}$ is the vector of observable variables which adequately describes the variables in ζ_t, only finite lags are involved in (8) and the future expectations do not affect the outcome, the empirical counterpart of (8) is: $E\left[\mathbf{y_t} \middle| \mathbf{y_{t-1}}, \mathbf{y_{t-2}}, \ldots \mathbf{y_{t-k}}\right] = \sum_{i=1}^{k} \mathbf{A_i y_{t-i}}$ which is exactly the VAR described in (7) if we take conditional expectations and $\vartheta_\mathbf{t} = \mathbf{y_t} - E\left[\mathbf{y_t} \middle| \mathbf{y_{t-1}} \ldots \mathbf{y_{t-k}}\right]$ is an innovation process to the available information.

Hendry (1995, p. 312) argues that "Although economic theory may offer a useful initial framework, theory is too abstract to be definitive and should not constitute a strait-jacket to empirical research: theory models should not simply be imposed on the data." In this sense we do not pursue a strict analysis of the econometric model by imposing the "strait-jacket" in the model derived in the next section. We do not assume therefore any a priori relationship between the variables in hope we can gain economic intuition from the data with respect to the demand for money and the monetary system as a whole. Nevertheless despite we follow a data driven analysis here we do not assume that economic theory has no role in the process, rather it guides the analysis throughout given that the long run relationships are all identified on the grounds of the theoretical models for money demand.

The core of the argument is to follow a progressive strategy in the sense that knowledge of the economic structure that underlie the economy is not necessary prior the development of the analysis, however given that the structure exists it is possible to determine it following this progressive strategy.

In the light of the results of the unit root tests presented in last section that shows that most of the variables in the two sub-periods appeared to be I(1) we then reparameterize the system under consideration for the progressive analysis given that the presence of integrated variables in the system implies that nonoptimal inference can result if we do not account for them. Therefore we reparameterize equation (7) into:

$\Delta \mathbf{y_t} = \sum_{i=1}^{p-1} \Pi_i \Delta \mathbf{y_{t-i}} + \Pi \mathbf{y_{t-p}} + \varpi \mathbf{D_t} + \vartheta_t$ (9) where: $\Pi_i = -\left(\mathbf{I_n} + \sum_{j=1}^{i} \mathbf{A_j} \right)$ and $\Pi = -\left(\mathbf{I_n} + \sum_{j=1}^{p} \mathbf{A_j} \right) = -\mathbf{A}(1)$ is the matrix of long-run responses.

Notice therefore that we are considering the system as equation (9) to which the derived SEM is contrasted. The advantage in using (9) is that we can investigate the presence of cointegration between the variables in $\mathbf{y_t}$ by testing the rank (*r*) of Π following Johansen (1995). Apart from non-stationarity like trends or level shifts presented in $\mathbf{D_t}$, if Π has full rank then all variables in $\mathbf{y_t}$ are I(0) stationary, if Π has rank, 0<*r*<*N* then there exist linear combinations of variables in $\mathbf{y_t}$ which are stationary, finally if rank of Π is zero then all variables in $\mathbf{y_t}$ are I(1) and $\Delta\mathbf{y_t}$ is I(0).

It is worthwhile to notice that equation (9) represents a I(0) parameterization of the system in (7) and it is essential that the system presents a congruent representation of the available information so it can be considered as coherent statistical basis for further assessments.

The class of SEM that we consider here has the form $\Theta \mathbf{f_t} = \mathbf{u_t}$ (10) where Θ is an $n \times N^*$ matrix and $N^* = Np \times r$ and $\mathbf{f_t}$ is the companion form of (7) given by: $\mathbf{f_t} = \Gamma \mathbf{f_{t-1}} + \omega_t$ (11) [8] where:

$$\mathbf{f_t} = \begin{pmatrix} \Delta \mathbf{y_t} \\ \vdots \\ \Delta \mathbf{y}_{\mathbf{t}-p+1} \\ \beta \mathbf{y_{t-p}} \end{pmatrix} \text{ and } \Gamma = \begin{pmatrix} \Pi_1 \; \Pi_2 \ldots \Pi_{p-1} - \alpha\beta' - \alpha \\ I \quad 0 \; \cdots \; 0 \qquad 0 \quad 0 \\ \cdots \; \cdots\cdots\cdots \qquad \cdots \quad \cdots \\ 0 \quad 0 \; \cdots \qquad \beta' \quad I \end{pmatrix} \text{ and } \omega_t \sim IN(0, \Omega)$$

In contrasting the SEM models we use the concept of encompassing formalized in Mizon and Richard (1986). Consider two rival SEM of the form $H_1 : \Theta_1 \mathbf{f_t} = \mathbf{u_{1t}}$ and $H_2 : \Theta_2 \mathbf{f_t} = \mathbf{u_{2t}}$ which are over identified relative to the congruent statistical system (11). Let τ_2 denote the vector of parameters in Θ_2 and let τ_p be what H_1 predicts τ_2 to be if H_1 were the DGP.

Then H_1 encompasses H_2 if and only if $\tau_2 - \tau_p = 0$. From this condition is possibly to derive that the VAR (H_1), say, encompasses H_2 since H_2 is a reduction of it, so H_1 predicts what τ_2 is to be.

In the context of the general to specific modelling strategy the question of interest is that if the SEM encompasses the system since, if it does, a simpler model nested within the general model (system) is accounting for the characteristics of a more general model. The VAR provides then the framework within which we access the properties of the SEM.

It is worthwhile to notice that according to Hendry and Mizon (1993) we can circumvent the problem of establishing encompass theorems about the SEM that usually are difficult because of exogeneity assumptions about rivals SEM may differ, by assuring that the system under analysis is closed and that $E[\mathbf{u_t f'_{t-1}}] = 0$ (12).

Furthermore the condition in (12) implies that $E[\Theta \mathbf{f_t f'_{t-1}}] = \Theta\Gamma E[\mathbf{f_{t-1} f'_{t-1}}] = 0$ (13) since $\Theta \mathbf{f_t} = \Theta \Gamma \mathbf{f_{t-1}} + \Theta \omega_t = \mathbf{u_t}$. But indeed to condition (13) be valid $\Theta\Gamma = 0$ (14) must be true which is a condition to absence of dynamic misspecification in the VAR in the I(0) space $(\Theta \omega_t = \mathbf{u_t})$.

Consequently, if condition (14) holds, so the SEM is congruent, the SEM is a valid reduction of the VAR and parsimoniously encompasses the VAR. Therefore, parsimoniously encompassing the VAR and being congruent is a sufficient condition to the SEM to encompass rival

[8] Notice that equation (5) is an alternative reparameterization of equation (4) presented here only to facilitate the notation.

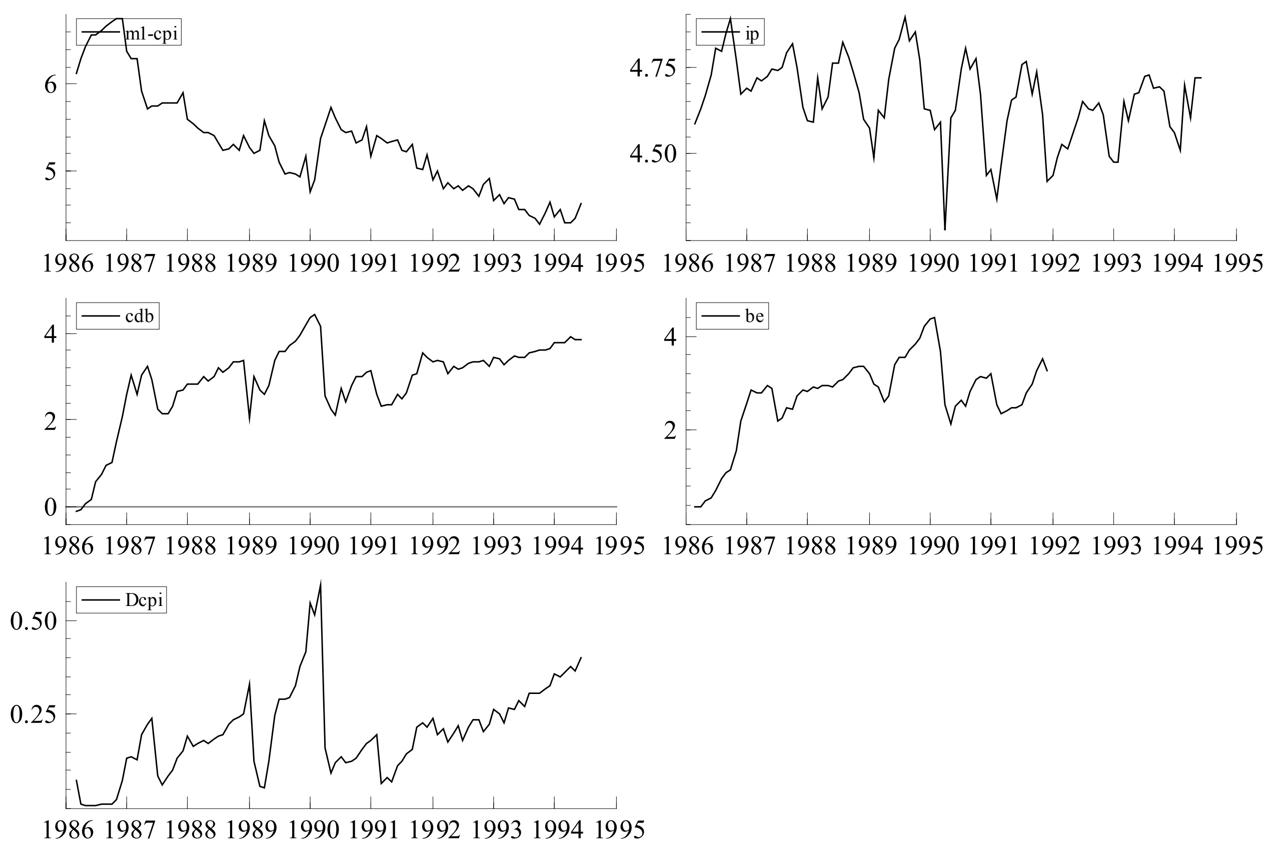
models (Hendry and Mizon, 1993). It is worthwhile to notice that condition (14) can be tested given that it coincides with the known condition for the validity of over-identifying restrictions.

Still, nevertheless to be investigated the fact that many models could satisfy the encompassing property derived from (14). However as noticed in Hendry and Mizon (1993) policy regime changes will induce changes in the parameters of the different SEM (Γ_i) destroying the observational equivalence and mutual encompassing which can only be assessed in a constant parameter world, therefore only the representation that corresponds to the actual structure of behaviours will remain constant. Such assertive is very interesting in our case where a sequence of policy regime changes took effect, so if we can derive a constant SEM for the period it will be unique given the information set.

3) Empirical Results

The sample data are monthly seasonally unadjusted for the period 1986 (2) to 1991(12) due an interruption in one of the series. *m1* is the log of M1, defined as paper money held by the public plus demand deposits, *cpi* is the log of the consumer price index, *ip* is the log of the industrial production index both as defined in Juselius (2002)[9], *be* is the log of the bill of exchange interest rate to the payee and finally *cdb* is the log of the interest rate paid in the certificate of deposits. The choice of the two interest rates follows their use in previous analysis as Cardoso (1983) and Gerlach and Simone (1985). Given these definitions we construct then the real money series as: $m1-cpi$, the inflation rate as: Δcpi, where Δ stands for the first difference operator. Figure 1 contains full sample time plots of the modelled variables: $m1-cpi, ip, be, cdb, \Delta cpi$.

Figure 1 Full Sample Time Plots



In table 1 and 2 we present the Augmented Dickey Fuller and Phillips Perron unit root tests respectively for the sample. The presence of stabilization plans comprise a further challenge to the analysis since the series present a sequence of structural changes in their levels. According to Cati

[9] The author would like to thank Katarina Juselius for providing the dataset used in Juselius (2002). The source for the *CDB* and *BE* is the Institute of Applied Economic Research (IPEA) at: www.ipeadata.gov.br

et ali (1999) the presence of structural changes would bias the conventional unit root tests towards a non-rejection of the null of unit root. When analysing inflation in Brazil, Cati et ali found exactly the opposite, namely a bias towards a rejection of the null hypothesis Nevertheless in our sample there is no evidence against the null hypothesis of unit root that justifies the use of the corrected tests despite the sequence of stabilization plans as tables 1 and 2 show.

Table 1 Augmented Dickey Fuller Test (1986/4 – 1991/12)

Variable	Lag	t-value	Critical Value (5% / 1%*)	Constant	Trend/Seasonal
$m1-cpi$	0	-1.981	-3.44 / -4.02	Yes	yes / no
ip	0	-4.388	-3.44 / -4.02	Yes	yes / yes
cdb	0	-2.656	-3.44 / -4.02	Yes	yes / no
be	0	-2.271	-3.44 / -4.02	Yes	yes / no
Δcpi	0	-2.752	-3.44 / -4.02	Yes	yes / no

* The asymptotic critical values are as tabulated in Maddala and Kim (1998) for a sample size of 100 observations.

Table 2 Phillips Perron Test (1986/4 – 1991/12)

Variable	Lag	Z statistic	Critical Value (5% / 1%)‡	Constant	Trend
$m1-cpi$	1	-9.232	20.7 / 27.4	yes	yes
ip	3	-61296.52	20.7 / 27.4	yes	yes
cdb	1	-11.038	20.7 / 27.4	yes	yes
be	1	-9.989	20.7 / 27.4	yes	yes
Δcpi	1	-18.948	20.7 / 27.4	yes	yes

‡ The asymptotic critical values are as tabulated in Maddala and Kim (1998) for a sample size of 100 observations.

As a general conclusion the evidence found in the data indicates that they are non-stationary except perhaps for the industrial production index and in particular well represented as I(1)[10].

We estimate initially a VAR (2) with the following variables $m1-cpi, ip, cdb, be \text{ and } \Delta cpi$. The VAR also included centered seasonal dummies, an unrestricted constant and a restricted trend so we avoid the unlikely presence of a quadratic trend in the levels. The model also includes the following unrestricted dummies:

$D3 = \begin{cases} 1 & t = 1989(1) \text{ to } 1989(4) \\ 0 & otherwise \end{cases}$ corresponding to period when the summer plan actually took place.

$dfm(3) = \begin{cases} 1 & 1989(1) \\ 0 & otherwise \end{cases}$ corresponding to first month of which the Summer actually took place

$dfma4 = \begin{cases} 1990(6) \\ 0 \quad otherwise \end{cases}$ corresponding to the first month after the end of the Collor plan (fourth plan),

$dfm4 = \begin{cases} 1990(3) \\ 0 \quad otherwise \end{cases}$ corresponding to the first month of which the Collor plan actually took place. [11]

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[10] We do not discard the possibility of the data being well described as I(2) as well, however this hypothesis is investigated using a system rather than a univariate analysis.

[11] The precise dates observed for the plans are the same as the in Cati et ali (1999) paper. The result obtained here was derived from initially setting these dummies as defined $D(i), dfm(i)$ and $dfma(i)$ for each one of the plans (Cruzado, Bresser, Summer, Collor and Collor I) but only those presented here were significant.

In table 3 we present the diagnostic statistics for the system. The individual diagnostic tests for the system show the presence of autocorrelation and non normality in the residuals for the Δcpi equation and autocorrelation in the residuals of the equation for the industrial production index. In contrast, at the system level the tests suggest that there is no departure from the null hypothesis of no autocorrelation, normality and homocedasticity. Having in mind that Pc-Give does the individual tests using the system residuals as they were the residuals for each one of the individual equations the interpretation of the individual tests is compromised. At best according to Doornik and Hendry (1997) the individual tests are usually valid only when the remaining equations are problem free, so we decide carry on the analysis based on the results of the system test.

The results of the cointegration statistics based on the trace test presented in table 5 and the eigenvalues of the companion matrix (unrestricted) presented in table 4 admit an opposite interpretation. The trace test suggests that there are three cointegrating vectors whereas the third eigenvalue of the companion matrix seems to be a bit far from one which would suggest that we have only two unit roots and possibly only two cointegrating vectors. However since the hypothesis of rank 2 being the last significative statistics is strongly rejected we carry on the analysis based on the hypothesis of 3 cointegrating vectors. Imposing the rank condition on the model and re-estimating generates the eigenvalues of the companion matrix presented in the second column in table 4, which shows that the number of unit roots is equal to two with the third eigenvalue being far from the unit, so we rule out the hypothesis of the data being I(2).

Table 3 Diagnostic Tests VAR 1986(4)-1991(12)

Test\Equation	$m1-cpi$ (p-value)	ip (p-value)	cdb (p-value)	be (p-value)	Δcpi (p-value)	System (p-value)
AR 1-5	1.15	2.91*	0.53	1.09	5.53**	1.18
	(0.35)	(0.02)	(0.75)	(0.38)	(0.00)	(0.23)
Normality	0.56	2.91	2.91	6.75	6.48*	13.79
	(0.75)	(0.23)	(0.23)	(0.03)	(0.03)	(0.18)
ARCH	0.25	0.47	0.10	0.47	0.72	
	(0.93)	(0.79)	(0.99)	(0.80)	(0.60)	
Hetero	0.52	0.34	0.23	0.21	0.66	0.25
	(0.93)	(0.99)	(0.99)	(0.99)	(0.83)	(1.00)

*indicates rejection at 5% level and **at 1% level.

Table 6 shows the cointegrating vectors after imposing and testing over-identifying restrictions using the LR test.

The first cointegrating vector shows the interest rate on the bill of exchange cointegrated with inflation admitting the interpretation of a Fisher relationship between the nominal interest rate and inflation with a small but significant trend. A possibly explanation for the rate of interest of the certificate of deposit absence is that in several occasions the Brazilian Central Bank changed the scope of the operations with CDBs and BEs[12].

Nevertheless, the operations with CDBs seem to have been more affected than those with the bill of exchange. As observed in Andima (1997) in 1989 the market share for the CDB

[12] Indeed several interventions took place during this period and all were valid for both CDB and BE. The first one in January 1986 fixing the minimum period for investment in 90 days at market determined rates or market rates plus monetary correction. Then in February 1986 the period was reduced to 60 days but only for those investments with ex-ante interest rates. In December 1986 the period was extended to 90 days again but the investment could have the same nominal yield as the Brazilian Central Bank Bills plus negotiable interest rates. These bills were negotiated in the open market and should give a closer nominal correction to the rates of CDB and BE. In November 1987 nevertheless the CDB and BE rates were again linked to official indexation rates. Finally in May 1989 the minimum period of investment was reduced to 30 days.

underwent a significant reduction since this type of investment could not follow the high real interest rates offered by the overnight operations after the change in the monetary correction index in November 1987 and possibly was no longer reflecting the real inflation rates.

Table 4 Five Largest Eigenvalues Companion Matrix

Eigenvalues r unrestricted	Eigenvalues $r = 3$
0.8898	1.0000
0.8898	1.0000
0.6519	0.7154
0.6519	0.5299
0.4425	0.5299

Table 5 Cointegration Statistics VAR (1986/4 – 1991/12)

R	0	1	2	3	4	5
Trace Test	200.29	124.26	68.73	23.91	4.74	
p-value	0.000	0.000	0.000	0.085	0.639	
Eigenvalue		0.668	0.552	0.477	0.242	0.006

Table 6: Cointegrating Vectors and Adjustment Coefficients VAR 1986(4) – 1991(12)

Cointegrating Vectors	$\hat{\alpha}_i$ (se)	$i = 1$	$i = 2$	$i = 3$
CIa: $-0.197be + \Delta cpi_t + 0.001t$	$m1 - cpi$	3.937 (0.737)	-0.179 (0.057)	0.550 (0.108)
CIb: $m1 - cpi - 5.747ip_t + 4.799\Delta cpi_t$	ip	0	0	0
CIc: $cdb - 4.508\Delta cpi - 1.930ip_t - 0.012t$				
LR test of Restrictions	cdb	-7.035 (0.781)	0	-1.321 (0.133)
Equilibria and Feedback: $\chi^2(9) = 5.205 \quad [0.8160]$	be	0	-0.376 (0.522)	-0.221 (0.049)
Equilibria only: $\chi^2(2) = 0.005 \quad [0.997]$	Δcpi	0	-0.110 (0.014)	0

The second cointegrating vector represents a long run demand for real money being positively influenced by increases in the economic activity represented by the industrial production and negatively affected by increases in inflation. Both coefficients have therefore the correct signal.

Despite all the instability of the period the long run relationship appears to be remarkably stable as can be noticed in figure 2 where we depicts the time plot of the three cointegrating vectors.

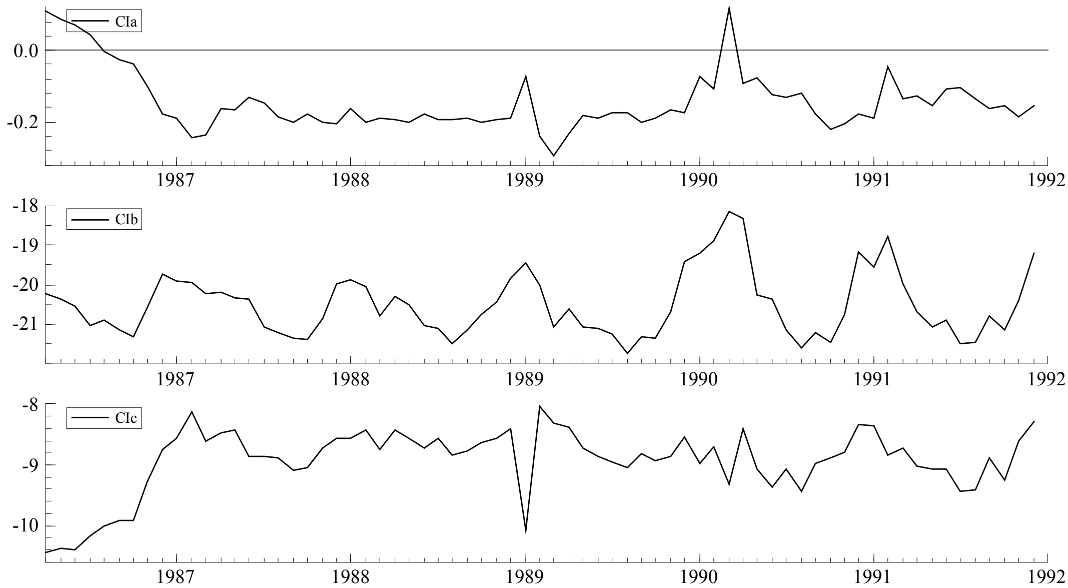
We consider the identification of this vector that assumes the interpretation of a money demand equation a remarkable result in the sense that in the 1980s a debate in the literature took place with respect the money demand in Brazil. The focus was the instability in the parameters of the estimated money demand equations as found by Rossi (1989).

In contrast, for the period posterior to the Cruzado Plan the attempts in the literature were very restricted to a Cagan specification given the high levels of inflation. Nevertheless, using a different approach it was possible identify a long run money demand equation leading to a much richer analysis than that allowed by the Cagan model in the sense that the real money demand equation is linked to the level of activity in the economy and not only to inflation or the rate of growth in M1 as in Engsted (1993). Further, the dynamic properties of the SEM investigated below allow a much more detailed analysis of the monetary system in Brazil in that period. Such results point out that the approach adopted here is more adequate than the Cagan model in describing empirically the phenomenon observed in Brazil.

Finally, the third vector has a difficult interpretation. Theoretically we expect that increases in the real interest rate would lead to a reduction in the economic activity reflected in the industrial production index but indeed with this cointegrating vector it would lead to an increase in the economic activity.

Nevertheless, we need to consider here that the Brazilian Central Bank followed a very loose monetary police during the years of 1986, 1987, and 1990 with negative real interest rates and 1988 when the real interest rates were close to zero what possibly influenced the long run relationship expressed by the vector.

Figure 2 Cointegrating Vectors VAR 1986/4 – 1991/12



The adjustment coefficients show that the output is weakly exogenous for the parameters of the cointegrating vectors a result that has no counterpart in the SEM. The real money equation reacts to the three vectors but only error corrects to the second one which exactly represents the long run money demand. An interesting result appears in the equation for Δcpi where it error corrects only to the money demand.

In the equation for the bill of exchange surprisingly there is no reaction to the first vector whereas in the equation for the interest rates in the certificate of deposits it reacts to the first and third vectors the latter reinforcing the rule of the real interest rate based on cdb proxied by this vector. A result that is difficult to interpret is the reaction to the first vector since this vector links be and Δcpi only. It might be related to the fact that in the previous period the long run relationship between the interest rate and inflation included cdb as well, a result that did not persisted in the long run in the second period but can be represented by the equation for cdb error correcting to the first vector. In the sequence we estimate a VEC including the three cointegrating vectors.[13]

The misspecification tests presented signals of autocorrelation in the residuals of the equation for $\Delta\Delta cpi$ which led us to reestimate the system excluding Δip_{t-1} from $\Delta \mathbf{y}_{\mathbf{t}-1}$ since this variable was not significant in the whole system according to the F-test. Testing the reduction led to the results presented in table 7. The diagnostic tests show that there are signals of autocorrelation and heteroscedasticity in the equation for $\Delta\Delta cpi$ and non-normality in the equation for Δbe. In contrast

[13] It is worthwhile to notice that in both cases the trend was not significant so the variables present a long run growth given by the fact that the constant is not restricted to lie on the cointegration space.

at the system level the VEqCM appears to be congruent with the information available with normal residuals, no signal of autocorrelation and heteroscedasticity.

Table 7 Diagnostic Tests Reduced VEqCM 1986/4 – 1991/12

Test\Equation	$\Delta m1 - cpi$ (p-value)	Δip (p-value)	Δcdb (p-value)	Δbe (p-value)	$\Delta\Delta cpi$ (p-value)	System (p-value)
AR 1-5	0.37	1.44	0.38	0.56	2.46	1.23
	(0.86)	(0.23)	(0.86)	(0.72)	(0.048)*	(0.14)
Normality	1.98	1.98	3.83	7.96	4.24	11.89
	(0.37)	(0.37)	(0.15)	(0.018)*	(0.12)	(0.23)
ARCH	0.40	0.24	0.54	0.36	0.84	
	(0.85)	(0.93)	(0.74)	(0.87)	(0.53)	
Hetero	0.49	1.17	0.24	0.35	2.24*	0.53
	(0.92)	(0.35)	(0.99)	(0.98)	(0.03)	(1.00)

*indicates rejection at 5% level

The reduction constituted a valid simplification in the system[14], where the number of parameters was reduced from 120 to 115 and constitutes therefore the basis from which the SEM is tested.

The SEM derived imposes a total of 19 restrictions which were not rejected based on the results of the LR test ($\chi^2(19) = 8.90$) and a total reduction of 19 parameters. The diagnostic tests are shown on table 8 whereas the final SEM is presented in table 9.

The results shown on table 8 indicate that the SEM has no signal of autocorrelation, non-normality and heteroscedasticity at the system level. The results at individual equations level in contrast to the system, show signals of misspecification in the equation for $\Delta\Delta cpi$ which are restricted to the presence of heteroscedasticity and autocorrelation with this last test being rejected at a 5% confidence level. We nevertheless opted to carry on the analysis using the specified SEM on the basis of the system level tests.

In figure 3 we present the test for parameter instability based on the Chow break point test and one step residuals. Remarkably there is no signal of instability in the parameters over the period under consideration. Again we qualify this result as remarkable given that in the period analyzed in the test the economy underwent three stabilization plans, one short lived hyperinflation plus the financial embargo enforced in the Collor Plan. Considering the instability in the period when the president stepped down in the middle of a political crisis and the upcoming of a new stabilization plan in 1994 in general the SEM is congruent with the information available and parsimoniously encompass the VAR.

The SEM short-run dynamic shows that the equation for $\Delta m1 - cpi$ is affected positively by changes in the bill of exchange interest rate, a signal that has a difficult interpretation. A possible interpretation lies in the fact that this interest rate is catching the effects of growth in the economy in the presence of low levels of M1 holdings in such way that agents would need more money in their demand accounts to apply in a different asset which would be otherwise protected from inflation in interest bearing accounts.

The signal for the rate of growth in inflation is as expected negative. The signal for the rate of growth in the interest rates for the certificate of deposits seems to reflect simply the substitution between M1 and a fixed income investment which had its maturity period reduced in relation to the previous period in line with inflation rates growth.

[14] The reduction test led to the following result:
SYS(29) --> SYS(32): F(5,41) = 1.7221 [0.1512]

Table 8 Diagnostic Tests SEM 1986/4 – 1991/12

Test\Equation	$\Delta m1-cpi$ (p-value)	Δip (p-value)	Δcdb (p-value)	Δbe (p-value)	$\Delta\Delta cpi$ (p-value)	System (p-value)
AR 1-5	0.89	2.07	1.00	1.45	2.63*	1.28
	(0.49)	(0.08)	(0.42)	(0.22)	(0.037)	(0.09)
Normality	2.63	1.98	3.64	4.63	4.18	11.37
	(0.26)	(0.37)	(0.16)	(0.09)	(0.12)	(0.32)
ARCH	0.28	0.09	0.76	0.47	0.86	
	(0.91)	(0.99)	(0.57)	(0.79)	(0.51)	
Hetero	0.54	1.48	0.42	0.53	2.62**	0.61
	(0.91)	(0.16)	(0.96)	(0.91)	(0.0089)	(0.99)

*indicates rejection at 5% level and ** at 1% level.

Table 9 SEM 1986/4 – 1991/12[15]

$$\underset{(SE)}{\Delta m1-cpi} = \underset{0.57}{0.94} - \underset{0.06}{0.22}\,\Delta cdb_{t-1} + \underset{0.08}{0.29}\,\Delta be_{t-1} - \underset{0.15}{0.63}\,\Delta\Delta cpi_{t-1} +$$

$$+\underset{0.70}{3.97}\,CIa_{t-1} - \underset{0.05}{0.21}\,CIb_{t-1} + \underset{0.10}{0.52}\,CIc_{t-1} + \hat{\varpi}\mathbf{D_t} \quad \hat{\sigma} = 0.090$$

$$\underset{(SE)}{\Delta ip} = -\underset{0.29}{0.69} - \underset{0.09}{0.59}\,\Delta\Delta cpi_{t-1} - \underset{0.18}{0.49}\,CIa_{t-1} - \underset{0.03}{0.07}\,CIc_{t-1} + \hat{\varpi}\mathbf{D_t} \quad \hat{\sigma} = 0.056$$

$$\underset{(SE)}{\Delta cdb} = -\underset{0.83}{11.50} - \underset{0.19}{0.43}\,\Delta m1-cpi_{t-1} - \underset{0.04}{0.13}\,\Delta be_{t-1} - \underset{0.47}{6.62}\,CIa_{t-1} -$$

$$-\underset{0.09}{1.19}\,CIc_{t-1} + \hat{\varpi}\mathbf{D_t} \quad \hat{\sigma} = 0.22$$

$$\underset{(SE)}{\Delta be} = -\underset{0.87}{8.32} - \underset{0.16}{0.19}\,\Delta m1-cpi_{t-1} - \underset{0.03}{0.34}\,CIb_{t-1} - \underset{0.05}{0.14}\,CIc_{t-1} +$$

$$+\hat{\varpi}\mathbf{D_t} \quad \hat{\sigma} = 0.21$$

$$\underset{(SE)}{\Delta\Delta cpi} = -\underset{0.26}{2.37} - \underset{0.010}{0.017}\,\Delta cdb_{t-1} - \underset{0.07}{0.29}\,\Delta\Delta cpi_{t-1} - \underset{0.21}{0.50}\,CIa_{t-1}$$

$$-\underset{0.01}{0.09}\,CIb_{t-1} - \underset{0.03}{0.06}\,CIc_{t-1} + \hat{\varpi}\mathbf{D_t} \quad \hat{\sigma} = 0.045$$

Overall the impact of shocks in $\Delta m1-cpi$ appears to be restricted to $\Delta m1-cpi$ only as we can infer from the accumulated impulse response functions in figure 4.

The equation for Δip shows a negative relationship with the growth rate in inflation and is certainly expressing the nominal impacts of inflation only. It is also represented in the impulse response functions where the dynamic properties of the estimated model indicate a negative fast response and posterior stabilization to a shock in $\Delta\Delta cpi$ with the impact reaching approximately half of the standard deviation for the equation to Δip. The findings of the cointegrating VAR were not reproduced with the equation for Δip since it reacts to the first cointegrating vector and not surprisingly to the third one.

The equation for Δcdb basically depicts this variable as negatively related to the rate of growth in real money reinforcing the interpretation that investing in fixed income would represent an alternative to holding money. The negative signal in the interest rates for the bill of exchange

[15] The reader should notice that the vector $\mathbf{D_t}$ comprises a different set of variables for each equation. We use the same notation only to save space. Indeed for all equations it comprises the centered seasonal dummies but for the first equation it includes also all dummies. For the second equation it comprises the *dfm4,dfma4* and *D3* dummies. For the third equation it includes only *dfm3* and *D3.* For the fourth equation the vector comprises all dummies, and finally in the last equation *dfm4,* and *D3* only.

coefficient represents the substituting effect between the two interest rates and finally the negative signal in Δip has no clear interpretation.

Figure 3 One Step Residuals and Breakpoint Chow Test SEM 1986/4 – 1991/12

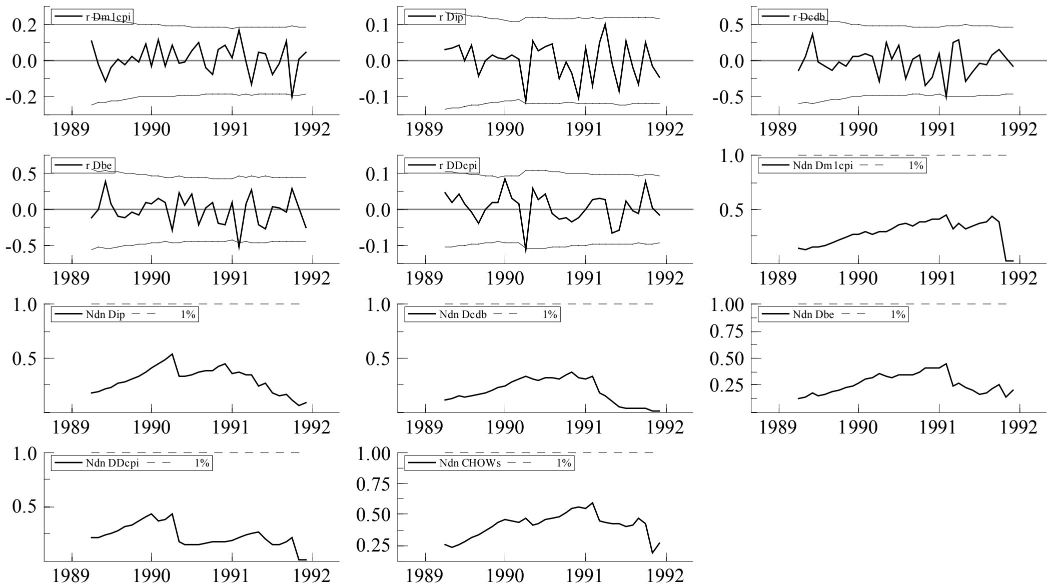
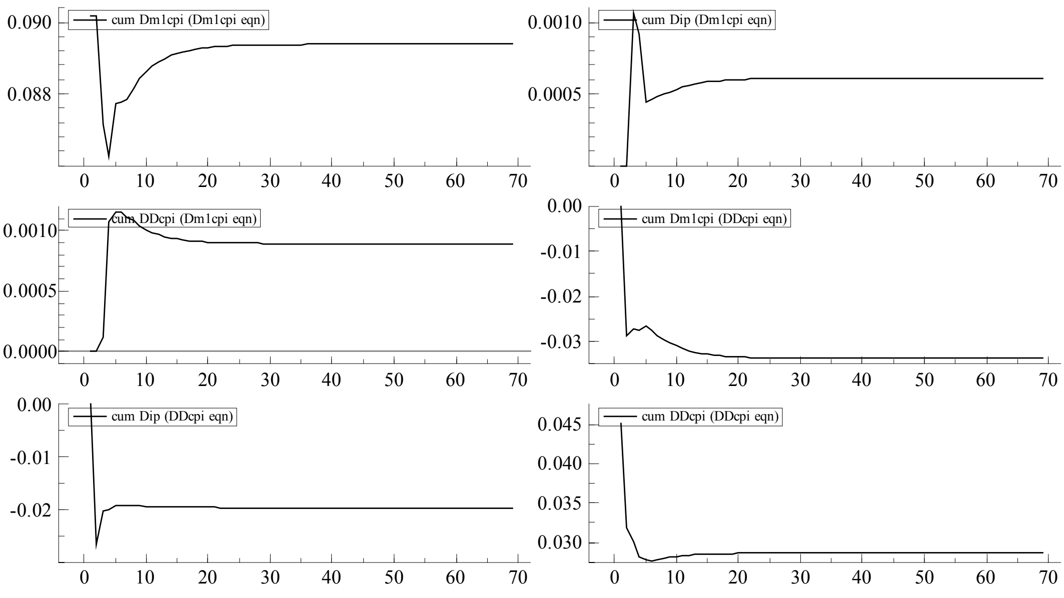


Figure 4 Accumulated One Standard Deviation Impulse Responses Functions Shock To (From) SEM 1986/4 – 1991/12



The equation for Δbe has a difficult interpretation since it is not clear why the coefficient for $\Delta m1 - cpi$ should have a negative signal, nevertheless this variable is only marginally significant implying therefore that Δbe is basically driven by the second and third cointegrating vectors and the deterministic variables.

Finally in the equation for $\Delta\Delta cpi$ the negative signal in $\Delta\Delta cpi_{t-1}$ depicts the presence of memory in the process but possibly reflecting the sequence of attempts for bringing down inflation. Nevertheless when we consider the impulse response function the picture is reversed and shocks on $\Delta\Delta cpi$ present a positive impact which shows the contrasting short-run impacts of the stabilization plans and a long run effect of increasing inflation rates given their successive failures. The coefficient for Δcdb_{t-1} despite having a difficult interpretation is only marginally significant which led us to conclude that the equation for $\Delta\Delta cpi$ is basically driven by its past values and the cointegrating vectors. This result seems to be describing the presence of inertia in inflation despite all the attempts to break down this component with the stabilization plans. The cumulating impulse response functions show an initial impact stabilizing after less than ten periods in values close to the standard deviation of the $\Delta\Delta cpi$ equation.

Overall the SEM main strengths are the correct characterization of the growth in real money as a negative function of $\Delta\Delta cpi$ and the negative signal of Δcdb_{t-1} in this equation showing the trade off between fixed income and cash holding. The SEM also is able to disentangle the short and long-run effects in $\Delta\Delta cpi$ of a shock in $\Delta\Delta cpi$. In the short run the SEM correctly describes a negative effect reflecting the sequence of stabilization plans, whereas in the long run it shows increasing rates which dominated the period.

4) Nominal Wage Inflation

The role played by nominal wage inflation in the Brazilian high inflation is captured in a theoretical perspective by readdressing the Taylor's (1979) model as proposed in Novaes (1991).The model is composed by a wage rule, a mark-up price rule for prices, a monetary rule and an aggregate demand equation. The key assumption in modifying the original model is to assume that the wage rule follows a backward adjustment as well as the monetary rule which accommodates immediate past inflation. Following Novaes (1991) we write:

$$\Delta w_t = \Delta p_{t-1} + \gamma \Delta y_t \quad (15)$$

$$\Delta p_t = \left(\frac{\Delta w_t + \Delta w_{t-1}}{2} \right) \quad (16)$$

$$\Delta m_t = \Delta p_{t-1} + \varphi \left(\Delta p_t - \Delta p_{t-1} \right) \quad (17)$$

$$\Delta m_t = \Delta p_t + \Delta y_t + \varepsilon_t^d \quad (18)$$

In equations (14) through (16) low cases indicates variables in log so that they are expressed in their variation rates given the $\Delta = (1 - L)$ operator. w is the nominal wage, y is a measure of demand excess, p is the price index, m is money and ε is a random shock in the demand equation. Equation (15) represents the wage rule, (16) the price rule, (17) the monetary rule and finally equation (18) the aggregate demand equation.

The main features of the model are the monetary rule that simply accommodates in the totality the previous inflation rate but with some discretionary power in such way that the growth in money is adjusted by the acceleration in the inflation rate in the current period. Further given this hypothesis the model differently from the original one does not assume that the agents have rational expectations. Novaes in her paper solves the model defining a reduced form equation for inflation as a function of its own past since her objective was test persistence in inflation. We follow though an alternative route finding a reduced equation form for inflation as a function of w and y in solving the model. The justification is that originally Novaes carry out a univariate analysis of the time series data whereas we are interested in the equation for p derived from the SEM in a multivariate context. Solving the model yields then:

$$\Delta p_t = \left(\frac{1-\varphi}{2} \right) \Delta w_t - \left(\frac{\varphi}{2} \right) \Delta w_{t-1} + \left(\varphi\gamma - \gamma - 1 \right) \Delta y_t + \vartheta_t^d \quad (19) \text{ where } \vartheta_t^d = -\varepsilon_t^d$$

Equation (19) relates inflation to the nominal wage growth rate and the excess in the aggregate demand. We use the modified model of Novaes testing the restrictions on the model derived in last section.

We start the modelling exercise by testing the exogeneity of nominal wage inflation to the system with respect to the long run parameters in the cointegrating vector derived in the last section for the same period using the nominal wage index calculated for the São Paulo manufacturing industry. The other variables namely CIa, CIb, CIc and $\mathbf{y_t}$ are defined as above and the estimated coefficients are omitted for simplicity. Table 10 display the weak exogeneity test results.

Considering that the F-test only rejects marginally at 5% level but not at 1% level we carry on the analysis assuming that the nominal wage is weakly exogenous for the parameters in the three cointegrating vectors[16]. The following step consists in running the open VAR excluding Δip_t from the system as it had been done previously when this variable was not significant in any of the equations. Table 11 displays the diagnostic tests.

Table 10 Nominal Wage Weak Exogeneity Test[17]

$$\Delta nw = \underset{0.988}{2.1161} CIa - \underset{0.0733}{0.2593} CIb + \underset{0.1501}{0.3236} CIc + \sum_{i=1}^{3} \hat{\Pi}_i \Delta nw_{t-i} + \sum_{i=1}^{3} \hat{\Theta}_i \Delta \mathbf{y}_{\mathbf{t-i}} + \hat{\upsilon} \quad \hat{\sigma} = 0.0728$$

$$F(1,36) = 4.1390[0.0493]$$

Table 11 Diagnostic Tests Open VAR (1986/4 – 1991/12)

Test\Equation	$m1 - cpi$ (p-value)	ip (p-value)	cdb (p-value)	be (p-value)	Δcpi (p-value)	System (p-value)
AR 1-5	0.44	3.371*	0.66	0.56	1.59	0.93
	(0.81)	(0.012)	(0.65)	(0.73)	(0.18)	(0.65)
Normality	2.11	4.06	5.37	18.457**	2.62	11.81
	(0.35)	(0.13)	(0.06)	(0.000)	(0.27)	(0.30)
ARCH	0.37	0.62	0.38	0.09	0.21	
	(0.87)	(0.68)	(0.85)	(0.99)	(0.95)	
Hetero	0.51	0.35	0.71	0.54	0.32	0.33
	(0.92)	(0.98)	(0.77)	(0.91)	(0.99)	(1.00)

The diagnostic tests show that the system presents signs of autocorrelation in the industrial production equation residuals and non-normality in the be equation residuals. In contrast at the system level we do not find any evidence of non-congruence of the system with the data. The one step residuals and Chow breakpoint tests displayed in figure 5 show no instability in the parameter. Such result in conjunction to the diagnostic tests led us to conclude that overall the extended system which account for the effects of nominal wage inflation is a congruent representation of the data.

It should be noticed that equation (19) is a reduced form equation from the general model proposed and as it stands implies a theoretical equation that imposes a causality direction from nominal wage and a measure of excess demand to prices. In contrast the present analysis does not assume such causality with respect to the excess demand on the extent that the system used is a VAR where the demand variable, namely the industrial production, is modeled. The over-identified SEM derived using equation (19) as a benchmark is presented in table 12. The model consists of the system presented above where we impose restrictions aiming to identify a short run equation as (19). In particular the equation of interest is $\Delta\Delta cpi$ where this variable is a function of nominal wage inflation with the same lag structure as equation 19 and the long run cointegrating vectors identified in last section. Such result shows the nominal wage relevance in explaining the rate of growth in inflation rate and differently from the previous SEM $\Delta\Delta cpi$ lagged one period no longer enters in the equation clarifying the difficult interpretation given its negative signal.

It is worthwhile to notice also that nominal wage inflation enters in equation for Δip with the same lag structure as observed in the equation for $\Delta\Delta cpi$. The new model also preserves the findings in the SEM derived previously with respect to impacts of $\Delta\Delta cpi$ on Δip, namely a negative signal to $\Delta\Delta cpi$. The difference now is that it does not react to any cointegrating vector being a function of past and present wage inflation and past price inflation.

In the equation for Δcdb the model preserves the findings without including the rate of growth in the industrial production index that had a difficult interpretation. Again nominal wage inflation enters with the same lag structure as the other equations. Another gain in interpreting the econometric model comes from the equation for Δbe where real money no longer enters in the

[16] For more details with respect to the exogeneity test see Johansen (1994)

[17] Figures below coefficients are standard deviations and inside square brackets are p-values.

equation as it did in the previous SEM. Finally nominal wage once more enters with the same lag structure. As a final remark, the equation for $\Delta m1 - cpi$ preserves exactly the same structure observed before.

Figure 5 One Step Residuals and Breakpoint Chow Tests Extended System (1986/4-1991/12)

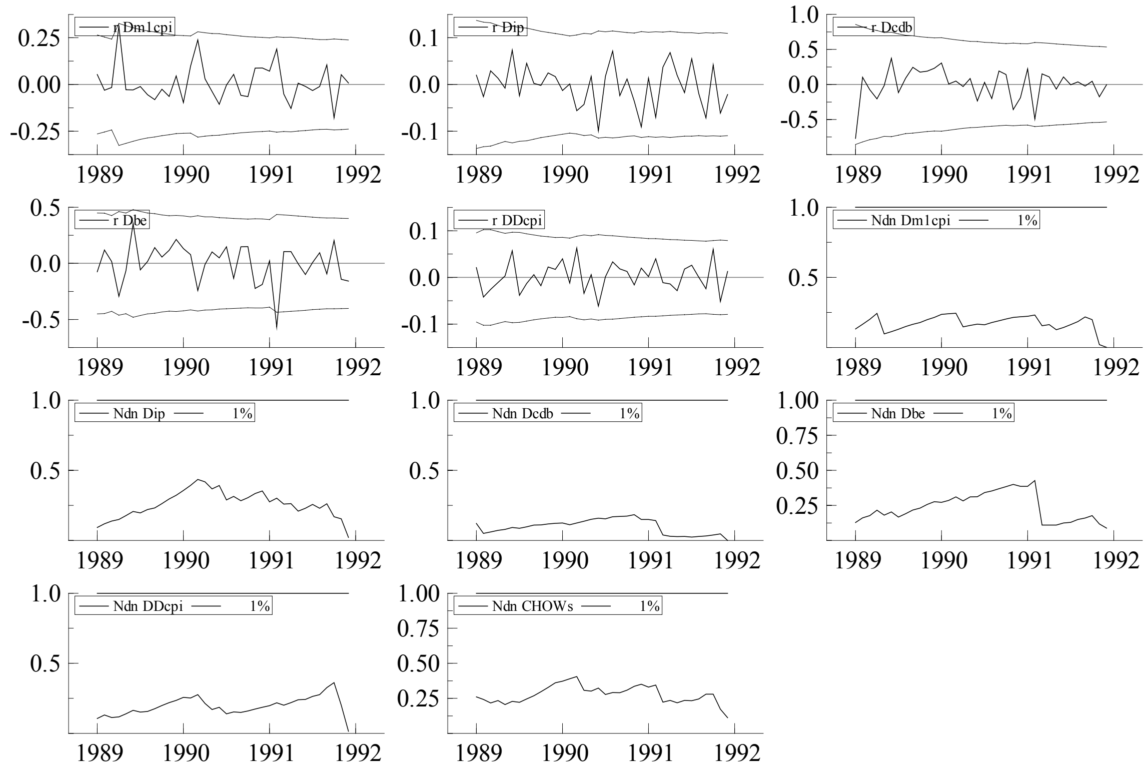


Table 12 SEM 1986/4-1991/12

$$\underset{SE}{\Delta m1 - cpi} = \underset{0.57}{1.37} - \underset{0.05}{0.21}\Delta cdb_{t-1} + \underset{0.07}{0.25}\Delta be_{t-1} - \underset{0.17}{0.41}\Delta\Delta cpi_{t-1} + \underset{0.63}{3.65}\, CIa_{t-1}$$

$$-\underset{0.04}{0.17}\, CIb_{t-1} + \underset{0.09}{0.48}\, CIc_{t-1} + \hat{\varpi}\mathbf{D_t} \quad \hat{\sigma} = 0.094$$

$$\underset{SE}{\Delta ip} = -9.625\times 10^{-5} - \underset{0.01}{}\underset{0.10}{0.44}\Delta\Delta cpi_{t-1} + \underset{0.067}{0.286}\Delta nw - \underset{0.074}{0.279}\Delta nw_{t-1} + \hat{\varpi}\mathbf{D_t}$$

$$\hat{\sigma} = 0.051$$

$$\underset{SE}{\Delta cdb} = -\underset{0.77}{11.31} - \underset{0.13}{0.27}\Delta m1 - cpi_{t-1} - \underset{0.04}{0.13}\Delta be_{t-1} + \underset{0.26}{0.93}\Delta nw - \underset{0.26}{0.76}\Delta nw_{t-1}$$

$$-\underset{0.43}{6.36}\, CIa_{t-1} - \underset{0.08}{1.17}\, CIc_{t-1} + \hat{\varpi}\mathbf{D_t} \quad \hat{\sigma} = 0.081$$

$$\underset{SE}{\Delta be} = -\underset{0.85}{7.58} + \underset{0.23}{1.04}\Delta nw - \underset{0.23}{0.91}\Delta nw_{t-1} - \underset{0.03}{0.30}\, CIb_{t-1} - \underset{0.05}{0.16}\, CIc_{t-1}$$

$$+\hat{\varpi}\mathbf{D_t} \quad \hat{\sigma} = 0.199$$

$$\underset{SE}{\Delta\Delta cpi} = -\underset{0.19}{1.44} + \underset{0.05}{0.40}\Delta nw_t - \underset{0.04}{0.36}\Delta nw_{t-1} - \underset{0.16}{0.25}\, CIa_{t-1} - \underset{0.01}{0.05}\, CIb_{t-1}$$

$$-\underset{0.02}{0.03}\, CIc_{t-1} + \hat{\varpi}\mathbf{D_t} \quad \hat{\sigma} = 0.031$$

The vector $\mathbf{D_t}$ comprises a different set of variables for each equation. Indeed for all equations it comprises the centered seasonal dummies but for the first equation it includes also *dfm4,dfma4* and *D3* as defined in chapter 5. For the second and fourth equations it comprises *dfm4,* and *dfma4*. For the third equation it includes only *dfm3*, and finally in the last equation *dfm4,* and *D3* only.

The present model also preserves the basic structure concerning the impulse response functions (omitted figure for saving space) meriting attention the long run pattern in shocks to the rate of growth in inflation where the model describe some level of persistence. Nevertheless now the short run impact on $\Delta\Delta cpi$ is restricted to nominal wage inflation derived from a theoretical model making more precise the short run analysis than in the last section model where a negative signal on $\Delta\Delta cpi_{t-1}$ by assuming that it represented stabilization plans impact on inflation dynamic.

The diagnostic tests in table 13 show the presence of autocorrelation in residuals in all but the first equation and non-normality in the first equation only. In contrast at the system level there is no signal of non-normality or autocorrelation in the residuals. Comparatively since this model has less dummy variables than the model defined before and considering the performance at the system level we carry on the analysis considering that this model exploits in full all the information available given the present set of variables.

Table 13 Diagnostic Tests SEM (M2) (1986/4 – 1991/12)

Test\Equation	$m1-cpi$ (p-value)	ip (p-value)	cdb (p-value)	be (p-value)	Δcpi (p-value)	System (p-value)
AR 1-5	2.35 (0.058)	5.071** (0.001)	3.24* (0.015)	4.03** (0.004)	2.536 (0.04)	1.04 (0.43)
Normality	9.148* (0.010)	3.35 (0.19)	0.87 (0.65)	5.680 (0.058)	4.03 (0.13)	10.46 (0.40)
ARCH	0.12 (0.98)	0.24 (0.94)	1.27 (0.29)	0.30 (0.91)	0.68 (0.64)	
Hetero	0.72 (0.79)	0.61 (0.88)	0.79 (0.70)	0.60 (0.89)	0.50 (0.95)	0.51 (1.00)

5) Conclusion

The present paper investigates the long run properties of a small macro econometric model in explaining the demand for money and inflation in Brazil during the period of high inflation in the 1980's. We model the monetary system following a progressive strategy as derived in Hendry and Richard (1982), Clements and Mizon (1991), Hendry and Doornik (1994), Hendry and Mizon (1993), Hendry (1995). This strategy contrasts to those followed in the literature for the Brazilian case where in general the Cagan model or variants of it have been tested. We derive a SEM which parsimoniously encompasses the underlying VAR. The model has a relatively complex dynamics and despite all the instability represented by the short lived hyperinflation, three stabilization plans and a change in the president of the republic has no signal of parameter instability.

The equation for the money demand shows the rate of growth in real money reacting positively to changes in the output and negatively to changes in the inflation rate. This variable also error corrects to the long run equilibrium represented by the second cointegrating vector that exactly admits the interpretation of a long run money demand with real money cointegrating with inflation and output with the corrected signals.

The equation for the growth in the industrial production index shows it reacting positively to increases in the rate of growth of inflation, an unexpected result but which has a reasonable interpretation on the grounds that the combination of price freezing and low interest rates present in most of the plans had the effect of generate a rapid growth in the economy in the aftermath of the plan launching, however in the sequence with the return of the inflation and the end of the consumption bubble the economy faced a contraction explaining therefore the apparent contradiction observed.

Finally the equation for the rate of growth in inflation shows the presence of memory in the process with lagged $\Delta\Delta cpi$ but with a negative signal possibly representing the several attempts to bring down inflation. The major finding however is that the rate of growth in inflation error corrects to the second cointegrating vector which shows that there possibly had been an equilibrium level of real money, output and inflation itself in the economy and that departures from this equilibrium had an impact in the rate of growth of inflation. Such result again reinforces the link between inflation and output in the period.

The extended SEM emphasizes the importance of nominal wage inflation in determining the short run structure and more specifically in the equation for Δip that now is completely determined by the short run impacts of nominal wage and price inflation highlighting the demand pressures exerted by wage inflation in driving the growth in industrial activity. Indeed the short run model dynamics depicts clearly the growth in the industrial production and inflation rate being driven by past inflation (wage and prices) a result that is much more in line with the theoretical models that emphasized the role played by the backward looking mechanism represented by widespread use of indexation. Interestingly the new SEM presents basically the same structure in the long run as that observed in the SEM derived in section 3. Since the results from section 3 point out to the inertial pattern of inflation present in the long run, such behaviour only strengths the conclusion that the backward looking mechanism of indexation played a significant role in determining the inflation pattern.

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