FUNCTIONAL DISTRIBUTION, CAPITAL ACCUMULATION AND GROWTH IN A NON-LINEAR MACRODYNAMIC MODEL

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Abstract: It is developed a macrodynamic model of distribution, capital accumulation and growth in which investment is non-linear in distribution: at low (high) levels of wage share, the impact of a higher profit share on investment is negative (positive). This specification conforms with some of the empirical evidence for the rise and fall of the Golden Age in several advanced economies. As it turns out, whether the economy follows a wage-led growth regime or a profit-led one depends on the prevailing distribution. Indeed, a similar dependence applies, alongside with the relative bargaining power of capitalists and workers and the cyclical behavior of markups, to the dynamic stability properties of the economy.

Key Words: capital accumulation, distribution, growth

Resumo: É desenvolvido um modelo macrodinâmico de distribuição, acumulação de capital e crescimento em que o investimento depende não-linearmente da distribuição: para níveis baixos (elevados) de parcela salarial, o efeito sobre o investimento de uma maior parcela de lucros é negativo (positivo). Esta especificação conforma-se com a evidência empírica para a ascensão e queda da Idade do Ouro em várias economias avançadas. Em conseqüência, se a economia estará em um regime de crescimento dirigido pelos salários ou dirigido pelos lucros é algo que finda dependendo da distribuição prevalecente. De fato, uma dependência semelhante se aplica, ao lado do poder de barganha relativo dos trabalhadores e capitalistas e do comportamento cíclico dos *markups*, às propriedades de estabilidade dinâmica dessa economia.

Palavras-chave: acumulação de capital; distribuição; crescimento

JEL classification: O41; O11; E32

Código ANPEC: Área 5 – Crescimento, Desenvolvimento Econômico e Instituições

* I should be grateful to Amitava Krishna Dutt and Jaime Ros for valuable suggestions, and to the Brazilian National Council of Scientific and Technological Development (CNPq) for research funding.

1. Introduction

This paper develops a stylized dynamic model of capital accumulation, functional distribution and growth. Firms' desired investment is made to be non-linear in distributive (wage and profit) shares, which implies that whether it will rise or fall in response to a change in distribution depends upon the level of distribution. It is assumed that firms' desired accumulation will be lower for both high and low levels of wage share, it being higher for intermediate levels of distribution. While at high levels of wage share the impact of higher profitability on investment plans is positive, this impact becomes negative at low levels of wage share. Indeed, this non-linear specification conforms with some of the empirical evidence for the rise and fall of the Golden Age in most advanced economies. In turn, inflation dynamics is determined within a conflicting claims framework, meaning that price inflation occurs whenever workers and capitalists press claims in excess of available income.

As it turns out, whether the economy will experience a wage-led growth regime or a profit-led one depends on the prevailing distribution, with a similar dependence applying, alongside with the relative bargaining power of capitalists and workers and the cyclical behavior of markups, to the dynamic stability properties of the economy. Wage-led capital accumulation and growth will obtain for lower levels of wage share, while profit-led capital accumulation and growth will obtain for lower levels of profit share. Hence, the model does not rely on full capacity utilization being reached for a change in the capital accumulation and growth regimes to take place. Regarding dynamics, while a long-run equilibrium with wage-led accumulation and growth will be unstable, one with profit-led accumulation and growth will be stable provided workers' bargaining power is weak enough.

This paper is organized in the following way. Section 2 describes the structure of the model, whereas Section 3 analyzes its behavior in the short run. The behavior of the model in the long run is discussed in Section 4, while the final section presents a summary.

2. The structure of the model

The economy is a closed one and with no government activities, producing only one good for both investment and consumption. Two (homogeneous) factors of production are used, capital and labor, with the technology being one of fixed coefficients. Hence, labor employment is determined by production:

$$L = aX \tag{1}$$

where L is the employment level, a is the labor-output ratio and X is the output level.

Production is carried out by oligopolistic, price-maker firms. At a point in time, prices are given, having resulted from past dynamics. Firms will produce according to demand, it being assumed throughout that forthcoming demand is insufficient for them to produce at full capacity utilization at the ongoing prices.[1] Firms are also assumed to make

[1] Steindl (1952) claims that firms plan excess capacity so as to be ready for a sudden expansion of sales. First, the existence of fluctuations in demand means that the producer wants to be in a boom first, and not to leave the sales to new competitors who will press on her market when the boom is over. Second, it is not possible for the producer to expand her capacity step by step as her market

accumulation plans given by the following desired investment function:

$$g^d = \alpha + \beta(\sigma - \sigma^2) \tag{2}$$

where α and β are positive parameters of the desired investment function, g^d, which is desired accumulation as a ratio of the existing capital stock, K, while σ is the share of wages in income. While Rowthorn (1981) and Dutt (1984, 1992) follow Kalecki (1971) and Robinson (1962) and make desired investment to depend positively on the profit rate, we make it to depend on functional distribution instead. Rather than following Bhaduri & Marglin (1990) in making it to vary monotonically with the profit share, however, we make it to depend non-linearly on distribution by assuming that it will be lower for both high and low levels of wage share, it being higher for intermediate levels of distribution. Hence, this inverted-U relation implies that while at lower levels of profit share the impact of higher profitability on desired investment is positive, this impact becomes negative at higher levels of profit share. As we intend to focus on the impact of distribution, other relevant factors that affect desired investment are assumed to be captured by the autonomous component given by α.

Indeed, one can read some of the empirics of the rise and fall of the Golden Age in the advanced capitalist economies (e.g. Marglin & Bhaduri 1990; Bhaskar & Glyn 1995; Glyn 1997) as showing that the level of distribution matters for the direction and intensity of the profitability effect on investment. In Bhaskar & Glyn (1995) and Glyn (1997), for instance, it is shown that while profitability is important for most of the countries, there is no simple association between increasing profits and increasing investment. Indeed, the average empirical evidence for Europe reported in Marglin & Bhaduri (1990) shows that the corporate business net profit share in income fell almost monotonically from the early 1950s through the late 1960s, whereas the growth rate of business gross fixed capital rose almost monotonically from the early 1950s through the early 1960s, and then fell almost monotonically through the late 1970s. Moreover, the empirical investigation of investment trends conducted by Bhaskar & Glyn (1995) suggests that declining profitability accounted for a major part of the investment slowdown in manufacturing of several OECD economies after 1973. Glyn (1997), in turn, reports that though profitability recovered in most OECD countries in the 1980s, the response of manufacturing investment was very patchy. Indeed, Hein & Krämer (1997) reports that the decline in the wage share in France, Germany, U.K. and U.S. in the 1980s was not necessarily accompanied by an increase in the rate of capital accumulation. Still referring only to the case of the U.S., empirical estimates by Gordon (1995) for the 1955-1988 period support an inverted-U relationship between the profit rate and capacity utilization, a result that will likewise follow from the model developed in this paper.

The economy is inhabited by two classes, capitalists and workers. Following the tradition of Marx, Kalecki (1971), Kaldor (1956), Robinson (1962), and Pasinetti (1962), we assume that these groups behave differently regarding saving. Workers provide labor and earn only wage income, all spent in consumption. This assumption that workers as a

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grows because of the indivisibility and durability of the plant and equipment. Finally, there is the issue of entry deterrence: if prices are sufficiently high, entry of new competitors becomes feasible even when capital requirements are great; hence, the holding of excess capacity allows oligopolistic firms to confront new entrants by suddenly raising supply and driving prices down.

class do no saving does not, of course, rule out the possibility that individual workers might save. What this view amounts to is the assumption that for workers as a class the saving of some of them is matched by the dissaving of others. Besides, they are always in excess supply, the actual number of potential workers growing at the rate n, which is exogenously given. Capitalists receive profit income, which is the entire surplus over wages, and save all of it. Hence, the division of income is given by:

$$X = (W / P)L + rK \tag{3}$$

where W is the money wage, P is the price level, and r is the profit rate, which is defined as the flow of money profits divided by the value of capital stock at output price. From (1) and (2), labor share is given by:

$$\sigma = Va \tag{4}$$

where $V = (W / P)$ stands for the real wage. The profit rate can then be expressed as:

$$r = (1-\sigma)u = \pi u \tag{5}$$

where π is the share of profit in income and $u = X / K$ is the rate of capacity utilization. Since we assume that the ratio of capacity output to the capital stock is constant, we can therefore identify capacity utilization with the output-capital ratio.

The price level is given at a point in time, but over time it will rise whenever the desired markup of firms exceeds their actual markup. Now, to a higher desired markup by firms a lower implied wage share, σ_f, will correspond. Formally, we have:

$$\hat{P} = \tau[\sigma - \sigma_f] \tag{6}$$

where $\hat{P}$ is the proportionate rate of change in price, $(dP / dt)(1 / P)$, and $\tau > 0$ is the speed of adjustment. Hence, inflation dynamics is determined within a conflicting claims framework, inflation resulting whenever income claims of classes exceed available income. It is worth stressing that this framework does not limit the inflation-generating ability of conflicting claims to situations of full employment of labor and/or capital, the focus being instead on the compatibility of current claims with current income. Whether or not the economy is operating at full employment of resources, inflation occurs whenever workers and capitalists press claims in excess of available income. Therefore, the conflict inflation approach recognizes that income claims do not automatically show up as effective demand. In the model developed in this paper, desired shares in income by capitalists and workers are both made endogenous following Dutt (1992), from which we have actually drawn a lot of inspiration.[2]

The price is determined à la Kalecki (1971), being set by firms as a markup over prime costs:

$$P = (1+z)Wa \tag{7}$$

where z is the markup. In a closed economy all purchases and sales of intermediate raw materials cancel out, and we can think of the representative firm as vertically integrated

2 Unlike the model developed in Dutt (1992), however, as shown later on, this model does not rely on full capacity utilization being reached for profit-led accumulation and growth as well as multiple equilibria to obtain within the distributive domain.

using directly and indirectly a constant amount of labor per unit of final output. It follows that given labor productivity, $(1/a)$, the markup is inversely related to the wage share, so that the gap between the desired and the actual markup can then be measured by the gap between the actual and the firms' desired wage share. The desired markup by firms is taken to depend on their perception of the state of the goods market, it being assumed here that a higher level of capacity utilization, which reflects more buoyant demand conditions, will induce firms to desire a higher profitability. Formally, we have:

$$\sigma_f = \varphi - \theta u \qquad (8)$$

where φ and θ are positive parameters.[3]

At a point in time the money wage is given, changing over time in line with the gap between the wage share desired by workers, σ_w, and the actual wage share:

$$\hat{W} = \mu[\sigma_w - \sigma] \qquad (9)$$

where $\hat{W}$ is the proportionate rate of change in money wage, $(dW/dt)(1/W)$, and $\mu > 0$ is the speed of adjustment. The wage share desired by workers is assumed to depend on their bargaining power in the labor market, which rises with the employment rate:

$$\sigma_w = \rho + \lambda e \qquad (10)$$

where ρ and λ are positive parameters and e is the employment rate, L/N, which can be linked to the state of the goods market in the following way:

$$e = auk \qquad (11)$$

where k stands for the ratio of capital stock to labor supply, $k = K/N$, with N being the supply of labor.

Since the model is demand-driven, the equality between investment and saving will be brought about by changes in output through changes in capacity utilization. Assuming that capital does not depreciate, g, the rate of capital accumulation, which is the growth rate for this one-good economy, is given by:

$$g = r \qquad (12)$$

which follows from the assumptions that workers do not save and capitalists save all of their income.

[3] Rowthorn (1977) suggests that higher levels of capacity utilization allows firms to raise prices with less fear of being undercut by their competitors, who would gain little by undercutting due to higher capacity constraints. Gordon, Weisskopf & Bowles (1984) argues that marked-up prices are inversely related to the perceived elasticity of demand, which is a negative function of the degree of industry concentration and of the fraction of the firm's potential competitors who are perceived to be quantity-constrained and thus not engaged in or responsive to price competition. The conclusion is that in the downturn the markup will fall because the general fall in capacity utilization gives rise to a smaller share of the firm's potential competitors being perceived to be operating under capacity constraints, and hence to an increase in the perceived elasticity of demand facing the firm.

3. <u>The behavior of the model in the short run</u>

The short run is defined as a time frame in which the capital stock, K, the labor supply, N, the labor-output ratio, a, the price level, P, and the money wage, W, can all be taken as given. The existence of excess capacity implies that output will adjust to remove any excess demand or supply, so that in short-run equilibrium, $g = g^d$. Substituting from (2), (5), and (12), we can solve for the equilibrium value of u, given σ and parameters:

$$u^* = \frac{\alpha}{1-\sigma} + \beta\sigma \tag{13}$$

It is assumed a short-run adjustment mechanism stating that capacity utilization will change in proportion to the excess demand in the goods market, which implies that u^* is stable. Indeed, such stability is insured by investment being less responsive than aggregate saving to changes in capacity utilization – recall that desired investment does not depend on capacity utilization, while the denominator of the expression in (13) is positive throughout its relevant domain, $0 < \sigma < 1$. Since α is positive, the numerator of the expression for u^* will be positive as well.

A natural question to raise regards the impact of changes in the share of wages on capacity utilization:

$$du^* / d\sigma = u_\sigma^* = \frac{\alpha}{(1-\sigma)^2} + \beta > 0 \tag{14}$$

Hence, capacity utilization is wage-led despite desired investment being non-linear in distribution.[4] For lower levels of wage share, a rise in the wage share will raise effective demand by raising both investment and consumption – recall that capitalists save a higher proportion of their income than workers do. For higher levels of wage share, in turn, a rise in the wage share will raise consumption demand by more than the accompanying fall in investment, so that capacity utilization will rise as well.[5]

Now, our assumptions regarding saving behavior by capitalists and workers imply that the short-run equilibrium rates of profit and growth happen to be the same, as shown in eq. (12). Given capacity utilization, a rise in the wage share will exert a downward pressure on the rates of profit and growth. However, capacity utilization being wage-led makes for the possibility that a higher wage share generates a rise in capacity utilization that more than compensates the accompanying fall in the profit share. Hence, the non-linear desired investment function specified here allows us to derive precise distributional intervals within

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4 In the newer post keynesian model developed independently by Rowthorn (1981) and Dutt (1984, 1990), in turn, capacity utilization is wage-led because desired investment depends positively on the rates of profit and capacity utilization. However, it can be easily checked that capacity utilization would be wage-led even in case desired investment depended solely on the rate profit – but then growth would cease to depend on distribution.

5 It can be easily checked that capacity utilization being necessarily wage-led does not depend on the assumption that capitalists save all of their income. In case capitalists saved a fraction s of their income, eq. (12) would simply become $g = sr$, while eq. (13) would be changed accordingly. Indeed, a sufficient condition for a rise in the wage share to lead to a rise in effective demand in the model of this paper is that capitalists save a higher proportion of their income than workers do.

which the relation between functional distribution and growth is either positive or negative. All of this ambiguity is captured by the formal expressions for the short-run equilibrium growth, g^*, and for the corresponding partial derivative, g^*_σ, which, using eqs. (5), (12) and (13), are given by:

$$g^* = \alpha + \beta(1-\sigma)\sigma \quad (15)$$

$$dg / d\sigma = g^* = \beta(1-2\sigma) \quad (16)$$

Hence, eq. (15) reveals not only that capital accumulation is the engine of growth, but that their rates are actually the same. In turn, eq. (16) implies that there is a level of wage share, $\sigma^* = 1/2$, below (above) which a rise in the wage share will lead to a rise (fall) in the rates of accumulation and growth.[6] For lower levels of wage share ($\sigma < \sigma^*$) wage-led accumulation and growth will prevail, whereas for higher levels of wage share ($\sigma > \sigma^*$) it is profit-led accumulation and growth that will obtain.

The intuition behind the result that the rates of accumulation and growth have the same short-run equilibrium value is straightforward: a rise in the wage share, for instance, by raising consumption demand, will raise capacity utilization in the same extent that, by implying a fall in the profit share, it will put a downward pressure on the rate of profit. The ultimate impact of a rise in the wage share on the rates of profit and growth, therefore, will be the same as the impact of this rise on desired investment. For lower levels of wage share ($\sigma < \sigma^*$), the ultimate extent in which capacity utilization will rise by more than the profit share will fall in response to a rise in the wage share, which is actually the growth rate, is therefore the same as the extent in which desired investment will increase. For higher levels of wage share ($\sigma^* > \sigma$), in turn, even though a rise in the wage share will end up raising capacity utilization by raising consumption by more than it lowers investment, it will do so in an extent that is lower than the one in which the profit share will fall; and how lower it will do so is as large as that fall in investment.

Therefore, the meaningful subset of the distributive domain can be divided into two regions. In the first one, comprised by lower levels of wage share ($\sigma < \sigma^*$), and to which we refer as LWS region in what follows, capacity utilization, capital accumulation and growth are all directly related to the wage share. In the second one, which comprises higher levels of wage share ($\sigma^* > \sigma$), and to which we refer as HWS region in what follows, even though capacity utilization is still directly related to the wage share, changes in the profit share dominate changes in capacity utilization because capital accumulation is profit-led. In the HWS region, therefore, growth is inversely related to the wage share.[7]

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6 One can easily check that the investment function does not have to be so rigidly parameterized, which makes for its maximum value to occur at $\sigma^* = 1/2$, to generate such a change in the growth regime. Indeed, eq. (2) could had been specified as $g^d = \alpha + \beta(c\sigma - \sigma^2)$, with $0 < c < 2$, which would make for $\sigma^* = c/2$, and the same qualitative results derived so far would follow. Admitedly, the choice of the specification in eq. (2) was a matter of convenience for some other calculations.

7 Hence, the double-sided relation between distribution and growth is explored in this paper from a demand-based perspective that differs from the mainsteam one. While there is no special connection

4. The behavior of the model in the long run

For the long run, we assume that the short-run equilibrium values of the variables are always attained, with the economy moving over time due to changes in the stock of capital, K, the supply of labor, N, the price level, P, and the money wage, W. One way of following the behavior of the system over time is by examining the dynamics of the short-run state variables σ, the wage share, and k, the ratio of capital stock to labor supply, and this is the analytical alternative pursued here. From the definition of these variables, and denoting time-rates of change by overhats, the corresponding state transition functions are:

$$\hat{\sigma} = \hat{W} - \hat{P} + \hat{a} \qquad (17)$$

$$\hat{k} = \hat{K} - \hat{N} \qquad (18)$$

Substitution from (10) and (11) into (9), and from the resulting expression into (17), along with substitution from (6) into (8), and from the resulting expression into (17), will yield:

$$\hat{\sigma} = \mu(\rho + \lambda auk - \sigma) - \tau(\sigma - \varphi + \theta u) \qquad (19)$$

where u is given by eq. (13). Since we abstract from technological change, the labor-output ratio remains unchanged throughout, which implies that the dynamics of the wage share is ultimately governed by the dynamics of the real wage.[8]

Substituting from (5) into (12) and the resulting expression into (18), we obtain:

$$\hat{k} = (1 - \sigma)u - n \qquad (20)$$

where u is given by eq. (13), while n is an exogenous growth rate of labor supply.

Equations (19) and (20), after using eq. (13), constitute a planar autonomous two-dimensional system of non-linear differential equations in which the rates of change of σ and k depend on the levels of σ and k, and on parameters. The matrix M of partial derivatives for this dynamic system is given by:

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between the growth model developed by Solow (1956) and the determination of distribution, a more recent neoclassical literature has provided an array of explanations for a relation between income equality and growth – for instance, personal inequality would lower growth by raising redistributive government expenditure and therefore distortionary taxation, by increasing sociopolitical instability, or by reducing private investment in human capital. See Aghion, Caroli & Garcia-Peñalosa (1999) and Barro (2000) for a survey of this literature, and Forbes (2000) for an empirical study that found a positive relation between inequality and growth instead. Chen (2003), in turn, obtained empirical evidence for an inverted-U relation between personal income inequality and growth.

[8] Macrodynamic models of capital accumulation, functional distribution and growth in which labor-saving technological innovation plays a fundamental role can be found in Lima (2000, 2004). In the former, technological innovation depends non-linearly on market concentration, along the lines of a possibility raised in the neo-schumpeterian literature on the relation between industrial dynamics and technical change. In Lima (2004), in turn, technological innovation is made to depend non-linearly on distribution itself, on the presumption that the latter determines both the incentives to innovate and the availability of funding to carry it out. In both cases, the non-linear nature of the model gives rise to multiple equilibria and endogenous, self-sustaining fluctuations.

$$M_{11} = \partial\hat{\sigma} / \partial\sigma = \mu(\lambda aku_{\sigma}^{*} - 1) - \tau(1 + \theta u_{\sigma}^{*}) \quad (21)$$

$$M_{12} = \partial\hat{\sigma} / \partial k = \mu\lambda au^{*} > 0 \quad (22)$$

$$M_{21} = \partial\hat{k} / \partial\sigma = g_{\sigma}^{*} \quad (23)$$

$$M_{22} = \partial\hat{k} / \partial k = 0 \quad (24)$$

Eq. (22) shows that an increase in the capital-labor supply ratio, by raising the employment rate, will raise the wage share desired by workers, σ_w, which will raise the rate of growth of money wage. Eq. (24) shows that since an increase in k does not affect either σ or u, there is no effect on the rate of accumulation, and hence no effect on the rate of growth of k.

Let us now turn to those partial derivatives whose signs are ambiguous. Eq. (21) shows that the impact of a change in the wage share on its own rate of change is mediated by the accompanying impact on capacity utilization. The reason is that both the wage share desired by workers and the wage share implied by firms' desired markup depend on capacity utilization. While σ_f depends directly on the state of the goods market, σ_w depends directly on the state of the labor market. Now, given the fixed-coefficient nature of the assumed production technology, an increase in capacity utilization in the short run will necessarily be accompanied by an increase in employment. Hence, the sign of this partial derivative will depend on the relative bargaining power of workers and capitalists. As for the sign of $\partial\hat{k} / \partial\sigma$, eq. (23) shows that it is governed by the impact of changes in the wage share on the growth rate.

We now have all the elements for a qualitative phase-diagrammatic analysis of the (local) stability properties of this dynamic system. The way we proceed is by analyzing the stability of an equilibrium position in each one of the two regions into which we divided the relevant domain. Now, eq. (16) shows that the growth rate is quadratic in the wage share, and with n being exogenously given, the equation describing the $\hat{k} = 0$ isocline is quadratic in the wage share as well. Hence, there may be up to two real values for the wage share in the (k-σ)-space at which a corresponding vertical $\hat{k} = 0$ isocline would be located. Given this geometry, we analyze separately the stability of the equilibrium solution in each one of those two regions were one of the $\hat{k} = 0$ isoclines to be located there.

In the LWS region ($\sigma < \sigma^{*}$), capacity utilization, capital accumulation and growth are all directly related to the wage share. Hence, a higher wage share will exert an upward pressure on its own rate of change by raising capacity utilization and thus employment, which will raise the wage share desired by workers. However, this same rise in capacity utilization will also raise the markup desired by firms, which will then exert a downward pressure on the rate of change of the wage share by lowering the wage share 'desired' by firms. In turn, with the growth rate being directly related to the wage share within this region, the rate of growth of k will rise with the wage share to make for $M_{21} > 0$. In case $M_{11} < 0$, the resulting steady-state solution will be a saddle-point. Indeed, the resulting steady-state will be a saddle-point anyway, the reason being that $Det(M) < 0$ no matter the sign of M_{11} -- that is, no matter the relative bargaining power of capitalists and workers.

Let us analyze the situation in which $M_{11} < 0$, meaning that the rate of change in prices is more responsive than the rate of change in nominal wages to changes in capacity utilization. The dynamics for this situation is shown in Figure 1. Given that $M_{12} > 0$, the slope of the $\hat{\sigma} = 0$ isocline, given by $-(M_{11} / M_{12})$, is positive. Since $\partial\hat{\sigma} / \partial k$ is positive, $\hat{\sigma}$ undergoes a steady increase as k increases, so that the sign of $\hat{\sigma}$ is negative (positive) to the right (left) of the $\hat{\sigma} = 0$ locus, which explains the direction of the horizontal arrows. The slope of the assumed $\hat{k} = 0$ isocline, in turn, given by $-(M_{21} / M_{22})$, is equal to zero, with σ_1 being the corresponding value of the wage share. Since $\partial\hat{k} / \partial\sigma > 0$, $\hat{k}$ undergoes a steady increase as σ increases, thus implying that the sign of $\hat{k}$ is negative (positive) to the left (right) of the $\hat{k} = 0$ isocline, which then explains the direction of the vertical arrows.

As Figure 1 shows, the economy has a long-run equilibrium with $\hat{\sigma} = \hat{k} = 0$ at E_1, with this long-run equilibrium being a saddle-point. If the economy happens to be below the downward-sloping separatrix, it will (eventually) move over time with falling levels of k and σ, and thus with falling rates of capacity utilization and growth. But if the economy happens to be above the separatrix, it will (eventually) move over time with increasing levels of k and σ, and thus with increasing rates of capacity utilization and growth.

In case $M_{11} > 0$, which means that the rate of change in nominal wages is more responsive than the rate of change in prices to changes in capacity utilization and employment, the $\hat{\sigma} = 0$ isocline will be downward-sloping instead, whereas the separatrix will again be downward-sloping. This equilibrium situation is portrayed in Figure 2.

Hence, the resulting steady-state solution will not be stable in the subset of the relevant domain in which wage-led accumulation and growth ($g_\sigma^* > 0$) obtains. In this subset stability is observed only on the stable branch of the saddle point – the separatrix – and it is not obtainable as a matter of course, this being the reason why a saddle point is classified as an unstable solution. Since the determinant of this two-dimensional system is given by $-M_{21} = -g_\sigma^*$, stability is automatically ruled out in case wage-led growth obtains.

In the HWS region ($\sigma > \sigma^*$), capacity utilization is still directly related to the wage share, so that the sign of $\partial\hat{\sigma} / \partial\sigma$ is ambiguous again. However, changes in the profit share dominate changes in capacity utilization, which implies that accumulation and growth are inversely related to the wage share. This makes for $M_{21} < 0$, with the effect that $Det(M)$ becomes positive. The possibility of a saddle-point is therefore ruled out, and whether the equilibrium solution with $\hat{\sigma} = \hat{k} = 0$ will be stable or unstable depends on whether M_{11} is negative or positive, respectively. Hence, stability (instability) obtains when the rate of change in prices is more (less) responsive than the rate of change in nominal wages to a change in capacity utilization generated by a change in the wage share, which makes for a negative (positive) $Tr(M)$.

Figure 3 pictures a situation in which $M_{11} < 0$, which makes for $Tr(M) < 0$. Given that $M_{12} > 0$, the slope of the $\hat{\sigma} = 0$ isocline, which is given by $-(M_{11} / M_{12})$, is positive. Since $\partial\hat{\sigma} / \partial k$ is positive, $\hat{\sigma}$ undergoes a steady increase as k increases, so that the sign of $\hat{\sigma}$ is negative (positive) to the right (left) of the $\hat{\sigma} = 0$ locus, which explains the direction of the horizontal arrows. The slope of the assumed $\hat{k} = 0$ isocline, in turn, which is given by $-(M_{21} / M_{22})$, is equal to zero. Since $\partial\hat{k} / \partial\sigma < 0$, $\hat{k}$ undergoes a steady decrease as σ increases, thus implying that the sign of $\hat{k}$ is positive (negative) to the left (right) of the

$\hat{k} = 0$ isocline, and that the direction of the vertical arrows is as shown. In this situation, as Figure 3 confirms, the economy will have a long-run equilibrium solution with $\hat{\sigma} = \hat{k} = 0$ at E_1, it being a stable focus.

Figure 4, in turn, pictures a situation in which $M_{11} > 0$, thus implying that $Tr(M)$ is positive. The resulting equilibrium solution will be unstable due to the rate of change of nominal wages being more responsive than the rate of change in prices to changes in capacity utilization. Since $M_{12} > 0$, the slope of the $\hat{\sigma} = 0$ isocline becomes negative. $\partial\hat{\sigma} / \partial k$ being positive, $\hat{\sigma}$ undergoes a steady increase as k increases, so that the sign of $\hat{\sigma}$ is positive (negative) to the right (left) of the $\hat{\sigma} = 0$ locus, which explains the direction of the horizontal arrows. As Figure 4 shows, the economy will have a long-run equilibrium solution with $\hat{\sigma} = \hat{k} = 0$ at E_1, it being an unstable focus.

In the HWS region of the relevant distributive domain, therefore, stability properties depend on the relative strength of the nominal wage change effect with respect to the price change effect. To the extent that it is a region in which wage-led capacity utilization *cum* profit-led capital accumulation and growth prevails, a long-run equilibrium located there will be stable (unstable) in case the price (nominal wage) change effect is stronger than the nominal wage (price) one – that is, in case capitalists' (workers') relative bargaining power is stronger. While in the LWS region the prevalence of wage-led growth prevents stability altogether, in the subset defined by higher levels of wage share, the HWS region, in which profit-led growth obtains, the stability requirement is that the nominal wage change effect is relatively weaker than the price change one.

Now, to see how sensitive are these stability requirements to the assumption that firms' desired markup is directly related to capacity utilization, let us briefly examine the general implications of assuming an inverse relationship instead.[9] In the LWS region, where wage-led accumulation and growth obtains, saddle-point equilibrium would still be the only possibility, the reason being that assuming that desired markup is inversely related to capacity utilization will not remove the source of instability of an equilibrium solution located in the LWS region, namely, the prevalence of wage-led accumulation and growth leading to $Det(M) < 0$.

In the HWS region, eq. (21) shows that a rise in capacity utilization due to a rise in the wage share may lead to a fall in inflation – and thereby put an upward pressure on the rate of change of the wage share – in case desired markup is sufficiently countercyclical. For stability in the HWS region, which requires a negative $\partial\hat{\sigma} / \partial\sigma$, such a fall in the rate

9 Some arguments suggesting that markups may be countercyclical are the following. Minsky (1975) argues that the fall in sales in a downturn forces firms to raise markups to meet outstanding financial obligations. Kalecki (1954) argues that since the markup depends partially on the ratio of overheads to prime costs, the rise in this ratio in downturns causes markups to rise. On the neoclassical side, Rotemberg & Woodford (1992) claim that it is more difficult for oligopolistic firms to sustain collusive prices during booms, when the incentive for any firm to cut its price rises because it becomes more worthwhile to capture current sales than to maintain collusion in the future. Chevalier & Scharfstein (1996), in turn, argue that as capital-market imperfections constrain the ability of firms to raise external finance, liquidity-constrained firms will raise (lower) markups during recessions (booms). A survey of recent developments in the neoclassical literature on the cyclical behavior of markups can be found in Rotemberg & Woodford (1999).

of inflation would therefore have to be more than compensated by a fall the rate change of the nominal.[10]

5. Summary

This paper developed a dynamic model of capital accumulation, distribution and growth in which desired investment is non-linear in the distributive shares, a specification that conforms with some of the empirical evidence for the rise and fall of the Golden Age in most advanced economies. Despite capacity utilization is always wage-led, whether the economy experiences wage-led growth or profit-led growth depends on the prevailing distribution, with a similar dependence applying, alongside with the relative bargaining power of capitalists and workers and the cyclical behavior of markups, to the dynamic stability properties of the system. While wage-led growth obtains for lower levels of wage share, profit-led growth obtains for lower levels of profit share. Regarding dynamics, while a long-run equilibrium with wage-led growth is unstable no matter the cyclical behavior of markups, such an equilibrium with profit-led growth is stable under either cyclical behavior of markups provided workers' bargaining power is weak enough.

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[10] The non-linearity embodied in the investment function makes for the possibility of multiple equilibria within the relevant domain. As shown above, with an exogenously given growth rate of labor supply there may be up to two real values for the wage share at which a corresponding vertical $\hat{k} = 0$ isocline would be located in the $(k - \sigma)$-space, so that it is possible that a configuration with up to two equilibria obtains within the relevant domain. However interesting, we do not analyze this possibility in the compass of this paper, though.

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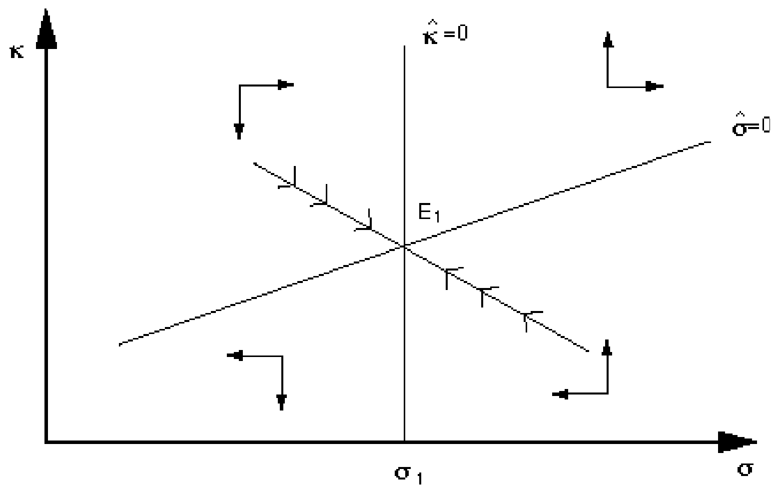

Figure 1. Long-run dynamics: saddle-point unstable equilibrium in the LWS region

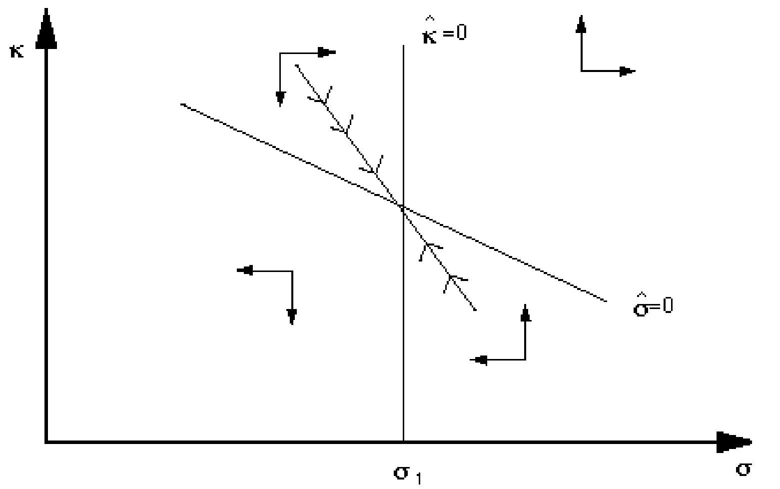

Figure 2. Long-run dynamics: saddle-point unstable equilibrium in the LWS region

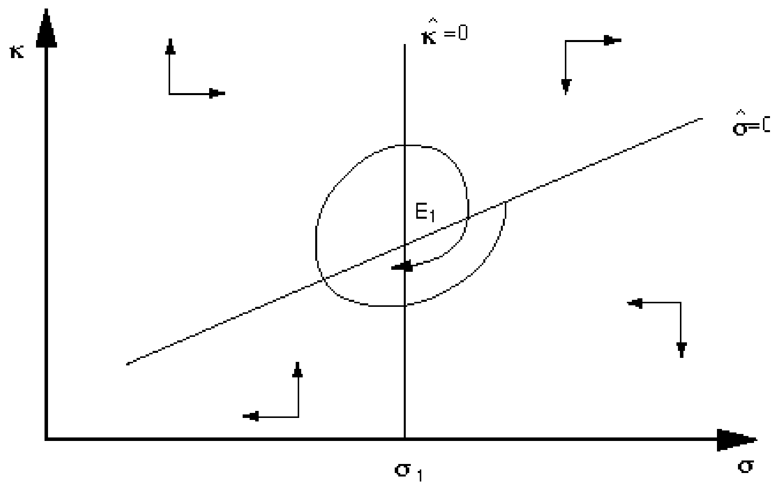

Figure 3. Long-run dynamics: stable equilibrium in the HWS region

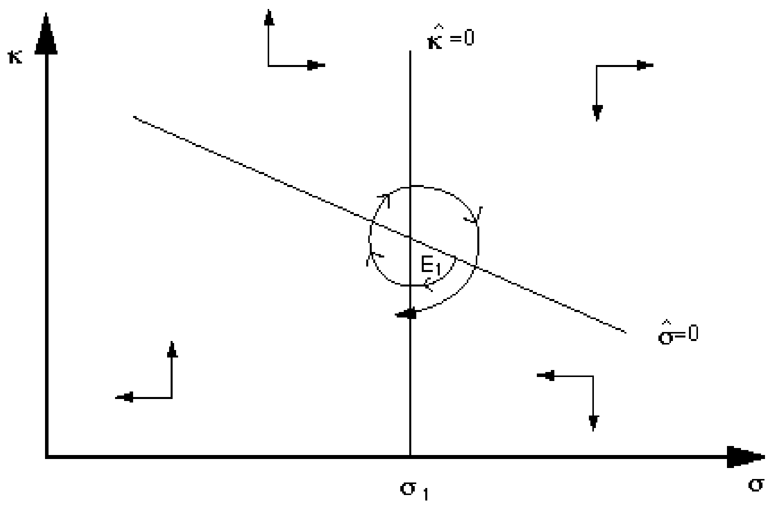

Figure 4. Long-run dynamics: unstable equilibrium in the HWS region