Income Inequality in a Job-Search Model With Heterogeneous Time Preferences*

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November 6, 2004

Abstract

This paper investigates the income inequality generated by a job-search process when different cohorts of homogeneous workers are allowed to have different degrees of impatience. Using the fact the average wage under the invariant Markovian distribution is a decreasing function of the time preference (Cysne (2004)), I show that the Lorenz curve and the between-cohort Gini coefficient of income inequality can be easily derived in this case. An example with arbitrary measures regarding the wage offers and the distribution of time preferences among cohorts provides some quantitative insights into how much income inequality can be generated, and into how it varies as a function of the probability of unemployment and of the probability that the worker does not find a job offer each period.

1 Introduction

A condition for a stochastic dynamic model to be of some use in the understanding of income distribution is that it delivers an endogenous distribution of incomes. Within this class of models, job-search models are certainly among the simplest.

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Pissarides (1974) uses a job-search model to compare the income distribution[1] of two cohorts[2] with different degrees of risk aversion. This author argues that cohorts with higher risk aversion will have a lower reservation wage and, because the wages of employed workers will have a greater range, the income distribution will be worse.

Pissarides does not explicit derive the stationary distribution generated by the interactions of the transition functions implicit in his model. Moreover, his work concentrates only on comparing (between two cohorts) within-cohort inequalities, not formally deriving a measure of inequality among cohorts.

In this work I complement Pissaride's analysis by pursuing answers to three different questions. First, instead of dealing with heterogenous degrees of risk aversion, I allow consumers in different cohorts to have different time-preference parameters. Second, in each cohort I concentrate the analysis on the invariant Markovian distribution of wages, rather than on the cross-sectional distribution. Third, I analyze inequality among cohorts, rather than within cohorts.

The inequality among cohorts is generated by the fact that more patient workers have higher reservation wages and higher average wages as well (see Cysne (2004) for a demonstration of this fact).

The paper proceeds as follows. Section 2 is used for the presentation of the general model, fully characterizing a the job-search problem within a cohort. Section 3 is dedicated to the derivation of the invariant-distribution between-cohort Gini coefficient of income inequality. Section 4 is used to make quantitative assessments of the model, based on an example with arbitrary measures for the wage offers and for the distribution of time-preferences among cohorts . Section 5 concludes.

2 The Model

The basic model presented here is the same as the one presented in Cysne (2004), which in turn draws on Stokey and Lucas's (1989) version of McCall's (1970) model. The givens of the model are the distribution of wage offers (the same for all workers and along all cohorts) obtained by the workers, the distribution of the time-preference parameter among cohorts of workers, the

[1] More rigorously, we are dealing here with wage inequality. However, transfers and capital income usually represent only a small fraction of most households' total income. For the United States, for instance, following the 1992 SCF (Survey of Consumer Finances), transfers and capital income account in average for only around 28% of the total income of the households surveyed. This percentage tends to be even lower in developing countries.

[2] In this work, a cohort is defined as a group of homogenous workers/consumers.

probability of layoffs and the probability that, each period, the worker does not get any job offer next period.

In the measurable space $\left([0,1),\mathcal{B}_{[0,1)},\mathcal{L}\right)$, $\mathcal{B}_{[0,1)}$, standing for the borelians in $[0,1)$ and $\mathcal{L}$ for the Lebesgue measure, consider a continuum of cohorts, indexed by j, each cohort with a large number of homogenous workers. Cohorts differ only in terms of their time-preference parameter. The distribution of time preferences $\beta_j \in [0,1)$ along these cohorts is determined by:

$$\beta_j = H^{-1}(j) \ , \ \ H'(.) > 0 \tag{1}$$

In equation (1), j has a uniform distribution in $[0,1)$ and H stands for a cumulative probability distribution of a random variable taking values in $[0,1)$.The isomorphism (1) allows us to put different probability measures m in the space where the time-preference parameters take value (also $([0,1),\mathcal{B}_{[0,1)})$). For instance, if H is the cumulative distribution function of a Beta (s,v) random variable, then the $\beta_j's$ will be distributed as a Beta (s,v). Note that having $H'(.) > 0$ allows us to identify each cohort j with its time preference parameter β_j.

For $0 < D < \infty$, consider also the second measurable space $\left(\Omega,\mathcal{B}_{[0,D]},p\right)$ and, in this space, the measure q induced by the wage function w: $\Omega \rightarrow [0,D]$. In the induced space $\left([0,D],\mathcal{B}_{[0,D]},q\right)$, denote by $F(t)$ the distribution function that ($q-a.e.$ -uniquely) determines the measure $q : F(t) = p(w \leq t)$.

By assumption, in each cohort, there are two states regarding the consumer's optimization problem: w and 0. State "w" corresponds to a job offer of w at hand, and state "0" to no job offer. In state w the worker can accept or turn down the offer. If he accepts it, by assumption he stays employed with that wage till he is laid off (which happens with probability θ). If he does not accept the offer or if he gets no offer, he remains in state 0. Being in state zero the only thing he can do is wait again for a job offer next period, which happens with probability $(1-\alpha)$. The individual is not allowed to search while in his job. The job offers are drawn from $[0,D]$ according to the measure q. q is known by all workers in all cohorts. Workers are not allowed to borrow or to lend. Their consumption c_t is equal to their income w_t in each period. Consumers in cohort j maximize the expected present value of their consumption $E\left(\sum_{t=0}^{\infty}\beta_j^t c_t\right)$. From now one I will only use the index j when necessary.

With $v(w)$ stating for the value function, $\bar{w}$, the reservation wage is determined by (Cysne (2004)):

$$\bar{w} = \frac{\beta(1-\alpha)}{1-\beta(1-\theta)}\int_{[\bar{w},D]}(w'-\bar{w})dF(w') \tag{2}$$

Following the analysis in Stokey and Lucas (1989, c. 10), the reservation wage $\bar{w}(j)$ divides $[0, D]$ into two regions: the acceptance region $A(\beta) = [\bar{w}(\beta), D]$ and the non-acceptance region $A^c(\beta) = [0, \bar{w}(\beta)]$.

Consider a new measure λ_t in $([0, D],\mathcal{B}_{[0,D]})$, representing the state of the state offer of workers of a certain cohort j (the j is omitted), in period t. This measure converges strongly to an invariant measure (Cysne (2004)). For $C \subset A$ this invariant measure reads:

$$\lambda(C) = \frac{(1-\alpha)q(C)}{\theta + (1-\alpha)q(A)}$$

in which case the average wage in each cohort j is:

$$w_A(j) = \int_{[\bar{w}(j),D]} \frac{(1-\alpha)w dF(w)}{\theta + (1-\alpha)q(A(j))} \tag{3}$$

where $\bar{w}(j)$ follows (2). As shown in Cysne (2004), w_A is an increasing function of the time preference parameter and, given (1), of j as well.

3 Income Inequality

The existence of different time preferences between cohorts leads to a between-cohort income inequality. The Proposition below provides an expression for the Gini coefficient as a function of the exogenous measure to be put into the time preference parameters.

Proposition 1 *The between-cohort Gini coefficient of income inequality associated with the problem outlined above is given by:*

$$G = 1 - 2\int_0^1 dm(\beta_j) \int_0^{\beta_j} w_a(u) dm(u) \tag{4}$$

Proof. The Lorenz curve $L(j)$ plots the percentage of total income earned by the economic agents of a certain cohort, when these cohorts are ordered from those with lower average income to those with higher average income. The Gini coefficient (G) of income inequality is then, by definition, given as a function of the Lorenz curve, by

$$G = 1 - 2\int_{[0,1)} L(j) dj. \tag{5}$$

A crucial point concerning the calculation of the Gini coefficient in this model, is that, as shown by Cysne (2004), there is an isomorphism linking the average wage w_A and the time-preference parameter β (because $w'_A(\beta) > 0$). Since (1) also defines an isomorphism, between j and β_j, there is (by composition) an isomorphism between the measure of the population j and the average wage $w_A(\beta_j(j))$, with $w'_A(j) > 0$. This fact implies that, by ordering the population by β_j we are, automatically, also ordering it by income, as required by the construction of the Lorenz curve.

Keeping (1) in mind, the measure of cohorts with time preference equal or less than a certain constant $a \in [0,1)$ is given by: $P(\beta_j \leq a) = P(H^{-1}(j) \leq a) = P(j \leq H(a)) = H(a)$. This is also equivalent to the measure of people with income less or equal than $w_A(\beta_j)$. The proportion of income earned by the $j\%$ poorer workers of the cohort (the Lorenz curve), therefore, can be written as $\frac{1}{\bar{w}_A}\int_0^{\beta_j} w_A(\beta_j)dm(\beta_j)$, where $\bar{w}_A = \int_0^1 w_A(\beta_j)dm(\beta_j)$. (4) follows trivially from (5). ■

4 Quantitative Assessments

This is as far as one can go without specifying the measures q (of wage offers) and m (of the distribution of time preferences among cohorts). In order to get some idea of the numbers generated by the analysis developed here, from now on I will assume that measure q is the Lebesgue measure in $[0,1]$ (this measure allows for closed-form solutions to the reservation wage, the average wage, and the Gini coefficient), and that m has a density with respect to the Lebesgue measure given by the density of the Beta $(114.5, 1.01)$ distribution. The parameters of this distribution have been chosen in order to make the (monthly) average equal to $114.5/(114.5+1.01) = 0.991\,3$. This corresponds to an yearly average time-preference of $0.\,900$. Some numerical results are presented in the example below.

Example 1 *This example assumes that in each period the workers face a probability θ of layoff, a probability α of not finding a wage offer and measures q and m specified as above. Using (2), (3) and (4), the Gini coefficient reads:*

$$G = 1 - 2\int_0^1 dm(\beta_j)\frac{\int_0^{\beta_j}\frac{(1-\alpha)(1-\left[\frac{1-u(\alpha-\theta)}{u(1-\alpha)}-\sqrt{(\frac{1-u(\alpha-\theta)}{u(1-\alpha)})^2-1}\right]^2)}{2\left[\theta+(1-\alpha)(1-\frac{1-u(\alpha-\theta)}{u(1-\alpha)}+\sqrt{(\frac{1-u(\alpha-\theta)}{u(1-\alpha)})^2-1})\right]}dm(u)}{\int_0^{1}\frac{(1-\alpha)(1-\left[\frac{1-u(\alpha-\theta)}{u(1-\alpha)}-\sqrt{(\frac{1-u(\alpha-\theta)}{u(1-\alpha)})^2-1}\right]^2)}{2\left[\theta+(1-\alpha)(1-\frac{1-u(\alpha-\theta)}{u(1-\alpha)}+\sqrt{(\frac{1-u(\alpha-\theta)}{u(1-\alpha)})^2-1})\right]}dm(u)}$$

The values of both the Gini coefficients, for different values of theta and alpha, are shown in Table 1 below:

Table 1

Gini Coefficient

		Theta	
		0	0.1
Alpha	0	0.0253	0.0094
	0.2	0.0270	0.0094

Note in Table 1 that the between-cohort inequality decreases when the probability of layoff increases. An increase in θ has two effects. First, it decreases the reservation wages of all cohorts, thereby making it more likely that low wage offers are taken by workers in some cohorts. Second, having workers unemployed more frequently, the average wages in all cohorts decrease, impoverishing all cohorts at the same time, and decreasing inequality.

The sensibility of the income distribution to the probability of not finding a job offer, though, in the range of values of theta and alpha in which we derived the results of Table 1, was of a different sign and very small. The basic difference is that even though an increase in alpha, as well as an increase in theta, impoverishes all cohorts, it has a different effect on the mass of workers in each cohort with a wage equal to zero.

A last comment with respect to the results in Table 1 is that, even though the heterogeneity of the time-preference parameters leads to income inequality among cohorts, the quantitative impact of this effect is very small.

5 Conclusions

In this paper I have investigated income inequality among cohorts of homogenous workers, when each cohort is characterized by a different degree of impatience. The work can be seen as being complementary to Pissarides's (1974), who used a job-search model to study within-cohort inequalities.

Using the result obtained by Cysne (2004), that the average wage in each cohort based on the invariant Markov measure is a decreasing function of impatience, I have shown how to derive the Lorenz curve and the between-cohort Gini coefficient of income inequality. Next, I have provided an example, based on arbitrary measures concerning the job offers and the distribution of time preferences among the different cohorts, in order to provide a quantitative assessment of the problem. The numbers obtained indicate that the degree of inequality is decreasing in the probability of unemployment and increasing in the probability that workers in each cohort do not find job offers next period, when unemployed. The probability of unemployment has shown to have a greater impact over the generation of income inequality than the probability of finding job offers. Finally, the inequality generated by the job-search process, based solely on heterogeneous degrees of impatience, has been shown to be very small, with Gini coefficients ranging from 0.94% to 2.70%.

References

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