The Interaction Between Unemployment Insurance and Human Capital Policies *

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Abstract

In the presence of an optimally designed unemployment benefit system we show that it is optimal for the government to encourage human capital acquisition. The driving force of this result is the complementarity between human capital and labor-market-oriented behavior. If policy includes inter-temporal transfers, the optimal level of investment in human capital is given at the point where, at the margin, expected return to human capital is identical to the risk free rate even though there is no full insurance at the optimum. Keywords: Unemployment Insurance; Educational Policy, JEL classification: J65, I28.

Resumo

Na presença de um sistema de seguro desemprego otimamente desenhado, mostramos que é sempre ótimo para o governo incentivar a aquisição de capital humano. O resultado é gerado pela complementariedade entre o capital humano e o comportamento em relação ao mercado de trabalho. Se as políticas públicas também envolvem transferências inter-temporais, o nível de capital humano ótimo é dado no ponto em que seu retorno é igualado ao do ativo livre de risco. Isto apesar de, no ótimo, não se ter seguro total. Palavras-chave: Seguro-desemprego, Política Educacional

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1 Introduction

Economists have long recognized the existence of a link between unemployment spells and human capital. On the one hand, unemployment episodes are associated with lower returns to schooling—at the limit, zero returns, if either agents do not take informal jobs while unemployed or if these jobs do not benefit from education. On the other, there is substantial evidence that, the more educated a person is, the less extent she is unemployed.\footnote{See, for example, Nickell and Bell (1997). Note however that this evidence is not present in countries where there is no unemployment benefit systems. We shall discuss this aspect of the evidence later.}

Actual policies also seem to take into account the relationship between unemployment and human capital investment decisions. Governments that have large unemployment insurance — henceforth, UI — programs are also those that have most significant participation in the funding of higher education.

It is apparent from the arguments that optimal unemployment insurance and educational policies may have important interactions. However, it is not clear how the facts cited in the first paragraph would lead to the policies cited in the second, since the two pieces of evidence offered in the first paragraph may indicate an ambiguous relationship between returns to schooling and overall unemployment risk. Fear of unemployment may lead to an over or under-investment in human capital depending on how education affects job opportunities. Moreover, once one incorporates the fact that unemployment is not independent of agents’ behavior, one should be suspicious that higher education may not causes a decrease in unemployment but rather the fact that other actions that reduce the probability of one remaining unemployed may be influenced by education creates endogeneity problems that drive the observed negative relationship between unemployment and the level of education.

To the best of our knowledge, Brown and Kaufold (1988) were the first to explore the relationship between human capital formation and unemployment insurance programs. They show that the presence of an unemployment insurance program may lead to increased investment in human capital by decreasing the human capital risk. Though this may have important consequences in guaranteeing that at least a small amount of unemployment insurance is likely to increase welfare, this says very little about how government should (or if it should) intervene in educational choices when a UI program is present.

The purpose of this paper is to discuss the interaction between educational and unemployment governmental policies emphasizing whether government should induce agents to get more or less education than they would privately choose.\footnote{We perceive our model as belonging to broader literature that deals with the effect of uncertainty on human capital policies: Eaton and Rosen (1980), Hamilton (1987), Andersson and Anderberg (2003), da Costa and Maestri (2004).}
We maintain the simple two period structure of Brown and Kaufold’s (1988) model, as well as their inclusion of an informal sector. Contrary to them, we emphasize the importance of incentives effects of unemployment insurance in determining the fraction of an agent’s productive life that she is unemployed. One of the nice things about incorporating this possibility is that it generates, as we shall see, a complementarity between education and search effort that endogenously produces the negative correlation between education and unemployment which empirical evidence we have referred to before. It also makes the encouragement of human capital formation an important ally for the UI program.

Our main finding is that it is always optimal for the government to encourage agents to obtain education than they would privately do. We show that education is to be encouraged, not because agents under or over-invest in it as one might be lead to believe from the our previous discussion of Brown and Kaufold (1988), but exactly because the complementarity of search effort and human capital signals a ‘good’ labor market attitude and helps separating ‘unlucky’ agents from those who just do not put enough effort in participating in formal markets. By emphasizing the complementarity between government policies and UI programs, our model may indicate one reason for the correlation between expenditures in UI programs and in public education. Indeed, the governments that decide to provide larger insurance networks may face more often individuals who claim that they do not find jobs when in fact they are not spending effort on the search for employment. Hence, it is possible to alleviate this problem by increasing the opportunity cost of the unemployment spell through the provision of more education.

Another important aspect of our paper is that we follow Brown and Kaufold (1988) in allowing for an informal sector where agents can be sheltered from taxes while benefiting from unemployment insurance or, earlier in life, where they can use the time not engaged in increasing their human capital. We show that our results are robust to the existence of an informal sector, not dependent on it.

The remainder of paper is organized as follows. Section 2 presents the economy and discusses agent’s choices absent government policies. Section 3 derives the optimal policies in a world where savings are controlled by the government. In section 4 we discuss the role of non-observed savings that arises when the government tries to implement the second-best inter-temporal transfers. This possibility is accompanied by some technical issues which are handled through a series of results that we present in the appendix. Section 5 concludes.

2 The Economy

We model a two period economy with an atom of identical agents. The two period assumption—also adopted by Acemoglu and Shimer (2001), Brown and Kaufold (1988) and Bailey (1978), to name a few—is mainly due to our emphasis in the interac-
tion between education and unemployment. Educational choices are usually long term choices—as compared to the length of unemployment spells—and mostly done early in life. They affect later choices regarding work since they change relative payoff of employment *vis à vis* unemployment, as emphasized by Brown and Kaufold (1988).

As for considering identical agents, Bailey (1978) has pointed out that non-observed heterogeneity and the possibility of self-selection issues may be the very reason for the non-existence of private unemployment insurance in the first place. Moreover, the fact that agents are identical means that we disregard the possible interactions between redistributive and insurance motives in government’s policy design. We do not do it for sake of realism but rather for simplicity. As in Bailey (1978), we remove heterogeneity to focus on some specific issues related to the incentive effects associated to the unemployment insurance program.

In the next few pages we spell out our model.

**Preferences** Agents are expected utility maximizers with temporary utility given by

\[ u(c) - \zeta (L - l), \]

where \( c \) is consumption, \( l \) is leisure and \( L \) is the agent’s time endowment. We assume that both functions are smooth with \( u' \), \( \zeta' \), \( \zeta'' > 0 \) and \( u'' < 0 \) and satisfy the usual Inada conditions.

There is another dimension of effort, not included in the description of temporary utility which is related to the struggle to remain in the legal markets. The fraction of time of an agent’s life she spends unemployed is a function of both the probability of her losing a job when employed and the probability that she gets a new job when unemployed.\(^3\) This other dimension of effort aims at capturing both aspects with a single parameter \( p \in [0, 1] \) that we associate to an agent’s attitude toward work: be it her capacity of working in some unpleasant environments or the time she spends in searching for new jobs. We take this to be a life-long choice made very early on an agent’s professional life.\(^4\)

Therefore, we write an agent’s expected utility as

\[ u(c) - \zeta (L - l) + E \left[ u(c) - \zeta (L - l) \right] - \varphi(p), \tag{1} \]

The choice of \( p \) as a parameter defined in the interval \([0, 1]\) is not accidental. We shall associate \( p \) to the probability that an agent is employed in any moment of her ‘adult’ life. This being the case, the expectation operator in (1) is with regards to the probability, \( p \).

**Technology** The economy has two sectors: a formal sector and an informal sector. Each sector produces goods with a linear technology that transforms one efficiency unit

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\(^3\)Equivalently, we could define it as a function of the length of her job tenures and unemployment spells.

\(^4\)There is strong evidence that this ‘labor oriented attitude’ is of great importance in defining labor market outcome. There is, however, some evidence that policies aimed at changing this attitude may be quite helpful, even at later stages of the working life. We shall neglect this dimension of policy.
of labor into one unit of output. We abuse notation slightly by using $Y$ to represent both, and normalize units in such a way that, in the first period of the agent’s life, one hour of her time provides one efficiency unit of labor. In the second period, however, an agent’s productivity will depend on whether she has acquired human capital and on whether she is working in the formal or informal sector.

Modelling an informal sector adds an important component of most development economies. Hence, when individuals cannot find any formal job they have access to a (less productive) informal sector job, in which case the income they produce is not observable by the government.

More specifically, in the informal sector one unit of time generates one efficiency unit $Y = L = \bar{L} - l$, where $L$ is working time, regardless of how educated the agent is. As for the formal sector we have $Y = w(h)L$ where $h$ is an agent’s human capital. We assume $w(0) \geq 1$ and $w' > 0, w'' < 0$. The fact that the productivity of labor is higher in the formal sector is not controversial, and, absent this, a formal sector would not exist.

The other two assumption, on the other hand are directly related to the role of education on agents’ labor market choices. The first assumption, as mentioned before, means that an agent is (weakly) more productive in the formal sector for all levels of human capital. The other assumptions guarantee that human capital increases productivity, at decreasing rates.

To acquire human capital an agent must dedicate some of her time at youth to studying, therefore sacrificing her leisure and/or her first period income. This means that, in the first period, and absent government intervention, $c = Y = \bar{L} - l - h$. Notice that we take foregone earnings to be the only cost of education.

**Agents’ choices** Absent government’s interventions the agent’s problem is

$$\max_{p,Y,h} \{ u(Y - s) - \zeta(h + Y) + pV^c(h,s) + (1 - p)V^u(s) - \varphi(p) \}$$

where

$$V^u(s) = \max_{Y^u} \{ u(Y^u + s) - \zeta(Y^u) \}$$

and

$$V^c(h,s) = \max_{Y^c} \left\{ u(Y^c + s) - \zeta \left( \frac{Y^c}{w(h)} \right) \right\}$$

Here, the problem of the agent need not be convex, in virtue of the interplay between $p$ and $h$. However, provided that the solution is interior, the first order conditions,

$$u'(Y - s) = \zeta'(h + Y),$$

$$\zeta'(h + Y) = pV^c(h,s)$$

and

$$V^c(h,s) - V^u(s) = \varphi'(p).$$
are necessary for the optimum of the agent’s problem.

Naturally, for there to be any investment in human capital formation it must be the case that \( \zeta'(Y) - pV_h^e(0,s) \geq 0 \). We shall assume this to be the case, since the other possibility is rather uninteresting.

We have not added a condition for the optimal choice of savings. In fact we have not even addressed the issue of inter-temporal transfer of income. We allow the government to control the agent’s savings which is what most of the literature that introduces dynamic features in agency problem does. Thus, education, \( h \), is the only form of inter-temporal transfers.

3 Optimal Policy

Notice that, even under the assumption that the solution is interior (which is restrictive but not overwhelmingly so) one cannot rely on the first order approach in the case where all these variables are beyond government’s control. However, this non-convexity will be of no consequence since we shall be assuming that the government directly controls the agent’s educational choice, \( h \), and savings—which we take to be 0 without loss in generality.\(^5\)

With regards to the assumption that the government controls savings, there are two sides of it. First, it may very well be the case that savings take the form of contracts or purchases of real assets for which observability is possible. This seems to be the underlying assumption adopted in most of the literature. Second, in the case discussed in the present section, at least with regards to savings, there is a good reason for believing that the possible non-observability of savings will not constrain the government’s policy. If credit markets are not well developed, and if one takes into account the fact that the bulk of one’s income comes later in life, the restriction that \( s \geq 0 \) should bind.\(^6\) This being the case we should not expect savings choice to destroy convexity over the relevant range of the main variables. We shall come back to this last possibility later on.

It is still the case that the government cannot observe the amount of work the agent supplies in the hidden economy, both when young, \( Y \) and when adult, \( Y_u \). But since the government controls \( h \) (and savings play no role) the problem of the agent is convex in the remaining choice variables, which means that the first order approach may be applied. Moreover, we assume that government can make a transfer \( \delta \) to the first-period

\(^5\)Notice that we are concerned with savings from youth, which is when educational choices are made, to adulthood. This is different from the issue of how savings made by one who is already a participant in the legal job market affect the design of unemployment insurance policies (e.g., Werning (2001), Kocherlakota (2004).)

\(^6\)This statement is precise in the case where we associate the life-cycle model to its certainty equivalence specification. If precautionary savings is important, however, it may not be optimal to bring future income to the present even if expected earnings are higher than current earnings.
of each agent life.

That is, the government solves the following problem,

$$\max_{h, p, y^e, Y^e, \omega, \delta} \left\{ V(h, \delta) + p \left( u(y^e) - \zeta \left( \frac{Y^e}{w(h)} \right) \right) + (1 - p)V^u(\omega) - \varphi(p) \right\}, \quad (2)$$

where $V(h, \delta) \equiv \max_Y \{ u(Y, \delta) - \zeta(h + Y) \}$, and $V^u(\omega) \equiv \max_{Y^u} \{ u(Y^u + \omega) - \zeta(Y^u) \}$, subject to the resource constraint,

$$p(Y^e - y^e) \geq (1 - p)\omega + \delta$$

and the incentive compatibility constraint

$$u(y^e) - \zeta \left( \frac{Y^e}{w(h)} \right) - V^u(\omega) = \varphi'(p). \quad [\lambda]$$

Since our main concern here is the educational policy, we differentiate the associated Lagrangian (the multipliers are as shown above) by $h$ to get,

$$(p + \lambda)\zeta' \left( \frac{Y^e}{w(h)} \right) \frac{Y^e}{w(h)^2} w'(h) = \zeta'(h + Y). \quad (3)$$

Assume, for now that the solution to the agent’s optimization problem is interior (the corner solution $h = 0$ is obvious). Then, absent government’s intervention, her optimal choice of education would be characterized by the first order condition

$$p\zeta' \left( \frac{Y^e}{w(h)} \right) \frac{Y^e}{w(h)^2} w'(h) = \zeta'(h + Y)$$

which means that the agent would under-invest in education, when compared to the second-best level chosen by the government.

Or, putting in a different perspective, the government should distort the agent’s choice as one can see by the wedge between private marginal costs and private marginal benefits of education presented in (3). The result obtains despite the fact that the agent is not able to smooth consumption across time by (dis)saving. The result is formally stated in the next proposition.

**Proposition 1** At the optimum human capital, $h$, is set above what an agent would privately choose.

This result arises from the fact that offering UI has a perverse incentive effect: people decide to exert less effort to remain in the legal market. In order to alleviate the moral hazard problem it is optimal for the government to encourage human capital formation.

The first order conditions with respect to $y^e$ and $h$ can be manipulated to obtain the following alternative representation of the wedge,

$$\frac{1}{1 + \lambda/p} = p \frac{u'(y^e)}{w'(c)} L^e w'(h) \quad (4)$$
where the private benefit of education appears in the right hand side, with the state-price
deflator adjusting for the risk involved in human capital investment.

Were the more hazard problem not present in this setting and we would have full
insurance and the optimal level of human capital would be found by equalizing its ex-
pected return to that of the risk free asset. We shall call this simply the ‘socially efficient'
level recognizing, however, that the level of labor supply and the probability at which
the expected return is measured need not coincide with the first best levels. What we
try to figure out next is how the prescription for the optimal level of education found in
this second best world compare with the socially efficient one.

From (4) one may not tell whether the level of investment is efficient. On the one
hand, agents adjust the return to risk as captured by the state price deflator in (4),
while on the other, the left hand side is less than one, while efficiency occurs when
\[ 1 = pL^e w'(h). \]
Hence, efficiency obtains if and only if
\[
\frac{u'(y^e)}{u'(c)} = \frac{1}{1 + \lambda/p}.
\]

This is where the assumption on the possibility of inter-temporal transfers, as repre-
sented by \( \delta \), become important. Taking the derivative of the government problem with
respect to \( \delta \) and using the envelope theorem yields: \( u'(c) = \mu \). Thus,
\[
\frac{u'(y^e)}{u'(c)} = \frac{p}{p + \lambda}.
\]

Hence, we are able to offer the following proposition.

**Proposition 2** Investment in human capital is driven to the socially efficient level.

Thus, in spite of the fact that agents choose a higher educational choice than the one
they would choose without government intervention, the human capital level is driven
to an efficient level. It is essential for this result the assumption that government has
full control over savings. In this case, it is possible to offer incentives for people in the
second period by increasing their second period income while employed, it can be done
either by increasing their productivity or by offering a monetary transfer, hence the cost
of opportunity of both policies should be the same.

In passing, it is also worth mentioning the fact that the marginal tax on labor income
is zero. This is immediate from the first order conditions with respect to \( y^e \) and \( Y^e \),
respectively,
\[
(p + \lambda) u'(y^e) = p\mu, \text{ and } (p + \lambda) \zeta' \left( \frac{Y^e}{w(h)} \right) \frac{1}{w(h)} = p\mu.
\]

As we shall see in section 4, this result does not hold if savings are not controlled by
the government.
3.1 Discussions and a Caveat

The inclusion of an informal market as part of the description of the economy may lead one to wonder how important this is for the results we obtain — and, consequently, how relevant this may be for developed economies. The answer is that results remain valid with the exclusion of the informal sector.

To see this just notice that when problem (2) is replaced with
\[
\max_{y,Y,h,p,Y^e,Y^u,\omega} \left\{ u(y) - \zeta (Y + h) + p \left( u (y^e) - \zeta \left( \frac{Y^e}{w(h)} \right) \right) + (1 - p) [u(\omega) - \zeta (0)] - \varphi(p) \right\},
\]
subject to the appropriately modified constraints, the exact same expressions, (3) and (4), are found.

Hence the result is not dependent on the existence of an informal market but is robust to it.

An important caveat for our main result, however, concerns the inter-temporal transfers. When arguing that the assumption that the government controls savings is not a very restrictive one, we used the fact that credit markets may not be generous enough to allow for negative savings when such long horizons are considered. With inter-temporal transfers the optimal policy involves
\[
u'(Y + \delta) < pu'(y^e) + (1 - p) u' (Y^u + \omega),
\]
This is nothing but a restatement of Rogerson's (1985) result, and is easily derived in our model by combining the first order conditions with respect to $\delta$, $\omega$ and $y^e$.\footnote{To be precise,}

The consequence of means that the non-negativity restriction on savings ceases to be important and non-observability can have important consequence for policy design.

There are two ways out of this difficulty. First we may argue that non-observability is not important so that, in practice, the government controls savings and all our results remain valid.

The second possibility is that non-observability is important and optimal policies should take this into account. The problem with this reaction is that without observability the problem that the agent solves is not convex, which means that the first order condition need not characterize the optimal choice of the agent. In the next section we have deal with this issue by discretizing the choice set and show that, even though hidden savings do alter other dimensions of policy — e.g., the marginal tax rate for labor income —, it is still the case that education should be encouraged.
4 Hidden Savings

Hidden savings are important for the design of unemployment insurance programs because they affect the costs of being unemployed and, thus the incentives to look for a new job. There is a growing literature dealing with the effects of hidden savings on the design of unemployment insurance programs.\footnote{E.g., Kocherlakota (2004), Werning (2002), Abrahan and Pavoni (2003).} Our two period model does not allow us to discuss the way in which savings affect the pattern of transfers along an unemployment spells. We share with the literature, however the concern with how incentives are affected by savings choices and how this feeds back into taxes and, ultimately, on human capital formation.

Because our model collapses an agent’s entire adult live in a single period, one may wonder whether allowing for savings would completely destroy the problem and make the discussion of an unemployment insurance program uninteresting. In fact, it may. Levine and Zame (2002) have shown that with infinite lives, purely idiosyncratic shocks the equilibrium allocation of an economy with unrestricted savings can be made arbitrarily close to the allocation with complete markets by making the discount rate sufficiently close to one. However, in an economy with finite lives and empirically sound discount rates, unemployment does matter even when precautionary savings are used to improve consumption smoothing. Without full insurance and with finite lives, consumption is history dependent and consumption is decreasing in the [expected amount of time] that one is unemployed.\footnote{With multiple periods, the point in time one is unemployed is also important. It may very well be the case that, at a given moment, an agent consumes more than another agent who have experienced more goods shocks. However, at any moment conditional on the current level of wealth the monotonicity is guaranteed.}

We first define the indirect utility functions,

\[
V^u(s, \omega) \equiv \max_{Y^u} \{u(Y^u + \omega + s) - \zeta(Y^u)\},
\]

and,

\[
V(s, k, h) \equiv \max_Y u(Y - s + k) - \zeta(Y + h),
\]

conditional on an agent being unemployed and employed, respectively.

The government’s program is

\[
\max \left\{ V(s, k, h) + p \left[ u(y^e + s) - \zeta \left( \frac{Y^e}{w(h)} \right) \right] + (1 - p)V^u(Y, s, \omega) - \varphi(p) \right\}
\]

subject to the resource constraint

\[
p(Y^e - y^e) \geq (1 - p)\omega + k
\]
and to the incentive constraint

\[(p, s) \in \arg \max_{(\hat{p}, \hat{s})} \left\{ V(\hat{s}, k, h) + \hat{p} \left[ u(y^e + \hat{s}) - \zeta \left( \frac{Y^e}{w(h)} \right) \right] + (1 - \hat{p})V^u(\hat{s}, \omega) - \varphi(\hat{p}) \right\}. \tag{7}\]

Non-observation of both savings and non-markets skills render the agent’s problem potentially non-convex, and makes the use of a first order approach unreliable.

There are some alternatives for dealing with the issue. Werning (2002) restricts preferences to a class where the first order approach is guaranteed to work. Abraham and Pavoni (2003) solve the model assuming that the approach works and then check whether, for the specific parametrization they have chosen, the first order conditions characterize a maximum at the optimal solution. In both cases, the models are substantially more complex than ours since these authors work with fully dynamic problems that demand the transformation of the problem into a recursive one. Our payoff is that we are able to adopt a different procedure that does not depend on a specific functional form of parametrization of the problem.

That is, we pursue a different path more suited to the problem we face. We discretize the effort space by making the domain of \( p \) to be the finite set \( \{0, p_1, ..., 1\} \) and proceed by analyzing the optimal strategies of deviation. This procedure, in some sense, mimics a numerical approach with an important advantage: all results derived herein do not depend on specific functional forms.

To begin with, let the worker’s savings problem, conditional on \( p \), be

\[\max_{\hat{s} \in \mathbb{R}_+} \left\{ V(\hat{s}, k, h) + p \left[ u(y^e + \hat{s}) - \zeta \left( \frac{Y^e}{w(h)} \right) \right] + (1 - p)V^u(\hat{s}, \omega) - \varphi(\hat{p}) \right\}. \tag{8}\]

which optimal value we shall denote \( W(k, h, y^e, Y^e, \omega, p) \). The restriction that \( \hat{s} \in \mathbb{R}_+ \) is due to the credit constraint.

Next, we define for the government a relaxed program as

\[
\max_{k, h, y^e, Y^e, \omega, \hat{p}} \left[ W(k, h, y^e, Y^e, \omega, \hat{p}) + \mu[\hat{p}(Y^e - y^e) - (1 - \hat{p})\omega - w] + \sum_{p < \hat{p}} \lambda(p)[W(k, h, y^e, Y^e, \omega, \hat{p}) - W(k, h, y^e, Y^e, \omega, p)] \right]
\tag{9}\]

where instead of considering the complete set of incentive compatibility constraints, we only consider those that guarantee that the agent does not choose a lower level of effort than the optimum, \( p^* \).

Because the government faces fewer constraints, the solution to (9) is not inferior to the solution to the government’s problem (5) when constraint (7) is considered. What we show in the appendix is that if \( (k^*, h^*, y^e_*, Y^e_*, \omega^*, p^*) \) solves (9) then, at this solution, there is no strategy with \( p > \hat{p} \) (and associated optimal choices) that yields higher expected utility for the agent. Therefore the solution to (9) solves government’s problem (5) subject to (6) and (7). This proposition shows that after solving the relaxed problem
no strategy associated to a higher effort can yield a better payoff. Intuitively, when the government designs the optimal mechanism, it can neglect the possibility of agents spending too much effort to search for a job.

The next two lemmas, proven in the appendix, are stated here to facilitate the intuition regarding some of the results that follow.

**Lemma 1** *In all strategies that contemplate a lower level of effort than the optimum the agent chooses no more savings than she chooses when she makes the socially optimal effort.*

**Lemma 2** *In all strategies that contemplate a lower level of effort the agent chooses at least the same labor supply in the first period.*

To grasp the logic behind these results, one should recall that savings are complementary with deviant behavior (lower than optimal effort, \( p \)), since agents who do not make enough effort to remain in the formal markets have a higher expected marginal utility of consumption. But higher savings also imply a higher propensity to work in the first period, which helps understand lemma 2: agents who plan on exerting a less intensive search for a job choose to work more on informal activities from very early in their lives. Moreover, as some recent papers in the UI literature show, they will hold always higher savings.\(^{10}\)

The fact that savings are higher off the equilibrium path will drive some of the prescriptions found in section 3 to differ from some of the prescriptions found herein.

**Educational Policy** We begin the discussion with our main concern in this paper: educational policy. What we show is that the qualitative results regarding educational policy are not altered by the possibility of hidden savings.

To do this we first write the first order necessary condition with respect to \( h \),

\[
-\zeta'(h + Y^*) + p^* \zeta'(L^e) L^e \frac{w'(h)}{w(h)} + \sum_{p < p^*} \lambda(p)[\zeta'(h + Y(p)) - \zeta'(h + Y^*)] \\
+ \zeta'(L^e) L^e \frac{w'(h)}{w(h)} \sum_{p < p^*} \lambda(p)[p^* - p] = 0. \tag{10}
\]

The envelope theorem was used in (8) to find the partial derivative of \( W \) with respect to \( h \).

It is apparent from lemma 2 that the third and the forth terms in (10) are positive. Therefore,

\[
p^* \zeta'(L^e) L^e \frac{w'(h)}{w(h)} < \zeta'(h + Y^*).
\]

\(^{10}\)This result may not be robust to the introduction of multiple unemployment episodes.
The inequality above shows that the optimal policy requires the creation of a wedge between optimal private and social schooling choice, which we formalize in the next proposition.

**Proposition 3** At the optimum, \( h^* > h^o \), where \( h^o \equiv \arg \max_h W(k, h, y^e, Y^e, \omega, p^*) \).

What the government must induce is a choice of \( h \) that is higher than the private optimum. The rationale is once again that, by forcing agents to get more education, the government raises the costs of free riding on the unemployment benefit program.

**Taxation and UI** Next, we investigate the consequences of hidden savings for optimal labor income taxes and the unemployment benefits. To accomplish this we start by taking the first order conditions with respect to \( y^e, \omega \) and \( k \), respectively,

\[
\mu = u'(c^*_e) + \sum_{p < p^*} \lambda(p) \left[ u'(c^*_e) - \frac{p}{p^*} u'(c^e(p)) \right],
\]

and

\[
\lambda = u'(c^*_h) + \sum_{p < p^*} \lambda(p) \left[ u'(c^*_h) - \frac{1-p}{1-p^*} u'(c^h(p)) \right],
\]

and

\[
\mu = u'(c^*_0) + \sum_{p < p^*} \lambda(p) \left[ u'(c^*_0) - u'(c_0(p)) \right].
\]

Next, combining the three first order conditions above, we get

\[
u'(c^*_0) - p^* u'(c^*_e) - (1-p^*) u'(c^*_h) = \frac{\sum_{p < p^*} \lambda(p) [u'(c_0(p)) - p u'(c^e(p)) - (1-p) u'(c^h(p))]}{1 + \sum_{p < p^*} \lambda(p)}
\]

From lemmas 1 and 2, in the appendix, and the expression above, \( u'(c^*_0) = E u'(c^*_1) \), (expectation is with respect to probability \( p^* \)). Then, \( p < p^* \Rightarrow s^* < s(p) \). This being the case, the first order conditions with respect to \( Y^e \) and \( w^e \) yield

\[
\frac{\zeta' (L^e)}{w(h)} \left\{ p^* + \sum_{p < p^*} \lambda(p) [p^* - p] \right\} = p^* u'(c^*_e) + \sum_{p < p^*} \lambda(p) [p^* u'(c^*_e) - u'(c^e(p))],
\]

which implies

\[
\frac{\zeta' (L^e)}{w(h)} - u'(c^*_e) = \frac{\sum_{p < p^*} \lambda(p) p \left[ \zeta' (L^e) / w(h) - u'(c^e(p)) \right]}{p^* + p^* \sum_{p < p^*} \lambda(p)} \geq 0.
\]

This can be written more compactly as

\[
\Phi(p^*) = \kappa (p^*) \sum_{p < p^*} \pi(p) \Phi(p) \geq 0.
\]  (11)

where

\[
\kappa (p^*) \equiv \frac{\sum_{p < p^*} \lambda(p) p u'(c^e(p))}{p^* + p^* \sum_{p < p^*} \lambda(p)} u'(c^*_e), \quad \pi(p) \equiv \frac{\lambda(p) p u'(c^e(p))}{\sum_{p < p^*} \lambda(p) p u'(c^e(p))}.
\]
and

\[ \Phi(p) \equiv \frac{\zeta'(L^c)}{w(h)w'(c^e(p))} - 1 \]

The marginal tax rate on labor income \( \Phi(p^*) \) is proportional to the (implicit) marginal tax rate on those who deviate by putting less effort than \( p^* \). The following assumption is sufficient to guarantee that the inequality in (11) is strict, which means that the marginal tax rate on labor income is negative.

**Assumption p:** There exists an (arbitrarily small) \( p > 0 \) such that \( \varphi(p) = 0 \).

This assumption guarantees that even if one does not make any effort to find a job there is a positive probability that she will find a job at the legal markets. With this assumption the following proposition 4—which is in contrast with what one would find in the setup of section 3—obtains.

**Proposition 4** Under assumption p, the marginal tax rate on labor income is negative at the optimum.

What is interesting about this last result is the fact that it was not present in the case where savings were observed. Nor is it part of any optimal unemployment insurance scheme derived in the literature. One should first notice that the unemployment insurance program does not affect labor supply conditional on one’s participation in the legal markets. This reasoning generates the prescription of zero marginal taxes in the framework of section 3.

What is new here is the fact that differences in savings affect the propensity to work conditional on one’s being in the legal markets. This allows the subsidy on work to create a disincentive to save that characterize deviating behavior.

This result may not be robust to relaxing the two period formulation. However, the result would also be valid under the assumption, made by most of the literature, that once a worker gets a job there is no longer any incentive problem. The crucial point here is that the possibility of a worker who is employed making some kind of effort to remain in this job is neither included in our formulations nor in the standard specification of the dynamic problem.

**5 Conclusion**

This paper investigates the interaction between unemployment insurance and educational policy.

In a two period model that subsumes life-long choices agents’ employment status is affected by labor market attitude, which, in its own turn is dependent on the relative cost of being unemployed. Education is important in this world not only because it
raises the expected income of agents but also because it affects the opportunity cost of unemployment. It is this latter effect that plays the most prominent role in our model.

The main result obtained in the paper is that both unemployment insurance and educational policies are complementary, i.e., in order to alleviate the moral hazard which is inherent to UI programs it is always optimal for the government to distort agents choices toward more investment on human capital, thus increasing opportunity cost of the unemployment spells.

Another, very interesting finding is that, despite our being in a second best world, at the optimum, the expected benefit of education is equal to its expected cost: a type of production efficiency result in our setup. This, however, requires the government to be able to make optimal inter-temporal transfers. The problem is that, as in Roger-son (1985), optimal policies require the expected marginal utility of consumption to be higher than marginal utility of consumption in the first period. This raises all types of questions about observability of savings and the potential non-convexities that arise when observability is not assumed.

We deal with non-observable savings and show that encouragement of education is robust to this modification in our main setup.

The model is very simplistic and aims only at capturing the interplay between these two forces. Extending the model to multiple periods is unlikely to change the main results though it may shed some light on some aspects of the dynamic of human capital accumulation, and on how this affects the design of optimal unemployment insurance contracts.

A Appendix

Lemma 3 At a fixed $s$, $c^e$ is decreasing and $c^u$ is increasing in transfers.

Proof. For the first part we just note that $c^e = Y^e + s - \omega$. Since $Y^e$ is chosen by the government, we have $dc^e/d\omega = -1 < 0$. For the second, note that $c^u = Y^u + s + \alpha \omega$ which means that $dc^u/d\omega = \alpha + dY^u/d\omega$. Then, $u'(Y^u + s + \alpha \omega) - \zeta'(Y^u) = 0$, which implies

$$\frac{dY^U}{d\omega} = -\alpha \frac{u''(c^u)}{u''(c^u) - \zeta''(Y^u)}$$

and

$$\frac{dc^u}{d\omega} = \alpha \left[ 1 - \frac{u''(c^u)}{u''(c^u) - \zeta''(Y^u)} \right] > 0.$$  

Lemma 4 $c_0$ is decreasing and $c_u$ and $c_e$ are increasing in $s$.

Proof. The proof follow the steps of lemma 3. 

Lemma 5 $c^*_e \geq c^*_u$ in any relaxed program.
Proof. We will consider the relaxed program and we will prove the lemma by showing that, if \( c_e^* < c_u^* \), a redistribution of income from the unemployment state to the employment increases welfare and is incentive compatible. Define \( (c_0^*, c_e^*, c_u^*, Y_u^*, Y_e^*, s^*) \equiv \)

\[
\begin{align*}
\text{arg max } & u(c_0) - \zeta(Y_0) + \arg\max \left[u(c^e) - \zeta \left( \frac{Y_{e^*}}{w(h)} \right) \right] + (1 - p^*)[u(c^u) - \zeta(Y_u^*)] - \varphi(p^*) \\
\text{s.t. } & c^e = y^{e^*} + s, c^u = Y^u + s + \omega, \text{ and, } c_0 = Y_0^* - s + k; \\
\end{align*}
\]

(12)

and \( (c_0(p), c_e(p), c_u(p), Y_0(p), Y_u(p), s(p)) \equiv \)

\[
\begin{align*}
\text{arg max } & u(c_0) - \zeta(Y_0) + p \left[u(c^e) - \zeta \left( \frac{Y_{e^*}}{w(h)} \right) \right] + (1 - p)[u(c^u) - \zeta(Y_u^*)] - \varphi(p^*) \\
\text{s.t. } & c^e = y^{e^*} + s, c^u = Y^u + s + \omega, \text{ and, } c_0 = Y_0^* - s + k \\
\end{align*}
\]

(13)

Since \( p^* \) maximizes the relaxed program we should have, for all \( p < p^* \),

\[
\begin{align*}
& u(c_0^*) - \zeta(Y_0^*) + p^* \left[u(c^e^*) - \zeta \left( \frac{Y_{e^*}}{w(h)} \right) \right] + (1 - p^*)[u(c^u^*) - \zeta(Y_u^*)] - \varphi(p^*) \geq u(c_0(p)) \\
& -\zeta(Y_0(p)) + p \left[u(c^e(p)) - \zeta \left( \frac{Y_{e}}{w(h)} \right) \right] + (1 - p)[u(c^u(p)) - \zeta(Y_u^*)] - \varphi(p) \quad (14)
\end{align*}
\]

Now, the fact that choices in (13) are optimal when the probability is \( p \) guarantees that

\[
\begin{align*}
& u(c_0^*) - \zeta(Y_0^*) + p \left[u(c^e^*) - \zeta \left( \frac{Y_{e^*}}{w(h)} \right) \right] + (1 - p)[u(c^u^*) - \zeta(Y_u^*)] - \varphi(p) \leq u(c_0(p)) \\
& -\zeta(Y_0(p)) + p \left[u(c^e(p)) - \zeta \left( \frac{Y_{e}}{w(h)} \right) \right] + (1 - p)[u(c^u(p)) - \zeta(Y_u^*)] - \varphi(p) \quad (15)
\end{align*}
\]

From (14) and (15) we have

\[
\begin{align*}
& u(c_0^*) - \zeta(Y_0^*) + p^* \left[u(c^e^*) - \zeta \left( \frac{Y_{e^*}}{w(h)} \right) \right] + (1 - p^*)[u(c^u^*) - \zeta(Y_u^*)] - \varphi(p^*) \geq \\
& u(c_0^*) - \zeta(Y_0^*) + p \left[u(c^e(p)) - \zeta \left( \frac{Y_{e}}{w(h)} \right) \right] + (1 - p)[u(c^u(p)) - \zeta(Y_u^*)] - \varphi(p),
\end{align*}
\]

which implies that

\[
\Delta p \left[\zeta(Y_u^*) - \zeta \left( \frac{Y_{e^*}}{w(h)} \right) \right] \geq \Delta p[u(c_u^*) - u(c_e^*)] + \varphi(p^*) - \varphi(p), \quad (16)
\]

where \( \Delta p = p^* - p \). Observe that the deviation strategies generally contemplate different choices of \( s \) and \( Y^u \), as long as \( c_e^* \neq c_u^* \). Notice, however, that, if \( c_e^* = c_u^* \), we have \( s(p) = s^* \) and \( Y^u(p) = Y_u^* \). Assume that \( c_e^* < c_u^* \). We, now, distribute income from the unemployment state to the employment until we have \( c_e^* = c_u^* \). This is feasible according to lemma 3. Denoting \( Y' \) the choice made by the truth-telling strategy after the reform,
we have \( Y^u > Y_u^* \), (see the proof of lemma 3). We shall prove that the reform does not violate incentive compatibility, i.e.,

\[
\begin{align*}
\quad u(\hat{c}_0) - \zeta(Y^*_u) + p^* \left[ u(\hat{c}_e^*) - \zeta \left( \frac{Y^e}{w(h)} \right) \right] + (1 - p^*) \left[ u(\hat{c}_u^* - \zeta(Y^*_u) - \varphi(p^*) \geq u(\hat{c}_0(p)) \\
- \zeta(\hat{Y}_0(p)) + p \left[ u(\hat{c}_e(p)) - \zeta \left( \frac{Y^e}{w(h)} \right) \right] + (1 - p) \left[ u(\hat{c}_u(p)) - \zeta(\hat{Y}_u(p)) \right] - \varphi(p)
\end{align*}
\]

(17)

Because \( \hat{c}_0 = \hat{c}_0(p) \), \( \hat{Y}_0 = \hat{Y}_0(p) \), \( \hat{c}_e = \hat{c}_e(p) \), \( \hat{c}_u = \hat{c}_u(p) \) and \( \hat{Y}_u = \hat{Y}_u(p) \), since \( c_e^* = c_u^* \), after the reform, inequality (17) collapses to

\[
\Delta p \left[ \zeta(Y^u) - \zeta \left( \frac{Y^e}{w(h)} \right) \right] \geq \varphi(p^*) - \varphi(p)
\]

(18)

Now, the right hand side of (18) minus the right hand side of (16) is \( \Delta p \int_{Y^*_u}^{Y_u^*} \zeta'(Y)dY > 0 \) and the left hand side of (16) minus the left hand side of (18) is \( \Delta p [u(c_u^*) - u(c_u^*)] > 0 \). Therefore, we conclude that the reform is incentive-compatible and increases welfare, since the utility is strictly concave. ■

**Proof of lemma 1.** Let \( s^* \) and \( s(p) \) be as defined in (12) and (13), respectively, for \( p < p^* \). Assume that \( s(p) < s^* \). (In which case \( s^* > 0 \)). From lemma 4, this implies \( c_0(s) > c_0^* \), which, in turn gives \( u'(c_0(p)) < u'(c_0^*) \). Now, \( u'(c_0(p)) \geq pu'(c_e(p)) + (1 - p)u'(c_u(p)) > pu'(c_e^*) + (1 - p)u'(c_u^*) > p^*u'(c_e^*) + (1 - p)u'(c_u^*) = u'(c_0^*) \), where we invoked lemma 4, once again to derive the first inequality. This is, however, a contradiction. ■

**Proof of lemma 2.** We have from lemma 1 that \( s^* \leq s(p) \) whenever \( p < p^* \). But, then, by an argument identical to the one used in the proof of lemma 3, one can easily show the result. ■

**Lemma 6** The resource constraint multiplier, \( \mu \), for the relaxed program, (9), is positive.

**Proof.** We first show that \( Y^e = 0 \) cannot be part of the solution to (9). First note that when \( Y^e = 0 \) the government only intervenes in the equilibrium of this economy by transferring resources across time. It is clear that at \( Y^e = 0 \) the welfare is lower than in the competitive equilibrium, when the government plays the same role of transferring resources. Hence, \( Y^e \) cannot solve the problem.

Consider, then, the case where solution with respect to \( Y^e \) is interior. Take the first order condition with respect to \( Y^e \),

\[
p^* \mu - \frac{1}{w(h)} \zeta' \left( \frac{Y^e}{w(h)} \right) \left[ p^* - \sum_{p < p^*} \lambda(p) (p^* - p) \right] = 0
\]

which, then, implies

\[
\mu = \frac{1}{w(h)} \zeta' \left( \frac{Y^e}{w(h)} \right) \left[ 1 - \sum_{p < p^*} \lambda(p) (1 - p/p^*) \right] > 0.
\]
Proposition 5 No constraint relative to a strategy that contemplates \( p > p^* \) is binding at the optimum.

Proof. First solve the relaxed problem and find the value \( p^* \) that solves (9). If there is no deviation strategy in which the agent chooses a higher level of effort and that yields at least the same utility level as the one associated with \( p^* \), then, we have proved our proposition. For every \( p \in \{0, p^1, ..., 1\} \) define

\[
W(p) \equiv \max_{(\hat{s}, \hat{Y})} \left\{ V\left(\hat{s}, \hat{Y}, \omega, h\right) + pV^e(\hat{s}, h, y^e, Y^e) + (1 - p)V^u\left(\hat{Y}, \hat{s}, k\right) - \varphi(p) \right\},
\]

and let \( \bar{W} \equiv \max_p W(p) \) and \( \bar{p} \equiv \max \{p; W(p) = \bar{W}\} \). Next, observe that \( \bar{p}(Y^{e*} - w^{e*}) - (1 - \bar{p})w^{us} - \omega^* > p^*(Y^{e*} - w^{e*}) - (1 - p^*)w^{us} - \omega^* \geq 0 \). Hence, resources are idle, which implies, from lemma 6, that \( \{w^{e*}, h^*, w^{us}, Y^{e*}\} \) is not a solution to the \( \bar{p} \)-relaxed program. This contradicts the assumption that \( p^* \) belongs in the solution of (9). ■

Proposition 6 At least one incentive compatibility constraint binds at the optimum.

Proof. Assume the contrary. It is clear that the government must provide full insurance. Hence, it is obvious that no agent would have any incentive to choose a positive effort. Therefore, this policy would not be feasible. ■

Lemma 7 At the optimum \( c^e > c^u \).

Proof. Recall the Lagrangian for the government’s problem,

\[
\mathcal{L} \equiv V(k, h, y^e, Y^e, \omega, \hat{p}) + \mu[\bar{p}(Y^{e} - y^e) - (1 - \bar{p})\omega - w] + \sum_{p < \hat{p}} \lambda(p) [V(k, h, y^e, Y^e, \omega, \hat{p}) - V(k, h, y^e, Y^e, \omega, \hat{p})].
\]

In this case,

\[
\left. \frac{\partial \mathcal{L}}{\partial \omega} \right|_{c^e = c^u} = \frac{\partial V_0}{\partial \omega} - \mu + \sum_{p < p^*} \lambda(p) \left[ \frac{\partial V(p^*)}{\partial \omega} - \frac{\partial V(p)}{\partial \omega} \right] = -\mu < 0.
\]

■

Lemma 8 At the optimum, there is \( p < p^* \) such that \( s(p) > s(p^*) \geq 0 \) and \( \lambda(p) > 0 \).
Suppose, however, that is, indeed, the case. So, let us suppose that $\lambda(p) > 0$ and that $s(p) = 0$ for all $p < p^*$ such that $\lambda(p) > 0$. Hence, it’s obvious that $u'(c^*_0) \geq u'(c_0(p))$, from lemma 2. From the first order condition, we have (if $s(p) = s(p^*) = 0$), $u'(c^*_u) = \mu - \sum_{p < p^*} \lambda(p)(p - p^*)u'(c^*_0)$, and $u'(c^*_e) = \mu - \sum_{p < p^*} \lambda(p)(p^* - p)u'(c^*_e)$. Therefore, from the strict concavity of $u(\cdot)$ we see that $c^*_u < c^*_e$. Hence, from lemma 1 and the assumption that $s(p) = 0$ [???] for all $p < p^*$ with $\lambda(p) > 0$, one can see that $u'(c^*_0) = u'(c_0(p))$ for all $p < p^*$ such that $\lambda(p) > 0$. Moreover, from the fact that $c^*_u < c^*_e$, it is clear that $E u'(c^*_0) < E u'(c_0(p))$. We will show that for $\varepsilon > 0$ sufficiently low, the policy $\{w^* - \varepsilon, w^* - \varepsilon, \omega^* + \varepsilon\}$ is welfare-improving and clearly does not violate the resource constraint. For $\varepsilon$ sufficiently low, $\Delta W \approx$

$$\varepsilon \left\{ u'(c^*_0) - E u'(c^*_0) + \sum_{p < p^*} \lambda(p)u'(c^*_0) - E u'(c^*_0) - u'(c_0(p)) + E u'(c_0(p)) \right\},$$

or

$$\Delta W \approx \varepsilon \left\{ u'(c^*_0) - E u'(c^*_0) + \sum_{p < p^*} \lambda(p)E u'(c_0(p)) - E u'(c^*_0) \right\} > 0,$$

which contradicts the optimality of the policy. ■

Proof of Proposition 4. From the lemma 8, there is $p < p^*$ such that $s(p) > s(p^*) \geq 0$ and $\lambda(p) > 0$. From the Spence-Mirrlees condition it is clear that $s(p) > s(p^*) \Rightarrow \Phi(p^*) < \Phi(p)$. Now, if the left hand side of (11) is zero, the right hand side is negative. Therefore, the marginal tax rate can not be zero.

Suppose, however, that $\Phi(p^*) < 0$. In this case,

$$0 < \frac{\sum_{p < p^*} \lambda(p)p}{p^* \left(1 + \sum_{p < p^*} \lambda(p)\right)} < 1.$$

The right hand side is less than the left hand side, hence this can not be the case. ■

References


