Inflation Target Zones as a Commitment Mechanism

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Abstract: In a simple new keyenesian model of monetary policy under discretion constraining the Central Bank to put inflation within a pre-specified Inflation Target Zone can eliminate the inflation bias and, at least for certain parameter ranges, significantly reduce the stabilization bias. Also, it is possible to investigate what is the optimal Inflation Target Zone for different economies. These seem to depend of the structural parameters in a non-linear and often non-monotonic way.

Keywords: Monetary Policy, Commitment Mechanism, Inflation Target Zone, Honeymoon Effect

Resumo: Em um modelo novo keynesiano simples de política monetária no qual o Banco Central age de forma discricionária, restringir o Banco Central a manter a taxa de inflação dentro de um Banda de Inflação pré-especificada é uma estratégia efetiva para eliminar o viés inflacionário e, ao menos para determinados parâmetros, reduzir significativamente o viés de estabilização. Além disso, é possível investigar qual a Banda de Inflação ótima para diferentes economias. A resposta para essa última pergunta parece depender dos parâmetros estruturais de forma não linear e, freqüentemente não monotônica.

Palavras Chaves: Política Monetária, Mecanismo de Comprometimento, Banda de Inflação, Efeito Lua de Mel.

JEL: E42, E52, E61

Área ANPEC: 3. Macroeconomia, Economia Monetária e Finanças

1. INTRODUCTION

Much of recent the analysis of monetary policy has centered on Central Banks's alleged inability to commit to an optimal, time consistent policy. Depending on the particular specification of the model, this can take the form of an inflation bias like in Kydland and Prescott's [Kydland & Prescott, 1977] and Barro and Gordon's [Barro & Gordon, 1983] analysis, where average inflation is too high, or the form of a stabilization bias as in New Keynesian models of monetary policy such as the one presented by Clarida, Gali and Gertler [Clarida *et al.* , 1999], where inflation and the output gap are too volatile. The reason for these biases is that the private sector makes decisions according to what it expects the Central Bank to do in the future, so that by being able to commit, the Central Bank can better influence the decisions of the private sector and generate better policy outcomes.

It would seem that this problem could be solved by having the Central Bank sign some contract where it would commit to following an optimal policy. However, such a contract would have to specify what the Central Bank would do in each contingency, including states that are observable only to the Central Bank, or if observable to other parties, that are not easily verifiable, in the sense that there could be enough room for legitimate disagreement that the Central Bank could not be credibly held into account for not reacting in the pre-specified way. Also, conditioning the policy of the Central Bank to easily observable variables as in a Taylor Rule [Taylor, 1993] will typically result in losses with respect to the optimal policy, as the Central Bank will not be allowed to use all relevant information when deciding its policy. Finally, even if all the relevant information was easily observed and verified by all parties, the contracts necessary to fully implement a first best policy would have to be very complicated, and their implementation could be effectively infeasible. Not surprisingly, in practice one does not observe such complicated contracts no more than one sees Central Banks following rigid policy rules.

This article purposes to analyze the effects of a widely used simple mechanism to manage the trade-offs, which is the use of an Inflation Target Zone[1]. The assumption is that the Central Bank is allowed to, and in fact will follow a discretionary policy so long as inflation remains within a certain pre-specified range. For simplicity, it is also assumed that the Central Bank can fully control inflation, although, as discussed in the conclusion, the model could be extended to allow for imperfect control. The numerical computations reported in this article indicate that the use of an Inflation Target Zone allows for elimination of the inflation bias and, in some cases, substantial correction of the stability bias to a point that will be made explicit shortly. This is because the existence of an Inflation Target Zone will in equilibrium affect the policy inside the band. The existence of a credible target zone dampens the effect of current shocks on expectations, because the private agents know that if further shocks occur, the Central Bank will only react to them so long as the band is respected. This effect is akin to the "Honeymoon Effect" identified by Krugman [Krugman, 1991] in the context of exchange rate target zones. Moreover, more favorable inflation expectations allow the Central Bank to react less to shocks within the band, where it follows a discretionary policy, thus further decreasing the volatility of inflation. However, one could easily exaggerate. Having the Central

[1] As Erceg [Erceg, 2002] points out, the definition of a target range as opposed to a point target seems to be adopted by most inflation targeting countries.

Bank commit to a pre-specified inflation rate without any room for variation is hardly optimal either. The reason is that the Inflation Target Zone is determined before the whole relevant information is revealed. In the presence of supply shocks it is optimal for a Central Bank which is not a strict inflation targeter to adjust inflation somewhat in the direction of the shock so that output does not vary too much. This suggests that there is in general a non-trivial optimal width for the inflation target band.

A large part of the literature has concerned itself with the design of mechanisms to make sure the Central Bank behaves in a less discretionary way, while allowing it to have enough independence to be able to make decisions that make the best use of information (including non-public information) it may have, thus reaching some kind of second or third best equilibria. A by now classic proposal [Rogoff, 1985] to reduce the inflation bias problem is that the Central Banker should give less weight to output gap variation than society at large, i.e., it should be relatively "weight" conservative. Svensson [Svensson, 1997] proposes that the Inflation Targeting framework reduces inflation by forcing the Central Bankers to persue a lower average rate of inflation than what would be socially optimal, thus compensating for the bias, i.e., he proposes that the Central Bank should be "Inflation Target" conservative. The numerical calculations done in this article show that the use of a Target Zone can effectively implement Svensson's suggestion by setting the center of the band low enough.

Also, as emphasized by Clarida et al. (1999), Rogoff's proposal is also able to at least in part correct the stability bias by implementing an optimal linear policy rule. There is no way such corrections can implement the first best policy arrangement, for this is history dependent and in a basic model such as the one that will be explored here none of these mechanisms can implement such a policy. The same is true for the Target Zone arrangement. What the numerical calculations show is that while the Target Zone mechanism falls short of providing the same gains as a assigning a suitable "weight" conservative Central Banker, at least for some parameter values it is able to generate very close outcomes

The adoption of an inflation target band is a common institutional arrangement which has the advantage of being implementable as one can easily verify whether inflation was or was not inside the band. One could think of the band as a contract in which the punishment for the Central Bank only occurs if inflation exceeds certain limits, and then it is quite harsh.

There are a number of papers that try to assess the effects of adopting such a contract. Walsh [Walsh, 2002] show that one can use a contract that will fire the Central Banker if the nominal GDP growth (or some modified version of it) exceeds a certain value to be specified. This contract eliminates the inflation bias, thus implementing what Walsh describes as an optimal policy. However, in Walsh's model there is no persistence in inflation shocks and inflation is only a function of past expectations of current inflation as in a "natural rate" type model, but not of current expectations of future inflation as in new keynesian models. Therefore, it does not allow for a discussion of the stability bias in monetary policy.

Another article which addresses the benefits of a target zone is Tetlow [Tetlow, 2000]. Tetlow departs from a somewhat different model of the behavior of the monetary authority but gets similar results. In his model, the Central Bank is assumed to follow a Taylor Rule type policy, but with a randomly varying inflation target, which he models as following a martingale. Tetlow shows that if one can reasonably assume that this inflation target is bounded above and below, there is also a positive effect in terms of inflation and output gap variability associated with some sort of honeymoon effect, akin to the one observed in Krugman's exchange rate target zone model.

The existence of such an effect was also proposed by Gerlach [Gerlach, 1994] and Amano, Black and Kasumovich. [Amano *et al.* , 1997]. However, in both cases the authors retain the flex-price approach of the Krugman model and keep policy exogenous. One of the results presented in this article is that by explicitly modelling the policy the honeymoon effect can yield an indirect effect on inflation and output gap variability which is probably more important then the direct impact of non-linearities implied by the honeymoon effect.

Finally, it is worth mentioning two articles that approach the issue from a different perspective. One is Erceg [Erceg, 2002] which regards the target zone as a confidence interval type statement of the variance that the Central Bank wishes inflation to have. In this interpretation the target zone does not have a "hard edge" in the sense that the Central Bank can allow inflation to obtain values out of the zone, but should be respected say, 95% of the time. However, as Erceg recognizes, target zones seem to be too narrow in that inflation only falls within their bounds less than 95% of the time. By allowing the target band to be more than a simple statement of intentions, the model presented in this paper presents a rationale for why the optimal zone may be "too narrow " by Erceg's criterion.

The other article is by Orphanides and Wieland [Orphanides & Wieland, 2000]. Their problem is to find what would justify a policy in which the Central Bank only pays attention to inflation when it is out of the target zone, otherwise focusing the policy on output gap stabilization. They propose a quadratic loss function which is nearly flat within the target band and the possibility of non-linearities in the Phillips Curve so that the inflation-output trade-off only becomes significant for high levels of inflation. While certainly interesting these investigations present a significant departure from the more standard framework and will not be persued here.

The article proceeds as following. In the next section, I will lay out the model and explain how it fits in the Inflation Targeting Framework, as exposed by Svensson [Svensson, 1997]. Section 3 shows how the existence of the band affects the discretionary, intra-band policy. Section 4 discusses how the location of target zone approach can be used to change the average inflation, thus dealing with the inflation bias problem. In section 5, the effect of different band-widths on the variances of inflation and the output gap is considered. Section 6 provides a discussion of some issues involved in the finding of the optimal Target Zone as a function of the model parameters. The last section concludes by summarizing policy implications, caveats in the analyisis and directions for future development of the model. The appendix discusses shows that the policy function of the Central Bank derived from the model can be recovered from a contraction.

2. THE MODEL

Svensson [Svensson, 1997] describes an Inflation Targeting Framework as a policy regime in which the Central Bank is assigned an Inflation Target, is given freedom to act on this target and is subsequently evaluated on the fulfillment of the target. In that same paper, Svensson shows that if one can set the target such that the Central Bank will persue an average long term inflation lower than the socially optimal, then one can let the Central Bank opperate under discretion so that the socially optimal average inflation will be attained. The Inflation Target Framework is thus interpreted as an institutional framework that makes Central Banks be more "inflation target" conservative then society at large.

The Inflation Targeting Framework is interpreted in this article in a related, but slightly different way. Here, the Central Banks not assigned an inflation target, but a target range, and is given freedom to act *so long as inflation remaisn within the band.* The kind of long-term assessment proposed by Svensson is substituted by a yearly assessment, that however is only apparent under certain extreme circunstances. The Central Bank is still unable to commit to an optimal policy within the band, but one can make it behave as if it had a preference for lower average inflation by chosing a lower Inflation Target Zone. However, as will become clear in part 4, the center of that Target Zone need not coincide with the long run level of inflation persued by the Central Bank. In that sense, the distinction between the two approaches is more than a change in focus, but has implications for how one should interpret the numerical targets adopted by different countries.

The underlying model has a Central Bank with a quadratic policy function and a forward looking Phillips curve. More specifically, the intertemporal Loss Function of the Central Bank is given by

$$L = E_t \left\{ \sum_{s=t}^{\infty} (1-\delta)\, \delta^{s-t} \frac{1}{2} \left[(\pi_t - \bar{\pi})^2 + \alpha (x_t - \bar{x})^2 \right] \right\} \tag{1}$$

Where as usual π_t and x_t stand for inflation and output gap respectively, $\bar{\pi}$ and $\bar{x}$ for the average levels of inflation and output gap which the Central Bank would like to attain in the long term, and E_t for the expectation operator conditonal on information available at time t. The Central Bank minimizes this loss function subject to a a simple, forward looking Phillips Curve[2]:

$$\pi_t = E_t [\pi_{t+1}] + \lambda x_t + u_t \tag{2}$$

Where $E_t [.]$ is the expectation operator conditional on information available in time t and u_t is a supply shock, which follows an AR(1) process:

$$u_t = \rho u_{t-1} + \eta_t \tag{3}$$

where η_t is an *i.i.d.* $N\left(0, \sigma^2\right)$

The further constraint added in this model is that the Central Bank has to keep inflation within a certain band, i.e.:

$$\pi_t \in [\pi^* - B; \pi^* + B] \tag{4}$$

Where π^* denotes the center or the location of the band, and B its width. Furthermore, for simplicity, it is assumed that the Central Bank can pick inflation and output at each period subject to the Phillips Curve above without any error

[2] This Phillips Curve is similar to the one derived from a Calvo style sticky price model [Calvo, 1983]. The difference is that the expected future inflation is not multiplied by the discount rate δ. This simplification is not of great consequence, as in any case δ would be calibrated to be very close to 1. A substantive consequence of this calibration is that the Phillips Curve becomes vertical in steady state, so that there is no long-run trade-off between inflation and output gap levels.

or lags. Note that in this model the decisions made by the Central Bank in each period do not affect its loss and hence its action in future periods. This means that if the Central Bank is unable to commit, it will take inflation expectations as given when chosing inflation and output. The dynamic minimization problem reduces to a sequence of static minimizations of the period loss functions[3]. If the Central Bank is allowed to chose output gap and inflation freely we have in such a framework the usual result that the desired inflation and ouptut gap will satisfy:

$$\pi_t^d - \bar{\pi} = -\frac{\alpha}{\lambda}\left(x_t^d - \bar{x}\right) \tag{5}$$

Where π_t^d and x_t^d stand for inflation and output desired by the Central Bank in the absence of the target band. However the Central Bank is constrained by the target band, so that in effect:

$$\begin{aligned} \pi_t &= \pi_t^d \text{ if } \pi_t^d \in [\pi^* - B, \pi^* + B] & (6)\\ \pi_t &= \pi^* - B \text{ if } \pi_t^d \leq \pi^* - B & (7)\\ \pi_t &= \pi^* + B \text{ if } \pi_t^d \geq \pi^* + B & (8) \end{aligned}$$

Substituting 5 into 2 yields

$$\pi_t^d - \bar{\pi} = \frac{\alpha}{\alpha + \lambda^2}\left[E_t\left[\pi_{t+1} - \bar{\pi}\right] + \lambda\bar{x} + u_t\right] \tag{9}$$

In this simple model, the state of the economy is completely summarized by u_t, so that we should be able to find π_t and π_t^d as functions of u_t.

One can find a stationary equilibrium $\pi_t(u_t)$ using the following algorithm:

1) Start with a $\pi^0(u_t)$ function which satisfies 4
2) Calculate $\pi^{d,1}(u_t)$ by plugging $\pi^0(u_t)$ in the right hand side of 9
3) Use 6, 7 and 8 to find $\pi^1(u_t)$
4) Iterate until convergence to a fixed point.

As shown in Appendix 1, this method is a contraction, so that it necessarily yields a unique reaction function to the Central Bank.

With $\pi(u_t)$ at hand it is straightforward to find $E[\pi(u_{t+1})|u_t]$ with the use of 3 and then $x(u_t)$ by inverting the Phillips Curve 2. With these functions at hand one can proceed to find the moments of inflation and output-gap through numerical integration.

When discussing optimal inflation target bands it will be important to differentiate the loss function adopted the Central Bank from the one appropiate for the society at large (or at least the one adopted by the designer of the target band, whoever that is). While it may be interesting to look at the case where these two coincide, these two are logically distinct. From the point of view of the designer of the target band, the weight that the Central Bank gives to the output gap and

[3] More precisely, the problem allows one to look for a Markov equilibrium of the game played between the Central Bank and its future incarnations, where the actions taken by the Central Bank are solely a function of the current state of the economy summarized by the shock u_t. This equilibrium is what is typically called "discretionary". This need not be the only solution. One could very well look for strategies that take past values of u_t as part of the state. This modification would allow for reputational effects analogous to the ones discussed by Barro and Gordon (1985) or to the implementation of a first best policy as in Woodford 's (2003) "time-less" commitment.

the target inflation and output gap levels are technology parameters, i.e., they are something that the designer has to take as given when deciding what policy parameter to chose. Moreover, allowing for the different loss functions allows one to evaluate whether in this kind of model there are gains to the Central Bank being more conservative than society at large, as proposed by Rogoff [Rogoff, 1985].

The loss function of the policy designer shall be given by[4]:

$$\begin{aligned}
\mathcal{L} &= E\left[\sum_{s=0}^{\infty}(1-\delta)\,\delta^{s}\left[(\pi_s-\hat{\pi})+\beta\,(x_s-\hat{x})^2\right]\right] \\
&= E\left[(\pi-\hat{\pi})^2\right]+\beta E\left[x^2\right]+\beta\left[(E\,(x)-\hat{x})^2\right] \\
&= Var\,(\pi)+(\hat{\pi}-E\,[\pi])^2+\beta Var\,(x)+\beta E\,[\hat{x}]^2
\end{aligned}$$

Where the stationarity of π_s and x_s and the fact that $E\,[x]=0$ are being used. $\hat{x}$ is the steady state output gap desired by the policy designer, which is also allowed to differ from the output gap desired by the Central Bank given by $\bar{x}$. Note, however, that $\hat{x}$ only enters as an additive constant in the Loss Function, so that it can also be ignored in what follows. $\hat{\pi}$ is the inflation desired by the Policy Designer, which is also allowed to be different from the one desired by the Central Bank, given by $\bar{\pi}$. As remarked by Svensson [Svensson, 1997], letting $\bar{\pi}<\hat{\pi}$ is another form of modelling Central Bank conservativeness. For the sake of simplicity, let us set $\hat{\pi}=0$ for the remaining of this paper. Note that all that matters for the policy designer is the distance from steady state inflation to its prefered long-term level $\hat{\pi}$, so that there is no loss in generality in doing this normalization.

Throughout the article, the resulting policy rule shall be compared with two benchmark cases. One is the optimal policy under discretion in the absence of the target band[5]. This is a natural benchmark, as it is with respect to this policy that the policy implied by the target zone should improve. The second benchmark will be given by the optimal linear policy.

These are given respectively by[6]:

$$\pi^{disc}\,(u_t)=\bar{\pi}+\alpha\bar{x}+\omega u_t=\bar{\pi}+\alpha\bar{x}+\frac{\alpha}{1+\alpha\,(1-\rho)}u_t$$

and

$$\pi^{opt}\,(u_t)=\omega^{*}u_t=\frac{\alpha^{c}}{1+\alpha^{c}\,(1-\rho)}u_t$$

where

$$\alpha^{c}=(1-\rho)\,\beta$$

[4]Note that as stated, the loss function that the policy designer minimizes does not incorporate any information about the state of the economy that the policy designer may have at the moment that the target band is set. In some sense, the optimization here takes place from a "timeless perspective" as advocated by Woodford [Woodford, 2003].

[5]For conciseness, this will be refered to hereafter as simply policy "under discretion" or "discretionary policy"

[6]For a derivation, see Clarida et al. [Clarida *et al.* , 1999]. The use of the optimal linear policy as a benchmark as opposed to the global optimal policy is justified by the fact that the global optimal policy is history dependent and there is no hope that the proposed mechanism can make the Central Bank adopt such a policy. Also, as seen below, the optimal linear policy does seem to dominate any policy implementable with the target band.

Note that average inflation in the discretionary case is given by:

$$E\left[\pi^{disc}(u_t)\right] = \bar{\pi} + \alpha\bar{x}$$

Also, note that the optimal linear policy corresponds to assigning a central bank with output-gap weight given by α^c and long-term inflation target given by $\bar{\pi} = -\alpha^c\bar{x}$, and that for $\rho > 0$ and $\bar{x} > 0$ this amounts to putting in place a Central Bank which has a lower output-gap weight α and a lower long-term inflation target $\bar{\pi}$ than the policy designer. These two requirements correspond respectively to Rogoff's and Svensson's suggestions.

The model can be normalized to reduce the number of free parameters without any loss in generality. One normalization is to set $\sigma = \sqrt{1-\rho^2}$. This sets the unconditional standard deviation of the u_t shocks to 1. The model can be renormalized by multiplying the Loss Function by $\frac{\sigma^2}{1-\rho^2}$ and the constraints by $\frac{\sigma}{\sqrt{1-\rho^2}}$. This means that if the normalized model implies an optimal band given by (π^*, B) then the actual optimal band should be $\left(\frac{\sigma}{\sqrt{1-\rho^2}}\pi^*, \frac{\sigma}{\sqrt{1-\rho^2}}B\right)$. Likewise, the resulting variances and squared inflation bias should be all multiplied by $\frac{\sigma^2}{1-\rho^2}$.

Another normalization is to note that if one can rewrite redefine variables so that the wellfare weight is reset to $\tilde{\alpha} = \frac{\alpha}{\lambda^2}$ and the pass-through from output-gap to inflation set at $\tilde{\lambda} = 1$. This allows us to focus on variations on α. Changes in this parameter will indicate both the effects of having a more conservative Central Bank or less costly short run trade-off between inflation and the output gap. The required redefinition is that x_t be the direct effect of the output gap on inflation, normally given by λx_t.

3. THE EFFECT OF THE BAND IN INTRA-BAND POLICY

Krugman [Krugman, 1991] identified what he called a "Honeymoon Effect" in the context of Exchange Rate Target Zones. His insight was that because the exchange rate is a forward looking variable, by commiting to intervene so that the exchange rate does not get out of certain bounds, the government can in effect reduce the variability of the exchange rate even around levels where no intervention takes place. Previous literature cited above has already suggested that the framework may be adapted to discuss Inflation Target Zones. However, as Amano et al. [Amano *et al.* , 1997] acknowledge, the application is not immediate. For one thing, an unadapted version of the standard exchange rate target zone would imply stability for the price level but not for inflation so that niceties about how variations in the price level are aggregated into annual inflation become relevant. By building on a rigid-price model such as the ones typically used in the analysis of monetary policy one can deal directly with inflation without having to worry too much about this kind of issue.[7].

Mathematically the model is in some ways similar to the standard exchange rate target zone models. It relies fundamentally on a forward looking equation, the Phillips Curve, which is analogous to the arbitrage equation derived in this kind of

[7] Although, for sure, it would also be interesting to see what this model would imply for the behavior of the Cental Bank in each quarter with the inflation target given on a yearly basis, as is usually the case.

model. The output gap x_t assumes the role of the money supply, i.e., the control variable of the government, and the supply shock u_t the role of the "fundamental" shock. Technical differences are the discrete time assumption as opposed to a continuos time one and that unlike the "fundamental" shock, the supply shock is assumed to be stationary[8].

On a more conceptual level, an important difference is that the Central Bank does not wait for inflation to reach the edge of the target zone to intervene as in the standard exchange rate target zone models, but will in general "lean against the wind" by decreasing the output gap as inflation increases. Moreover, the intra-band policy is endogenous to the size and position of the band. The "Honeymoon Effect" works here not only because it affects inflation expectations, but fundamentally because by affecting these expectations it also makes the policy adopted by the Central Bank within the target range less aggressive, creating a positive feedback (see equation 9).

In what follows, let's compare the policy with the band with the policy adopted under full discretion. This is done in Graphs 1a to 1d. The solid line represents the policy with the target band, whereas the dashed line represents the policy when there is no such band. The parameter values used to plot the graphs are $\alpha = 3$ and $\bar{x} = \bar{\pi} = 0$. The latter two equalities imply absence of inflation bias in the discretionary policy.. The left hand side graphs (1a and 1c) represent inflation as a function of the supply shock u_t, and the right hand side graphs its first derivative. For the upper graphs (1a and 1b), the shocks were calibrated to be fairly persistent, at $\rho = 0.8$[9].

What immediately stands out in graph 1a is that apart from flattening out the reaction of inflation to the shock in the points where inflation is forced against the edges of the bands, the behavior of inflation within the band is changed. The $\pi(u_t)$ schedule is both translated to the right and rotated downwards. This means that for a given shock the band implies a lower inflation level and for a given change in the shock, the band implies a weaker reaction. One can decompose the effects of the location and the width of the band. It turns out that shifting π^* with a very high B implies translation of the $\pi(u_t)$ without downward rotation and tightening B while keeping $\pi^* = 0$ implies downward rotation without any translation[10].

By inspecting graph 1a one cannot discern any non-linearity such as the one predicted by the Krugman exchange rate target zone model. This only becomes apparent when one looks at the marginal effect graph 1b. As predicted by Krugman's model, the first derivative of the effect of the shock on inflation decreases as the shock brings inflation closer to the edge of the band. The point where the derivative is largest does not coincide with the center of the band. This is an artifact of having the center of the band shifted. With $\pi^* = 0$ the picture would look symmetric.

[8] The use of discrete time and stationary shocks leaves this model closer to the models usually develped in the monetary policy literature. The almost universal adoption of a brownian motion process for the fundamental in exchange rate target zone models is typically justified by the claim that exchange rates seem to follow a random walk, but this certainly does not apply to inflation in most circunstances.

[9] This calibration will be used often in the article, and it corresponds to a half life of approximately 3 years for the shocks.

[10] These last results, like others in that paper, are not represented in graphs to avoid overextending, but I hope my explanations of the model is detailed enough that the reader can reproduce these for him or herself. Also, I will be happy to provide the Matlab code upon request. The same holds true for other claims made in later parts of the paper that are not represented graphically.

Graphs 1c and 1d are equivalent to 1a and 1b above, but with iid shocks ($\rho = 0$). The interesting result here is that by eliminating any source of persistence the slope of the inflation schedule does not change with the introduction of the band and the non-linearities within the band disappear. It can be demonstrated analytically why this is so. Note that with $\rho = 0$, inflation expectations don't depend on the state, so that the intraband policy is described by:

$$\pi_t^d - \bar{\pi} = \frac{\alpha}{\alpha + \lambda^2} \left[E\left[\pi_{t+1} - \bar{\pi}\right] + \lambda \bar{x} + u_t \right] \tag{10}$$

The slope of the intraband policy rule is given by $\frac{\alpha}{\alpha+\lambda^2}$ which is identically equal to ω when $\rho = 0$. The Honeymoon effect disappears. However, the band may still have an effect on the intercept of the policy by shifting $E\left[\pi_{t+1} - \bar{\pi}\right]$ around and thus correcting for eventual inflation biases. In effect, the translation caused by the shifted π^* is, if any thing, greater than before.

In the following two sections, it will be shown how the choice of π^* and B can be used to affect the the outcome in terms of average inflation and the variances of both inflation and the output gap. These are preciesly the arguments that enter the Loss Function of the policy designer, so that if we are to have any hope that the bands can be used to improve policy outcomes, they should better have some effect on these quantities. This is made with the help of Graphs 2a - 2d, that attempt disentangle how the location of the band and its width affect respectively average inflation and inflation and output-gap variances.

4. USING THE TARGET BAND TO CHANGE AVERAGE INFLATION

The original articles on dynamic inconsistency of monetary policy ([Kydland & Prescott, 1977]; [Barro & Gordon, 1983]) were mainly concerned with the issue of inflation bias. Theoreticians believe that the Central Banks may fall prey to inflation biases because the natural rate of unemployment is below what would be the optimal one. For this reason, Central Banks would desire to have the output-gap permanently at a level above its natural rate, thus pushing up steady state inflation above what would be optimal and, because of the long-term neutrality of the Phillips Curve assumed in these models, not achieving their desired goal in terms of the output gap or unemployment. Whether this characterization of Central Banks is realistic is open to disagreement, and at least one insider account [Blinder, 1998] disputes this notion.

In any case, in the absence of an Inflation Target Zone the present model does imply the existence of such a bias if $\alpha\bar{x}+\bar{\pi} \neq 0$. Svensson [Svensson, 1997] suggested that one can eliminate the inflation bias by picking an "inflation conservative" Central Bank, i.e., a Central Bank which as a lower long-run inflation target than society at large. In the case under analysis, this would mean having a Central Bank with $\bar{\pi} = -\alpha\bar{x}$. In this section the aim is to show that in the absence of concerns with the stabilization bias issue, the target band can be used as a mechanism to implement this.

When plotting graphs 1a and 1b an inflation bias was introduced by setting $\bar{x} = 1$ while keeping $\bar{\pi} = 0$. The other parameter values were $\beta = 1$ and $\rho = 0.8$. When plotting these graphs, the band was made very wide, with $B = 8$[11]. The somewhat surprising result is that even such a wide band can have a discernable

[11] Note that the maximum standard deviation that inflation attains in this class of models (given

effect on the loss function (plotted in 1a) and average inflation (plotted in 1b). This is an extreme result, which somewhat strains the credibility of the Markov Equilibrium concept used to derive it. It is nevertheless an instructive limiting case.

More to the point, another result is that while the bias under discretion is different for $\alpha = 1$ and $\alpha = 3$ (and given the parametrization, is in fact equal to α), the point that minimizes the loss functions and that sets the bias equal to 0 is $\pi^* = -1$ in both cases. In fact, it turns out that in general this point is given by $\pi^* = -\frac{\bar{\pi}+\alpha\bar{x}}{\alpha}$, where the numerator corresponds exactly to the inflation bias. Contrairiwise to what one would think at first, the center of the band should not be set to be symmetric to the bias, but to be symmetric to the bias divided by the weight α. In graph 1b α only has an effect on the slope of the average inflation schedule, with higher α corresponding to a steeper slope.

One final result not shown in the graph is that the variances of inflation and output-gap remain unchanged at their discretionary level, as one would expect. The full correction in the inflation bias while keeping the variances of inflation and output gap unchanged is exactly what is achieved by Svensson's proposal.

Svensson points out that a linear penalty mechanism also achieves this result, but faces "practical and political consequences" (p. 105), which leads him to propose that the inflation target framework shall insteady be interpreted as making sure that the Central Bank to aim for an appropiate $\bar{\pi}$. The same difficulties do not arise with the Target Zone mechanism, so much so that it is ubiquitous. In any case, it should be noted that the model used here differs in a number of details from Svensson's and that it is not entirely clear that the results presented in this paper would be robust to changes in these details. It is worth emphasizing that the π^* that implements Svensson's solution is in general different from Svensson's $\bar{\pi}$. This is a potential source of confusion as in the present paper the Inflation Target is more easily identified with π^* then with $\bar{\pi}$, as proposed by Svensson.

5. USING THE TARGET BAND TO CHANGE INFLATION AND OUTPUT-GAP VARIANCES

The monetary policy literature based on new-keynesian type models like the one used here has recognized that even if there is not an inflation bias problem, the fact that price-setting behavior is forward looking is enough to generate time inconsistency issues in policy making. This appears typically as inflation that is too variable. In particular it can be the case that both output and inflation are more variable than at the optimum. This behavior is what has been called the "stabilization bias". By managing the variances of inflation and the output-gap the Target Band can potentially at least ameliorate this issue.

Graphs 2c and 2d show how this is done. For plotting these graphs the parameter values were set at $\alpha = \beta = 1$, $\bar{x} = \bar{\pi} = 0$ and $\rho = 0.8$. Also, in all graphs $\pi^* = 0$. A result which is not shown is that under these conditions average inflation is not affected by the bandwidth. The same would not be true if there was some kind of inflation bias problem.

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the normalized variance of the shocks) is the one implied by the discretionary regime, given by ω. For the parametrizations used here this is 5/6 for $\alpha = 1$ and 15/8 for $\alpha = 3$. A band which is more than 8 standard deviations wide effectively means that only in extremely rare circunstances will inflation fall out of the target band.

Graph 2c shows how the loss function changes as a function of B and compares it with the loss when there is no band and when the optimal linear policy is implemented. The loss attains a minimum at a point close to $B = 0$, implying that the chosen parameter values require a fairly tight but non-trivial target band to achieve the optimal policy. It is not generally the case that the optimal band is positive and finite. For example, if $\alpha = \beta$ and $\rho = 0$ the optimal band is not to have any band at all and letting the Central Bank persue a discretionary policy, which amounts to the same thing as having $B \to \infty$. The reason why this is so will be discussed shortly.

Another interesting result is that the loss is at all points but for very large B's discernably smaller than the loss of the discretionary policy, but greater than the loss implied by the best linear policy,even if at the optimum it gets very close to that one. While it is not always true that it is better to have any band than to not have a band at all, it is the case for all the combinations of parameter values surveyed by the author that the loss implied by the Band is greater than the loss implied by the optimal linear policy. While I don't prove this result analytically, the robustness of the result is large enough that one can be fairly confident that it will always hold[12]. Also, the result that a well chosen bandwidth is able to put the loss at a level much closer to this lower bound than to the one implied by fully discretionary policy is fairly robust, although not always true.

The significance of the fact that loss implied by the optimal linear policy represents a lower bound on the loss function is that, as emphasized by Clarida et al. [Clarida *et al.* , 1999] in the absence of inflation bias, the optimal linear policy can be interpreted as assigning a Central Banker with output-gap weight given by $(1 - \rho)\,\beta$. This result implies that the use of the Band is not a substitute for the appointment of a suitably "weight conservative" Central Banker, even though it can approximate the outcome to a large extent. Also, this interpretation makes clear why with $\alpha = \beta$ and $\rho = 0$ there is no way the Band can be used to improve on the discretionary policy as in this case the discretionary policy and the optimal linear policy are the same.

Graph 2d shows how the variances of inflation and output-gap change with the size of the band. These behave exactly as one would expect, with inflation variance increasing and output gap variance decreasing with the bandwidth. However, inflation increases faster than the reduction in the output gap, this being the source of the stability bias. Because $\alpha = 1$, for large B both inflation and output gap variance are the same. At the optimal B, inflation variance is very low and output gap variance, very high.

6. DEFINING THE OPTIMAL BAND

The results shown above demonstrate that, at least for certain parameter values there is, given π^* an optimal non-zero and finite value for the bandwidth and for a given value of B, an optimal location for π^*. It seems natural to think that there

[12] However, this result should not be overemphasized, as it could depend critically on the linear-quadratic nature of the model. For an example of how a departure from such a model can generate target-band type behaviors as optimal policies see Orphanides and Wieland [Orphanides & Wieland, 2000]. Also, see Kim and Kim [Kim & Kim, 2003] for a discussion of how the linear quadratic approximation of a general non-linear model can lead to important reversals in the wellfare ordering of policies.

will be in general an optimal combination of B and π^*. Graph 3 shows a case where this is true in an interesting, non-trivial way. The graph was plotted with $\rho = 0.8$, $\alpha = \beta = 1$, $\hat{\pi} = \bar{\pi} = 0$ and $\bar{x} = 1$. The graph implies that the optimal policy design involves both a small B as implied in graph 2c when there was no inflation bias issue and π^* was fixed at zero, but also a strictly negative π^*, which is nevertheless smaller in absolute value than the bias, given by $\bar{x} = 1$.

Also, for small to medium values of B, the level curves tend to slope downwards with higher B 's being associated with more negative values of π^*. The intuition is straightforward. If, for example B is set at, say, 0.5 and π^* at -1 as is optimal when B is very large, this means that inflation will always be lower than its optimal long term level given by $\hat{\pi} = 0$. So for smaller B one needs a π^* closer to zero. For very large values of B the level curves flatten out. When the band-width is large, changes in B have very little impact in the variances, so that all that matters is the location of the target band, which affects the average inflation.

One could think that by agreeing to set π^* closer to zero to accomodate a smaller B the policy designer would be settling for some inflation bias, trading it off for the stability bias. Again, it turns out that for a very wide range of parameters, the optimal Target Band design always implies that the inflation bias disappears. Whether this is a consequence of the technological opportunities offered by the Target Band framework or of the specification of the preferences is not entirely clear and should be the object of further work.

Graphs 4a to 4c show how the optimal policy changes with $\bar{x}$ given $\alpha = \beta = 1$ and the two benchmark values of ρ, namely, $\rho = 0$ and $\rho = 0.8$. Consider first the $\rho = 0$ case. The optimal values of B are not shown, because as discussed in section 5 above, with $\alpha = \beta$ and $\rho = 0$ the optimal Target Zone is for it to be as wide as possible. However, as discussed in section 4, the Target Zone is still instrumental in correcting the inflation bias. This is done by letting $\pi^* = -\bar{x}$ and is in fact what appears as the optimal location for the band in Graph 4a. Graph 4c shows the probability of inflation reaching the edge of the band and, indeed it is indistinguishably close to zero.

Now look at what the results are for $\rho = 0.8$. First note that for $\bar{x} = 0$ the optimal band is centered at 0. The result that in the absence of inflation bias the band should be centered around the prefered level of long run inflation is robust to a variety of changes in the parameters. As $\bar{x}$ increases, the optimal width of the band increases slowly at first and then at around $\bar{x} = 1.5$ it starts to grow faster. This behavior is mirrored by the optimal location of the band, which at first departs only slowly from its initial value of π^* and then from $\bar{x} = 1.5$ starts to decrease at roughly one for one with the increase in $\bar{x}$. More interestingly, at some value of $\bar{x}$ between 3 and 3.5 the optimal band collapses to include only the point $\pi = 0$. It is as if the mechanism is able to deal with moderate values of inflation bias, but after a point enough becomes enough and it doesn't have any option better than just telling the Central Bank to stick to zero inflation whatever happens[13]. Graph 4c shows how in the absence of bias the probabilities of inflation reaching either side of the band are equal and smaller than 50%. As $\bar{x}$ increases the probability

[13] Strictly speaking the optimal design is no longer unique. Namely, it includes all designs that make sure that inflation is identically equal to 0 at all times. This can be achieved by setting $B = \pi^* = 0$, but also by setting, say $B = 1$ and $\pi^* = -1$. One possible interpretation for this result is that for very large values of B the Target Zone loses any impact on the variances of inflation and output so that the best it can do is to eliminate the inflation bias thus reaching a loss of 0.67, whereas by forcing inflation to be always equal to 1 the loss is just 0.5.

of inflation reaching the upper part of the band increases and the probability of reaching the lower part decreases. The optimal band in the presence of a moderate inflation bias will tend to be asymmetric not only in the sense that average inflation will exceed the center of the band, but also in the sense that it will reach the upper edge of the band more often than its lower edge.

The result discussed above should not be taken as a general result, but more as an example of how the strategies appropiate to deal with smaller or larger values of given parameters can change drastically depending on which range is considered. The model as it stands allows for few if any general conclusions about which design to chose as a function of the parameters of the economy. Rather, this decision should be made by considering the optimal designs at paramter ranges that appear reasonable to the economy under analysis. Similar exercises could be done for changes in α and ρ, and while I haven't encountered such drastic changes in policy strategies, it is fair to say that the way the optimal design reacts to changes in these parameters is less straightforward than one would think at first.[14]

7. CONCLUSIONS, CAVEATS AND DIRECTIONS FOR FUTURE WORK

Blinder [Blinder, 1998] commented about how economists often behave as in the joke in which one of them is confronted with some real world fact and asks himself or herself about whether it is also true in theory. This is in some sense exactly what was done in this paper. The use of Inflation Target Bands is fairly common among inflation targeting countries. Not only their location, but also their width varies from country to country and the latter is often regarded as being too "tight" in the sense that inflation hits its edges too often. The model presented in this paper provides a theoretical rationale for these real world facts. The use of an Inflation Target Bands is shown to be able to eliminate the inflation bias and, at least in some circumstances, significantly reduce the stability bias. Also, in this model, its optimal width and location depend in complicated ways from the underlying parameters of the economy, which may respond to their diversity. Finally, the tightness of the band emerges endogenously from the optimization of the policy designer, so that it is to be expected that whenever one thinks that the Band can be used to deal with the stability bias issue one would see inflation reaching its edges fairly often.

The numerical exercises showed above demonstrate that the use of a Target Band is able eliminate the inflation bias problem. In that sense it serves as a substitute for an "Inflation Target" conservative Central Banker or alternatively as a way of making a Central Bank who is not sufficiently conservative in that sense act as if it were.

Regarding the correction of stability bias the mechanism is less successful. Within the current setup it wouldn't be reasonable to expect that it would give incentives for the Central Bank to make its policy history dependent. However, the mechanism does not deliver the best non-history dependent policy either. In particular, it seems to fare consistently worse than the best linear non-history dependent policy. This is specially significant as that policy can be implemented by the choice of a suitably "weight" conservative Central Banker. Moreover, if such a Central

[14] The investigation performed on variations of these parameters do identify a pattern in the sense that the optimal bandwidth becomes tighter as both α and ρ increase, in the sense that the probability of inflation reaching the edge of the band increases. However, this doesn't necessarily translates into smaller B's, as higher α and ρ also tend to imply higher variance of inflation.

Banker is chosen, the Target Zone should not be used or at least that the band should be so wide that it only has a discernible effect on average inflation. This latter result should be taken with care as it very probably dependent on details of the underlying model.

In terms of empirical applications, the model implies there is an inflation bias and if the band is being set optimally, inflation should be on average above the center of the Target Zone, and one should see it crash the upper edge of the band more often than the lower edge. This would serve as a test both of the adequacy of this model, in case one is willing to maintain the hypothesis that the inflation bias exists, or as a test of the existence of an inflation bias in case one is willing to maintain the hypothesis that the model is a valid description of reality.

The main caveat is that the results were shown for a very simplified model. Future work should try to evaluate which of these results are robust to the introduction of a more realistic utility function which gives weight to other features of the distribution of inflation and output-gap apart from their mean and variance[15] or non-linearities in the Phillips Cuve, in the line of what is done by Orphanides and Wieland [Orphanides & Wieland, 2000]. More importantly, there should be some investigation about how well the Target Band fares when there is persistence not only in the shocks, but in the inflation and output-gap processes as well.

While technically more demanding, the extensions proposed above do not involve any significant conceptual change. In that aspect the main issue is the assumption the Central Bank has perfect control over inflation. Apart from not being true in practice, this assumption generates the unrealistic outcome that inflation never falls out of the band. An extension of the model that would deal with that would be to keep the punishment conditioned on observed inflation and have the Central Bank chosing accordingly to avoid a very high probability of missing the band. Such an approach is in fact the one taken by Walsh [Walsh, 2002]. Such a modeling strategy would probably call for larger bands to avoid the Central Bank being overconservative. The size of the bands would thus also be a function of the uncertainty faced by the Central Bank in its policy.

Also, one should take into account the fact that it is not always the case that the Target Band is set once and for all without regard for current conditions. In effect, in some cases (eg. Brazil), every year the "policy designer" decides in a discretionary way what the Target Band will be a few years in the future. This could be modelled by extending the state at each date to include future target bands that were decided in previous periods and deriving the policies of the Central Bank and the "policy designer" as a Markov equilibrium. The relevant policy parameter for a second level, "timeless" policy designer would no longer be the target range, which would become endogenous to the model, but the time lag with which the target range would have to be defined. A longer lag would probably allow for less opportunistic adjustments of the target range but on the other hand the range would not be adjusted to new information and circunstances.

Last, future work should try to assess what the optimal Inflation Target Zone should be for individual economies. This would require having the model extended to allow for greater realism, and than calibrating it with the appropiate parameter values. Such exercises are the more important given that the model does not generally allow for easy, intuitive, conclusions in terms of optimal policy design.

[15] For example, one could reasonably be worried with avoiding extreme events.

8. APPENDIX
PROOF THAT THE ALGORITHM TO FIND $\pi(U_T)$ IS A CONTRACTION

PROPOSITION 1. *Let the operator T be given by the algorithm presented in section 2. Then for $\lambda = 1$, T is a contraction with modulus $\frac{\alpha}{\alpha+1}$.*

We will show that T is a contraction by showing that it satisfies the Blackwell sufficient conditions. First, note that there is no loss in generality in focusing our attention in bounded $\pi(.)$ as for given finite B and π^*, $\pi(.)$ is necessarily bounded. To show that T is a contraction, one has to show furthermore that it satisfies the following two conditions:

i) T is monotone:

$$\pi \leq \pi' \Rightarrow T\pi \leq T\pi'$$

We have after setting $\lambda = 1$ that

$$\pi^{d,n+1}(u_t) = \frac{\alpha}{\alpha+1}\left(E_t\left[\pi^n(u_{t+1})\right] + \bar{x} + u_t\right)$$

By the properties of the expectations operator, if $\pi^{n\prime}(u_{t+1}) \geq \pi^n(u_{t+1})\ \ \forall u_{t+1}$ then it must be the case that $\pi^{d,n+1\prime}(u_t) \geq \pi^{d,n+1}(u_t)\ \ \forall u_t$. As $\pi^{n+1}(u_t)$ is a non-decreasing transformation of $\pi^{d,n+1}(u_t)$ it must be the case that $\pi^{d,n+1\prime}(u_t) \geq \pi^{n+1}(u_t)\ \ \forall u_t$, establishing the result.

ii) Discounting:

$$\exists\ \delta \in (0,1)\ \text{ s.t. } T(\pi + c)(u_t) \leq T(\pi) + \delta c,\ c \geq 0$$

Let $\pi^{n\prime}(u_t) = \pi^n(u_t) + c$ with $c > 0$. Then
...

$$\begin{aligned}\pi^{d,n+1\prime}(u_t) &= \frac{\alpha}{\alpha+1}\left(E_t\left[\pi^n(u_{t+1}) + c\right] + \bar{x} + u_t\right)\\ &= \pi^{d,n+1}(u_t) + \frac{\alpha}{\alpha+1}c \qquad (11)\end{aligned}$$

To see that this implies that $\pi^{n+1\prime}(u_t) \leq \pi^{n+1}(u_t) + \frac{\alpha}{\alpha+1}c$ note that

$$\begin{aligned}\pi^{n+1} &= \max\left(\min\left(\pi^{d,n+1}, \pi^* + B\right), \pi^* - B\right)\\ \pi^{n+1\prime} &= \max\left(\min\left(\pi^{d,n+1\prime}, \pi^* + B\right), \pi^* - B\right)\end{aligned}$$

If $2B < \frac{\alpha}{\alpha+1}c$ then we are done. Otherwise there are five relevant cases:

$$\begin{aligned}1 &: \ \pi^{d,n+1\prime}(u_t) < \pi^{d,n+1}(u_t) \leq \pi^* - B \text{ and}\\ 2 &: \ \pi^* + B \leq \pi^{d,n+1\prime}(u_t) < \pi^{n+1}(u_t)\end{aligned}$$

In these two cases $\pi^{n+1}(u_t) = \pi^{n+1\prime}$. The third case is

$$3 : \pi^{*} - B < \pi^{d,n+1\prime}(u_t) < \pi^{d,n+1\prime}(u_t) < \pi^{*} + B$$

In this case, $\pi^{n+1}(u_t) = \pi^{d,n+1}(u_t)$ and $\pi^{n+1\prime}(u_t) = \pi^{d,n+1\prime}(u_t)$ so that equation 11 holds. The last two cases are:

$$\begin{array}{rcl} 4 & : & \pi^{d,n+1\prime}(u_t) \leq \pi^{*} - B < \pi^{d,n+1}(u_t) \\ 5 & : & \pi^{d,n+1\prime}(u_t) \leq \pi^{*} + B < \pi^{d,n+1}(u_t) \end{array}$$

In both of this cases, $\pi^{n+1}(u_t) - \pi^{n+1\prime}(u_t) \leq \pi^{d,n+1}(u_t) - \pi^{d,n+1\prime}(u_t)$, establishing the result. So with $\delta = \frac{\alpha}{\alpha+1} < 1$ the discounting property is established.

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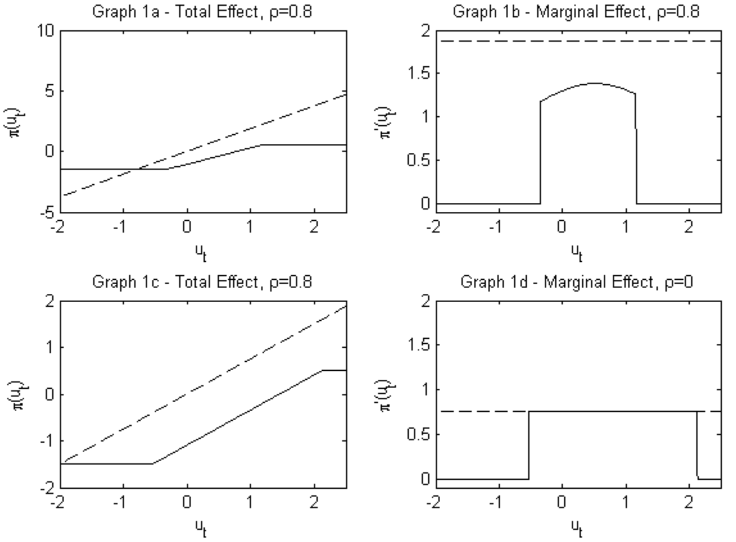
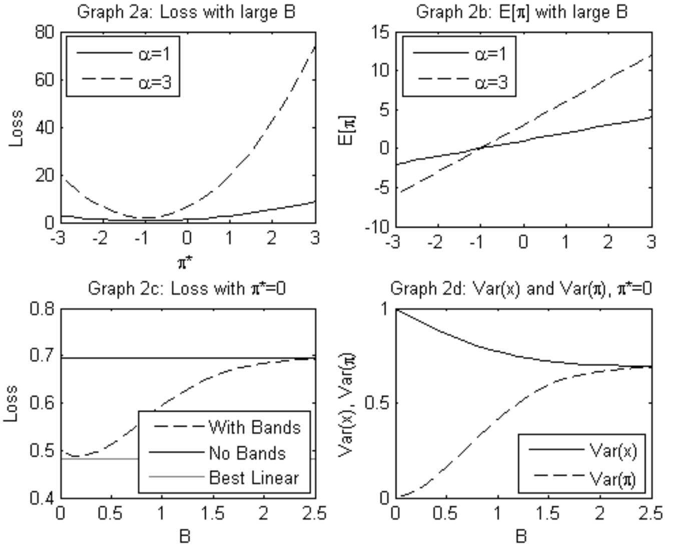
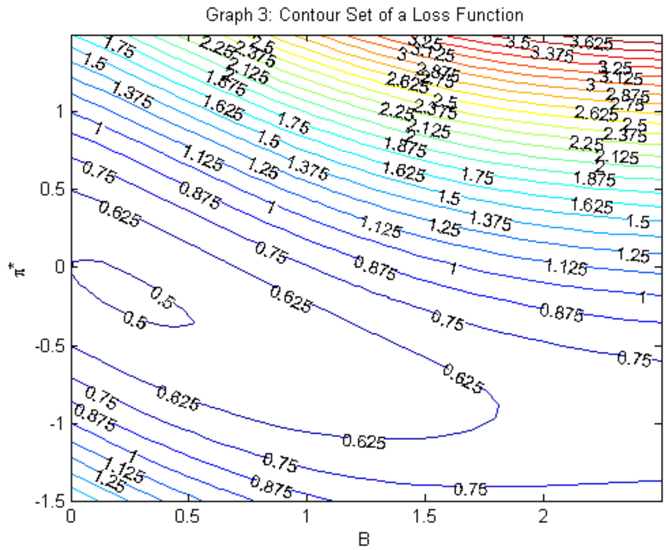
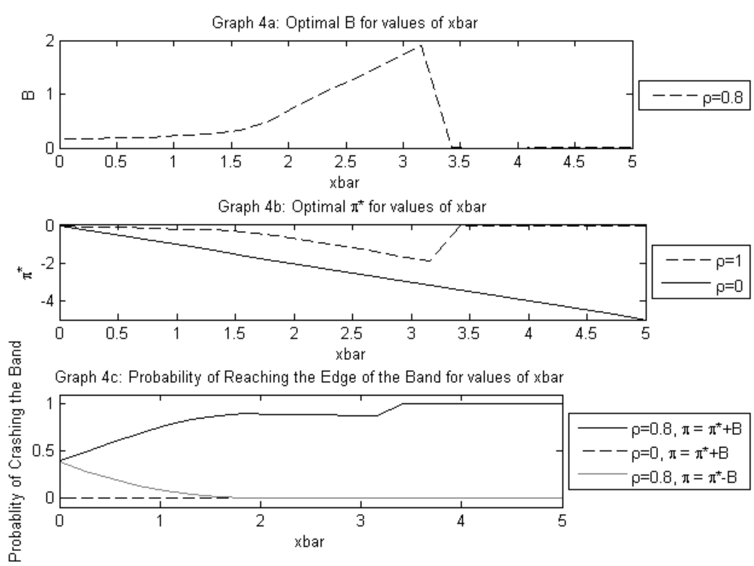

Graph 1a - Total Effect, ρ=0.8
Graph 1b - Marginal Effect, ρ=0.8
Graph 1c - Total Effect, ρ=0.8
Graph 1d - Marginal Effect, ρ=0
π(u_t)
π'(u_t)
u_t


Graph 2a: Loss with large B
Graph 2b: E[π] with large B
Graph 2c: Loss with π*=0
Graph 2d: Var(x) and Var(π), π*=0
α=1
α=3
Loss
E[π]
π*
With Bands
No Bands
Best Linear
Var(x)
Var(π)
Var(x), Var(π)
B


Graph 3: Contour Set of a Loss Function
π*
B


Graph 4a: Optimal B for values of xbar
B
xbar
ρ=0.8
Graph 4b: Optimal π* for values of xbar
π*
xbar
ρ=1
ρ=0
Graph 4c: Probability of Reaching the Edge of the Band for values of xbar
Probablity of Crashing the Band
xbar
ρ=0.8, π = π*+B
ρ=0, π = π*+B
ρ=0.8, π = π*-B