Estimating Potential Output and the Output Gap for Brazil*

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Abstract

This paper aims to estimate output gap for Brazilian economy through differents methodolies. We use traditional univariate techniques and propose a new semi-estructural methodology that combines HP filtering and the production function approach. In order to compare the different potential output estimates, we use a Phillips curve to predict free price inflation and a rolling forecast experiment as a test of forecast accuracy. Our results shows that the forecasts produced by the Local level and Watson models are even more inaccurate than those generated by the simplest univariate models. The main evidence is that the Beveridge-Nelson methodology outperforms all the models at all forecast horizons.

Resumo

Este artigo tem como objetivo estimar o produto potencial e o hiato do produto para a economia brasileira. Usamos técnicas univariadas tradicionais e propomos uma metodologia semi-estrutural que combina a filtragem de Hodrick-Prescott e a abordagem de função de produção. Para avaliar os resultados estimados do produto potencial, calculamos a inflação de preços livres prevista pela curva de Phillips e o erro quadrático médio em relação ao valor observado. Os resultados indicam que as previsões produzidas por modelos de componentes não observados são mais imprecisas que aquelas pelos modelos univariados mais simples. A metodologia de Beveridge-Nelson é mais eficiente em todos os horizontes de previsão.

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1 Introduction

Along the last fifteen years we have seen a worldwide adoption of inflation targeting regime as reported at Mishkin and Schmidt-Hebbel (2001). Those countries explicitly adopt a Taylor Rule reaction function. Although Blinder (1997) claims that Federal Reserve Bank (FED)'s strategy is different from the "deliberate desinflation" followed by inflation targeters, Bernanke (2003) asserts there is little difference between inflation targeting and FED's strategy, known as "constrained discretion".

In fact, FED has a legal mandate to pursue "maximum employment" and "stable prices". In line with this framework, European Central Bank (ECB) has a statutory objective of maintaining price stability, but without prejudicing this target, it shall support economic policies with a view to contributing to the achievement of high level of employment and sustainable and non-inflationary growth.

Those facts give evidence that both FED and ECB implicitly adopt a Taylor Rule type reaction function as described in Taylor (1993). Thereafter, it seems that all modern central bankers operate monetary policy with a view at the short-run trade-off between inflation and unemployment. Blinder reports that the Federal Open Market Committee (FOMC)'s process of decision involves a consensus reached meeting-by-meeting, based on analysis of the current macroeconomic situation and the near-term outlooks. Under these circumstances, in order to change the interest rate (the policy instrument), policemakers demand both estimations of expected inflation and potential output.

Potential output, the associated output gap and the natural rate of unemployment are all concepts whose have received increase attention over the past few years in central banks, international organizations and among academics researchers. Measures of potential output and output gap are useful to identify the scope for sustainable noninflationary growth and to allow an assessment of the stance of macroeconomic policies.

St-Aman and van Norden (1997) discussed some methodologies for estimating potential output and the output gap for the Canadian economy. Claus (1999) used a structural vector autoregression methodology to obtain a measure of potential output for the New Zealand economy, finding evidences that consumption increases in anticipation of higher future earnings. Kichian (1999) measured potential output using state-space models for the Canadian economy, finding that the obtained output gap can be quite useful in the formulation of monetary policy. Orphanides and van Nordem (1999) examined the reliability of alternative output detrending methods, with special attention to the accuracy of real-time estimates, they showed that ex post revisions of the output gap are of the same order of magnitude as the output gap itself, that these ex post revisions are highly persistent and that real-time estimates tend to be severely biased around business cycles turning points. Cerra and Saxena (2000) reviewed different methodologies that can be used to estimate potential output for Sweden, finding evidences of a large negative output gap. Proietti, Musso and Westermann (2002) evaluated unobserved components models based production function approach for estimating the output gap and potential output for the Euro Area, they concluded that this models can be valuable for growth accounting and for reducing the uncertainty surrounding the output gap estimates. Rennison (2003) used a Monte Carlo experiment to evaluate the ability of a variety of output gap estimators to accurately measure the output gap in a model economy, his evidences indicate that an estimator that combines Hodrick-Prescott filter with a Blanchard-Quah structural vector autoregression yields an accurate estimate. Doménech and Gómez (2003) proposed a new method to obtain estimates of the potential output, core inflation and the NAIRU as latent variables, using a standard Okum's law, a forward-looking Phillips curve and an investment equation. Ehrmann and Smets (2003) used a small forward-looking model of the euro-area economy to investigate the implications of incomplete information about potential output for the conduct of monetary policy; under optimal monetary policy, they found that output gap uncertainty leads to persistent deviations between the actual and the perceived output gap in response to supply and cost-push shocks. Arnold (2004) examined methods rely on purely statistical techniques (filtering, simultaneous econometrics models, multivariate time-series models) and others rely on statistical procedures grounded in economic theory (Solow growth model), highlighting the pros and cons of various approaches.

In July 1999, the Brazilian government officially adhered to an inflation targeting regime as described in Bogdanski, Tombini and Werlang (2000). This institutional change led the Central Bank of Brazil's researches to invest in developing tools that appropriately delivery estimations of potential output and of inflation to members of Brazilian monetary committee. This effort is highlighted in Alves (2001), Bryan and Cecchetti (2001), Alves and Muínhos (2003) and Silva Filho (2002). This paper goes in the same line, so we try to contribute to the debate addressing these two issues.

First, we estimate potential output paths for the Brazilian economy through several different techniques, including time series filtering - such as Kalman filter (KF) and Beveridge-Nelson decomposition (BN) - and semi-structural approaches – a Hodrick-Prescott (HP) filter constrained by some structure. Time series filtering techniques, that basically extract a trend from Gross Domestic Product (GDP), have been largely used as reported in Orphanides and Norden (1999) and Kichian (1999). Even though semi-structural approaches have been equally popular[1], as far as we know, our approach is the only that allows simultaneously setting paths for two latent variables - non-accelerating inflation rates of unemployment (NAIRU) and non-accelerating capacity utilization (NAICU) - to obtain output gap as by-product.

In a second stage, we will care about inflation. Through Ordinary Least square

[1] St-Amant and van Norden (1997) and Alves and Muínhos (2003) uses this approach.

techniques we will identify the best Phillips Curve type equation for each output gap path generated before.

The rest of the paper is organized as follows. Section 2 presents the econometric techniques considered to estimate potential output and output gap. Section 3 describes data used to estimate each model. Section 4 compares the inflation forecasts generated by a Phillips curve that is estimated using each output gap series. Section 5 concludes.

2 Econometric Approaches

Potential output is usually identified as the output trend. Many kinds of filtering have been proposed in order to decompose output into its high and low frequency components which were respectively assigned to output gap and potential output. The detrending methods considered in this paper are:

1. Deterministic Trends.
2. Moving Average.
3. The Hodrick-Prescott Filter.
4. The Beveridge-Nelson Decomposition.
5. Unobserved Component Models.
6. Hodrick-Prescott constrained to a Production Function

Next we briefly discuss each of these six groups and the variants of these methods which we apply.

2.1 Deterministic Trend

The first method we consider assume that the trend in (the logarithm of) output is well approximated as a simple deterministic function of time. The linear trend is the simplest of these models, it assumes that output may be decomposed into a cyclical component and a linear function of time:

$$y_t = \alpha + \beta \cdot t + c_t \tag{1}$$

where c_t is the output gap and y_t is our chosen measure of output (in logarithms).

2.2 Moving average

The second method assume that the logarithm of output may be decomposed into a trend component, τ_t, and a cyclical component c_t,

$$y_t = \tau_t + c_t \tag{2}$$

where τ_t is the moving average of the output $(\tau_t = (y_t + y_{t-1} + y_{t-2} + y_{t-3})/4)$.

2.3 The Hodrick-Prescott Filter

The Hodrick-Prescott Filter (1997), commonly called HP filter, is a simple smoothing procedure that has become increasingly popular because its flexibility in tracking the characteristics of the fluctuations in trend output. Output trend, y_t^g, derived using the HP filter is obtained by minimizing a combination of the gap between actual output, y_t, trend output and the rate of change in trend output for the whole sample of observations, T. Formally, The HP-filtered trend is given by

$$\{y_t^g\}_{t=1}^T = \arg\min \sum_{t=1}^{T} (y_t - y_t^g)^2 + \lambda \sum_{t=2}^{T-1} \left[\left(y_{t+1}^g - y_t^g\right) - \left(y_t^g - y_{t-1}^g\right)\right]^2 \tag{3}$$

and y_t^c is the resulting measure of the output gap,

$$\{y_t^c\}_{t=1}^T = \{y_t - y_t^g\}_{t=1}^T$$

λ is called the "smoothing parameter" and penalizes the variability in the growth component. The larger the value of λ, the smoother the growth component and the greater the variability of the output gap. As λ approaches infinity, the growth component correspond to a linear time trend. For quarterly data, Hodrick and Prescott propose setting λ equal to 1600.

2.4 The Beveridge-Nelson Decomposition.

Beveridge-Nelson decomposition is a detrending method using unobserved components. Output is assumed to contain unobserved permanent and temporary component consisting of a random walk with drift and a stationary autoregressive process.

Beveridge and Nelson (1981) consider the case of an ARIMA(p,1,q) series, y_t, which is to be decomposed into a trend and a cyclical component. For simplicity, we can assume that all deterministic components belong to the trend component and have been removed from the series. Since the first-difference of the series is stationary, it has an infinite-order MA representation of the form

$$\Delta y_t = \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} + \cdots = \xi_t \tag{4}$$

where ε is assumed to be an innovations sequence. The difference in the series over the next s periods is

$$y_{t+s} - y_t = \sum_{j=1}^{s} \Delta y_{t+j} = \sum_{j=1}^{s} \xi_t \tag{5}$$

The trend is defined to be

$$\lim_{s \longrightarrow \infty} E_t \left[y_{t+s}\right] = y_t + \lim_{s \longrightarrow \infty} E_t \left[\sum\nolimits_{j=1}^{s} \xi_t\right] \tag{6}$$

From equation 4, we can see that

$$E_t\left[\xi_{t+j}\right] = E_t\left[\varepsilon_{t+j} + \beta_1\varepsilon_{t+j-1} + \beta_2\varepsilon_{t+j-2} + \cdots\right] = \sum_{i=1}^{\infty}\beta_{j+i-1}\varepsilon_{t+j-i} \tag{7}$$

Since changes in the trend are therefore unforecastable, this has the effect of decomposing the series into a random walk and a cyclical component, so that

$$y_t = \tau_t + c_t \tag{8}$$

where the trend is

$$\tau_t = \tau_{t-1} + \xi_t \tag{9}$$

and ξ_t is a white noise.

2.5 Unobserved Component Models

Unobserved component models attempt to specify the time series properties of output and and use the resulting model to identify cyclic and trend component. The simplest unobserved component is the Local Level Model,

$$\begin{aligned} & y_t = \mu_t + \varepsilon_t \\ & \mu_t = \mu_{t-1} + \eta_t, \\ & \varepsilon_t \backsim iid\ N(0, \sigma_\varepsilon^2) \\ & \eta_t \backsim iid\ N(0, \sigma_\eta^2) \\ & E[\varepsilon_t, \eta_s] = 0, \forall t, \forall s \end{aligned} \tag{10}$$

It assumes that the observed output series y_t may be decomposed in a random walk component μ_t and a white noise ε_t. ε_t and the increments of the random walk are assumed to be mutually uncorrelated and follow independent Gaussian distributions. This implies that y_t follows an IMA(1,1), with the size of the MA term determined by the relative variances of ε and μ.

The second unobserved component model we consider is

$$\begin{aligned} & y_t = \mu_t + c_t \\ & \mu_t = \delta + \mu_{t-1} + \eta_t \\ & c_t = \alpha_1 c_{t-1} + \alpha_2 c_{t-2} + \varepsilon_t \\ & \varepsilon_t \backsim iid\ N(0, \sigma_\varepsilon^2) \\ & \eta_t \backsim iid\ N(0, \sigma_\eta^2) \\ & E[\varepsilon_t, \eta_s] = 0, \forall t, \forall s \end{aligned} \tag{11}$$

due to Watson (1986), where the output time series y_t is decomposed into a trend component , μ_t, and a cyclical component, c_t. The trend component, μ_t, is assumed to follow a random walk with drift and the cyclical component, c_t, is assumed to follow an AR(2) process, to allow for more persistence.

2.6 Hodrick-Prescott constrained to a Production Function

In order to impose some structure to the HP filtering, we assume that a Cobb-Douglas production function with constant return to scale is used to assess output and potential output.

$$
\begin{aligned}
y_t &= A_t(K_t c_t)^{\alpha_t}(L_t(1-u_t))^{(1-\alpha_t)} && (12)\\
\overline{y}_t &= A_t(K_t naicu_t)^{\alpha_t}(L_t(1-nairu_t))^{(1-\alpha_t)} && (13)
\end{aligned}
$$

where y_t is the output, $\overline{y}_t$ is the potential output, A_t is the productivity factor, K_t is the capital stock, L_t is the labor force, α_t is the income capital share, c_t is the capacity utilization, u_t is the rate of unemployment, $naicu_t$ is the non-accelerating inflation capacity utilization and $nairu_t$ is the non-accelerating inflation rate of unemployment

It is important to emphasize that α is not supposed to be constant, since there are evidences that α grew significantly during the last decade.

As stressed in Banco Central do Brasil Inflation Report (2003), "The potential output estimated from a production function (...) depends on the behavior of variables that are difficult to measure, such as capital stock and the amount of labor".

As an strategy to eliminate unnecessary data problems[2] and measuring errors generated on the estimation of capital stock[3] we define output gap as

$$
h_t = \ln\left(\frac{y_t}{\overline{y}_t}\right) \tag{14}
$$

and use equations 12 and 13 to assess the output gap and the potential output as

$$
\begin{aligned}
h_t &= \alpha_t\left[\ln(c_t) - \ln(naicu_t)\right] + (1-\alpha_t)\left[\ln(1-u_t) - \ln(1-nairu_t)\right] && (15)\\
\overline{y}_t &= \frac{y_t}{\exp(h_t)} = y_t\left(\frac{naicu_t}{c_t}\right)^{\alpha_t}\left(\frac{nairu_t}{u_t}\right)^{1-\alpha_t} && (16)
\end{aligned}
$$

The potential output and its unobserved components - nairu and naicu - are estimated by solving the following problem[4]:

[2] IBGE, which was in charge of measuring labor force, discontinued its series at December 2002. They created another one, based on a different methodology, that started at March 2002

[3] Morandi (2003) used the perpetual inventory method to measure capital stock at a yearly base. Alves and Muinhos (2003) runs a optimization process to create a quartely series for capital stock, using Morandi (2003) as benchmark.

[4] On the following problem Δ^2 represents the second centred difference. For instance, $\Delta^2 y_t = y_{t+1} - 2y_t + y_{t-1}$.

$$\min_{\{nairu_t\}_{t=1}^N,\{naicu_t\}_{t=1}^N,\{\alpha_t\}_{t=1}^N} \left\{ \begin{array}{c} \beta_1 \left[\sum_{t=1}^{N} (nairu_t - ru_t)^2 + \lambda \sum_{t=2}^{N-1} \left(\Delta^2 nairu_t\right)^2 \right] + \\ \beta_2 \left[\sum_{t=1}^{N} (naicu_t - cu_t)^2 + \lambda \sum_{t=2}^{N-1} \left(\Delta^2 naicu_t\right)^2 \right] + \\ \beta_3 \left[\lambda \sum_{t=2}^{N-1} \left(\Delta^2 \alpha_t\right)^2 \right] + \\ \beta_4 \left[\sum_{t=1}^{N} (\overline{y}_t - y_t)^2 + \lambda \sum_{t=2}^{N-1} \left(\Delta^2 \overline{y}_t\right)^2 \right] \end{array} \right\}$$

$$\begin{aligned} & s.t. \\ & \frac{1}{4}\sum_{j=1}^{4} \alpha_{4(T-1)+j} = \overline{\alpha}_T, \forall T \in \{1, ..., N/4\} \\ & 0 \leq \alpha_t \leq 1, \forall t \in \{1, ..., N\} \\ & \overline{y}_t = y_t \left(\frac{naicu_t}{c_t}\right)^{\alpha_t} \left(\frac{nairu_t}{u_t}\right)^{1-\alpha_t} \end{aligned} \tag{17}$$

It is worth noting that equation 16 appears as restriction of this problem. Without this restriction, the optimization defined in 17 would give the same solution as if the following four optimizations were solved separately

$$\min_{\{nairu_t\}_{t=1}^N} \left\{ \sum_{t=1}^{N} (nairu_t - u_t)^2 + \lambda \sum_{t=2}^{N-1} \left(\Delta^2 nairu_t\right)^2 \right\} \tag{18}$$

$$\min_{\{naicu_t\}_{t=1}^N} \left\{ \sum_{t=1}^{N} (naicu_t - c_t)^2 + \lambda \sum_{t=2}^{N-1} \left(\Delta^2 naicu_t\right)^2 \right\} \tag{19}$$

$$\min_{\{\overline{y}_t\}_{t=1}^N} \left\{ \sum_{t=1}^{N} (\overline{y}_t - y_t)^2 + \lambda \sum_{t=2}^{N-1} \left(\Delta^2 \overline{y}_t\right)^2 \right\} \tag{20}$$

$$\begin{aligned} & \min_{\{\alpha_t\}_{t=1}^N} \left\{ \sum_{t=2}^{N-1} \left(\Delta^2 \alpha_t\right)^2 \right\} \\ & s.t. \\ & \frac{1}{4}\sum_{j=1}^{4} \alpha_{4(T-1)+j} = \overline{\alpha}_T, \forall T \in \{1, .., N/4\} \\ & 0 \leq \alpha_t \leq 1, \forall t \in \{1, ..., N\} \end{aligned} \tag{21}$$

By definition[5], the optimizations presented in 18, 19 and 20 correspond to using the Hodrick-Prescott filter to the series $\{u_t\}_{t=1}^{N}$, $\{c_t\}_{t=1}^{N}$ and $\{y_t\}_{t=1}^{N}$ in order to extract their trend, respectively assigned as $\{nairu_t\}_{t=1}^{N}$, $\{naicu_t\}_{t=1}^{N}$ and $\{\overline{y}_t\}_{t=1}^{N}$.

The optimization presented in 21 was originally designed at Alves and Muinhos (2003). It aims to create an quarterly series, $\{\alpha_t\}_{t=1}^{N}$, using as benchmark an yearly series[6], $\{\overline{\alpha}_T\}_{T=1}^{N/4}$. As a restriction, it imposes that the average value of α_t during a given year T is $\overline{\alpha}_T$.

However, the results obtained when 16 is added is quite different from those generated when 18,19,20 and 21 are solved separately. It is easily seen that 18,19 and 20 does not allow the mean level of $\{nairu_t\}_{t=1}^{N}$, $\{naicu_t\}_{t=1}^{N}$ and $\{\overline{y}_t\}_{t=1}^{N}$ to be respectively different from the mean level of $\{u_t\}_{t=1}^{N}$, $\{c_t\}_{t=1}^{N}$ and $\{y_t\}_{t=1}^{N}$. This characteristic is extremely undesirable since it does not have any economic appeal[7]. Adding 16 corrects this problem.

Calibrating this model through quantifying the relative importance among problems - i.e., defining $\beta_{1,}$ $\beta_{2,}$ β_3 and β_4[8] - becomes an important task. As a initial adjust, we chose $\beta_1 = \frac{1}{\sigma_u^2}$, $\beta_2 = \frac{1}{\sigma_c^2}$, $\beta_3 = \frac{1}{0.25}$ and $\beta_4 = \frac{1}{\sigma_y^2}$ where σ_u^2, σ_c^2 and $\sigma_y^2$are respectively variances of the series $\{u_t\}_{t=1}^{N}$, $\{c_t\}_{t=1}^{N}$ and $\{y_t\}_{t=1}^{N}$. The idea behind this choice is to put all series at the same scale. In order to fully understand this idea, imagine that we had normalized the series $\{u_t\}_{t=1}^{N}$, creating the series $\{\widehat{u}_t\}_{t=1}^{N}$ that, being $\overline{u}$ and σ_{ru} respectively the mean and standard deviation of $\{u_t\}_{t=1}^{N}$, was given by:

.

$$\widehat{u}_t = \frac{u_t - \overline{u}}{\sigma_u}, \forall t$$

It is easily seen that solving

$$\min_{\left\{\widehat{nairu_t}\right\}_{t=1}^{N}} \left\{ \sum_{t=1}^{N} \left(\widehat{nairu}_t - \widehat{u}_t \right)^2 + \lambda \sum_{t=2}^{N-1} \left(\Delta^2 \widehat{nairu}_t \right)^2 \right\}$$

and estimating nairu by

$$nairu_t = \sigma_u . \widehat{nairu}_t + \overline{u}$$

[5] For further details about HP filtering, see Hodrick and Prescot (1997) and Araújo, Areosa and Rodrigues Neto (2003)

[6] 1-$\overline{\alpha}_t$ is estimated from the empirical average labor share obtained from national accounts (available at tab04.xls, extracted from sinoticas.zip at ftp://ftp.ibge.gov.br/Contas_Nacionais/Sistema_de_Contas_Nacionais/2000_2002/

[7] For instance, it is natural to think that during expantion (retraction) periods the output could stay sistematicaly above (under) the potential output.

[8] The problem would not change if we had normalized one of the coefficients (ex. $\beta_1 = 1$). We wrote the problem with four coefficient to be more explicit about our views.

is equivalent[9] to solving

$$\min_{\{nairu_t\}_{t=1}^N} \frac{1}{\sigma_{ru}^2} \left\{ \sum_{t=1}^{N} (nairu_t - u_t)^2 + \lambda \sum_{t=2}^{N-1} \left(\Delta_t^2 nairu_t\right)^2 \right\}$$

The same rationale can be used to explain β_2 and β_4.

Following the same idea, we would like to have $\beta_3 = \frac{1}{\sigma_\alpha^2}$, where σ_α^2 would be the variance of $\{\alpha_t\}_{t=1}^N$. However we do not have its variance, since $\{\alpha_t\}_{t=1}^N$ is one of the quarterly series that we want to estimate. Using the variance of $\{\overline{\alpha}_T\}_{T=1}^{N/4}$is not a good idea because it may underestimate the "real" value, once it does not take into consideration movements that occurs during an given year. However, we know that each value of α_t is between 0 and 1 and, consequently, its variance is between 0 and 0.25. Thereafter choosing $\sigma_\alpha^2 = 0.25$ can be interpreted as being the less restrictive about variability on $\{\alpha_t\}_{t=1}^N$.

Although we have used $\beta_1 = \frac{1}{\sigma_u^2}$, $\beta_2 = \frac{1}{\sigma_c^2}$, $\beta_3 = \frac{1}{0.25}$ and $\beta_4 = \frac{1}{\sigma_y^2}$ as our first set of parameters, we have run the optimization 17 with many other sets. Two sets were chosen to represent the obtained results. They emphasize the importance of calibrating this model, since the results generated are considerably different.

	β_1	β_2	β_3	β_4
Set 1	9.347	9.159	4	0.056
Set 2	9.347	9.159	12	0.056

Nevertheless, a more important issue must be considered. If we had an quarterly series for $\overline{\alpha}$, we would not need to built the series $\{\alpha_t\}_{t=1}^N$. Thereafter the problem described in 17 could be reduced to

$$\min_{\{nairu_t\}_{t=1}^N, \{naicu_t\}_{t=1}^N, \{\alpha_t\}_{t=1}^N} \left\{ \begin{array}{c} \beta_1 \left[\sum_{t=1}^{N} (nairu_t - ru_t)^2 + \lambda \sum_{t=2}^{N-1} \left(\Delta^2 nairu_t\right)^2 \right] + \\ \beta_2 \left[\sum_{t=1}^{N} (naicu_t - cu_t)^2 + \lambda \sum_{t=2}^{N-1} \left(\Delta^2 naicu_t\right)^2 \right] + \\ \beta_4 \left[\sum_{t=1}^{N} (\overline{y}_t - y_t)^2 + \lambda \sum_{t=2}^{N-1} \left(\Delta^2 \overline{y}_t\right)^2 \right] \end{array} \right\}$$

$$s.t. \qquad (22)$$

$$\overline{y}_t = y_t \left(\frac{naicu_t}{c_t}\right)^{\alpha_t} \left(\frac{nairu_t}{u_t}\right)^{1-\alpha_t}$$

[9] Equivalence in this context means that, not only it gives the same solution, but also the same value for the minimum.

In order to identify the effect of this change, we have chosen to run 22 assuming that α_t remained constant during every given year T, that is

$$\alpha_{4(T-1)+1} = \alpha_{4(T-1)+2} = \alpha_{4(T-1)+3} = \alpha_{4(T-1)+4} = \overline{\alpha}_T, \forall T \in \{1, .., N/4\}$$

3 Data

The following set of time-series (1995 to 2003) was used in our exercises, which from now on will be represented by the letter in brackets.

- Gross Domestic Product – GDP (y_t); Fundação Instituto Brasileiro de Geografia e Estatística (IBGE) - the official statistics bureau - is in charge of measuring this quarterly series.

- Unemployment Rate (u_t); IBGE produces the unemployment series that embraces the widest survey region. Nevertheless, IBGE has recently changed its measuring methodology, discontinuing its original series at December 2002 and starting another one at March 2002. In this paper, we extended this old series supposing that all variation occurred in the new series would also occur in the old one. We changed a monthly series into a quarterly one by considering the mean value of the observed values that occurred during each quarter.

- Capital Utilization Rate (c_t); Conselho Nacional da Indústria (CNI) produces this monthly series. We changed this series into a quarterly one by considering the mean value of the observed values that occurred during each quarter.

- Income Capital Share (α_t); We estimated an yearly series based on the empirical labor share obtained from national accounts (available at tab04.xls, extracted from sinoticas.zip at ftp://ftp.ibge.gov.br/Contas_Nacionais/Sistema_de_Contas_Nacionais/2000_2002).

- Domestic Consumer Price Index Inflation Rate (π_t); IBGE is in charge of measuring the IPCA - the official consumer price index used in the inflation targeting regime.

- Free Prices Inflation Rate (π_t^f); This series is obtained excluding the administrated price items from the IPCA headline index.

- Expected Free prices Inflation($E_t[\pi_{t+1}^f]$); Since we did not have a long series for this variable, we chose to build according to two different approaches. One of them considered that agents could perfectly forecast inflation. On the other approach, an error was added to simulate FOCUS[10] series.

[10] Survey group from Banco Central do Brasil

Next we briefly discuss the Phillips Curve specification used to compare the output gaps generated by each detrending method.

4 Comparative Analysis

The models presented in section 2 were estimated using full sample in order to generate potential output. Figure 1, presented in appendix 1, shows these estimations which can be classified in two groups: low and high variance trend. The former encompasses Watson, Hodrick-Prescott and deterministic trends. As we can notice in Figure 1 , when we used the observed values (HPCPF2a) instead of estimating a quarterly series for the income capital share (HPCPF2), the potential output becomes much more volatile.

After estimating potential output through different methodologies, we assessed the output gap using equation 14. In order to compare the output gaps generated from different methodologies, a comparative measure should be established. Following Proietti, Musso and Westermann (2002), we compare the proposed models on the basis of their accuracy in forecasting. If output gap represents a measure of inflation pressures, we expect it to increase accuracy in our inflation forecasts.

Analyzing the predictive power of each output gap series is a complex task. First of all, we should estimate a plain Phillips curve that captures the trade-off between inflation and unemployment. Based on Bodanski, Tombini and Werlang (2000), we have chosen the following specification:

$$\pi_t^f = \alpha_1 \pi_{t-1} + \alpha_2 E_t[\pi_{t+1}] + \alpha_3 h_{t-2} + \varepsilon_t, \tag{23}$$

where π_t^f is the log of free prices inflation, π_t is the log of headline inflation, $E_t[\pi_{t+1}]$ is the conditional expected value of inflation and h_t is the output gap.

It is important to highlight that the dependent variable in this equation is the free price inflation. This specification is in line with Banco Central do Brasil Inflation Report (2003) and it can be understood considering that a relevant part of inflation is due to administrated prices and consequently does not respond to monetary policy. It is also important to note that expected inflation takes an important part on this specification.

We used a rolling forecast experiment as a test of forecast accuracy as proposed in Proietti, Musso and Westermann (2002). For each output gap series, we estimated the Phillips curve changing the sample from 1995:1 - 2001:4 to 1995:1 - 2003:4 and, each time, forecasting up to 4 steps ahead. The mean square error of these forecasts was chosen as a measure of accuracy for each model.

Figure 2 shows the observed and forecasted free prices inflation presented in Table 2. Several features are readily apparent. First, the forecasts produced by the Local level and Watson models are even more inaccurate than those generated by the simplest univariate models. Second, the deterministic trend, moving average, Hodrick-Prescott, Beveridge-Nelson and production function models have strong short-term comovements, appearing to be moving upward and downward at roughly the same time, although the amount of these moves vary from one method to another. Third, despite having similar short-term movements, the different methods typically give rise to a wide range of different estimates of free prices inflation. The differences between highest and lowest estimates is 1.09 % in 2002:4 and 0.53 in 2003:4.

Table 1 - Mean Square Error Results

	1step	2 steps	3steps	4 steps
Deterministic Trend	0.000048	0.000040	0.000037	0.000042
Moving Average	0.000047	0.000039	0.000039	0.000041
Hodrick-Prescott	0.000050	0.000048	0.000048	0.000049
Beveridge-Nelson	0.000032	0.000034	0.000034	0.000038
Local Level Model	0.000162	0.000165	0.000155	0.000187
Watson(1986)	0.000138	0.000156	0.000147	0.000149
HPPF1	0.000062	0.000056	0.000060	0.000075
HPCPF2	0.000053	0.000054	0.000057	0.000059
HPCPF2a	0.000066	0.000075	0.000092	0.000101

HPCPF stands for Hodrick-Prescott filter constrained to a Production Function. The number 1 and 2 refers to the parameter set used. In HPCPF2a we used the observed income capital share series, as described in equation 23.

Table 1 reports the mean square forecast error relative to the free prices inflation, resulting from the rolling forecast of the equation (23), the target variable being the quarterly inflation rate of the free prices. The main evidence is that the Beveridge-Nelson methodology outperforms all the models at all forecast horizons.

Table 2 - Observed and forecasted free price inflation

	Quarter								
	2002.1	2002.2	2002.3	2002.4	2003.1	2003.2	2003.3	2003.4	2004.1
$\log(\pi^f)$	**1.58**	**0.62**	**2.53**	**6.15**	**4.02**	**1.60**	**0.64**	**1.25**	**2.16**
Deterministic Trend	1.67	1.27	1.60	6.42	4.80	0.71	1.13	0.10	1.99
Moving Average	1.58	1.16	1.48	6.32	4.79	0.65	1.03	0.18	2.01
Hodrick-Prescott	1.61	1.19	1.35	5.78	4.66	0.77	1.15	0.19	1.74
Beveridge-Nelson	1.58	1.19	1.80	6.19	4.45	0.93	0.95	0.15	1.80
Local Level Model	1.20	1.08	2.24	3.00	2.23	1.73	0.46	1.16	1.19
Watson(1986)	1.43	1.18	3.11	4.00	2.25	0.89	0.08	0.67	0.48
HPPF1	1.36	1.32	1.48	5.42	4.59	0.88	1.48	0.05	1.53
HPCPF2	1.55	1.17	1.35	5.57	4.61	0.79	1.22	0.16	1.69
HPCPF2a	1.31	1.22	1.39	5.33	4.50	0.78	1.38	-0.03	1.47

HPCPF stands for Hodrick-Prescott filter constrained to a Production Function. The number 1 and 2 refers to the parameter set used. In HPCPF2a we used the observed income capital share series, as described in equation 23.

5 Conclusion

In this paper we have calculated different measures of potential output and output gap for the Brazilian economy, using different models largely used in the literature. In order to compare the output gaps generated from different methodologies, we used a forward-looking Phillips curve and a rolling forecast experiment as a test of forecast accuracy.

Our findings are the following. The potential output estimated by the different models can be classified in two groups: low and high variance trend. The forecasts produced by the unobserved components models are even more inaccurate than those generated by the simplest univariate models. The deterministic trend, moving average, Hodrick-Prescott, Beveridge-Nelson and production function models have strong short-term comovements, appearing to be moving upward and downward at roughly the same time. The main evidence is that the Beveridge-Nelson methodology outperforms all the models at all forecast horizons.

References

[1] Alves, S. A. L. (2001), "Evaluation of the Central Bank of Brazil Structural Model's Inflation Forecasts in an Inflation Targeting Framework," Banco Central do Brasil Working Paper Series n. 16, www.bcb.gov.br

[2] Alves, S. A. L.and M. K. Muinhos (2003), "Medium-Size Macroeconomic Model for the Brazilian Economy," Banco Central do Brasil Working Paper Series, n. 64, www.bcb.gov.br.

[3] Araújo, F., M. B. M. Areosa and J. A. Rodrigues Neto, "r-filters: a Hodrick and Prescott Filter Generalization", Banco Central do Brasil Working Paper Series, n. 69, www.bcb.gov.br.

[4] Arnold, R. (2004), "A Summary of Alternative Methods for Estimating Potential GDP," CBO Paper.

[5] Bernanke, B. (2003), "Constrained Discretion and Monetary Policy," http://www.federalreserve.gov.

[6] Beveridge, S. and C. R. Nelson (1981), "A New Approach to Decomposition of Economic Time Series into Permanent and Transitory Components with Particular Attention to Measurement of 'Business Cycle'," Journal of Monetary Economics, 7, 151-174.

[7] Blinder, A. S. (1997), "Distinguished Lecture on Economics in Government: What Central Bankers Could Learn from Academics and Vice-versa," The Journal of Economic Perspective, Vol. 11, n 2, 3-19.

[8] Bogdanski, J., A. A. Tombini andS. R. C. Werlang, (2000), "Implementing Inflation Targeting in Brazil," Banco Central do Brasil Working Paper Series n. 1, www.bcb.gov.br.

[9] Bryan, M. F. and S. T. Cecchetti, (2001), "A Note on the Efficient Estimation of Inflation in Brazil," Banco Central do Brasil Working Paper Series n. 11, www.bcb.gov.br.

[10] Cerra, V. and S.C. Saxena (2000), "Alternative Methods of Estimating Potential Output and the Output Gap: An Application to Sweden," IMF Working Paper, 00/59.

[11] Claus, I (1999), "Estimating Potential Output for New Zealand: a Structural VAR Approach," Reserve Bank of New Zealand, Discussion Paper Series.

[12] Doménech, R. and V. Gómez (2003), "Estimating Potential Output, Core Inflation and the NAIRU as Latent Variables," mimeo University of Valencia.

[13] Ehrmann, M. and F. Smets (2003), "Uncertain Potential Output: Implications for Monetary Policy," Journal of Economic Dynamics and Control, 27, 1611-1638.

[14] Hodrick, R. and E. Prescott (1997), "Post-war Business Cycles: An Empirical Investigation," Journal of Money,Credit and Banking, 29, 1-16.

[15] Banco Central do Brasil Inflation Report, December 2003, p. 117, www.bcb.gov.br.

[16] Kichian, M. (1999), "Measuring Potential Output within a State-Space Framework," Bank of Canada Working Paper, 99-9.

[17] Mishkin, F. S. and K. Schmidt-Hebbel (2001), "One Decade of Inflation Targeting in the World: What Do We Know and What Do We Need to Know?," National Bureau of Economic Research Working Paper n. 8397 (July).

[18] Morandi, L. (2003), "Estimativa do estoque de capital fixo Brasil - 1950/2000," Rio de Janeiro: IPEA / Dimac, 2003. (Working paper, no prelo).

[19] Orphanides, A. and S. van Norden (1999), "The Reliability of Putput Gap Estimates in Real Time," Board of Governors of the Federal Reserve System.

[20] Proietti, T., A. Musso and T. Westermann (2002), "Estimating Potential Output and the Output Gap for the Euro Area: a Model-Based Production Function Approach," EUI Working Paper ECO No. 2002/9.

[21] Rennison, A. (2003) "Comparing Alternative Output Gap Estimators: a Monte Carlo Approach," Bank of Canada Working Paper, 03-8.

[22] Silva Filho, T. N. (2001), "Estimating Brazilian Potential Output: A Production Function Approach," Banco Central do Brasil Working Paper Series, n. 17, www.bcb.gov.br.

[23] St-Amant, P. and S. van Norden (1997), "Measurement of the Output Gap: A Discussion of Recent Research at the Bank of Canada", Technical Report n. 79, Bank of Canada.

[24] Taylor, J. B. (1993), "Discretion versus Police Rules in Prectice," Carnegie-Rochester Conference Series on Public Police, 39, December, 195-214.

[25] Watson, M. W. (1986), "Univariate Detrending Methods with Stochastic Trends," Journal of Monetary Economics, 18, 49-75.

Appendix 1 - Figures

Figure 1 - Potential output estimated by different methodologies

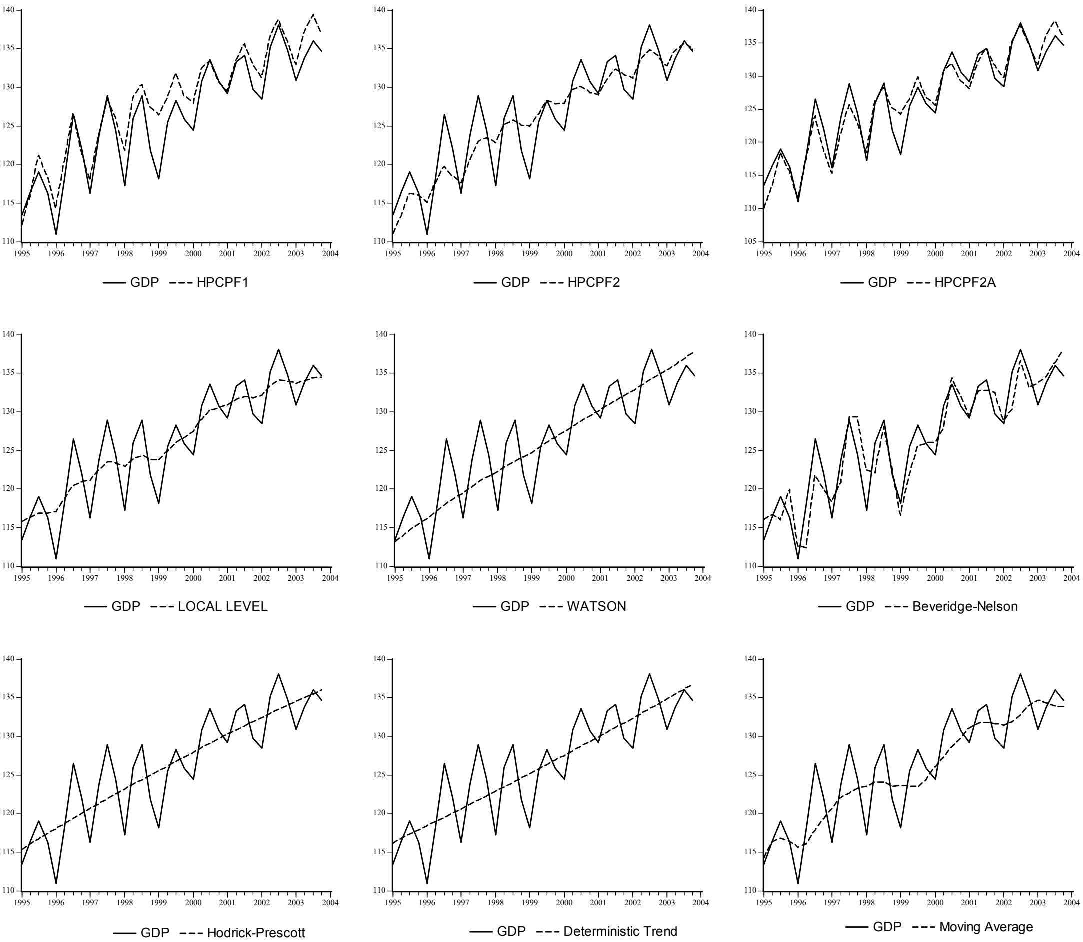

Figure 2 - Free prices inflation forecast