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A Simple Model of Demand-Led Growth and Income Distribution

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Abstract: this paper presents a one-sector demand-led model where capital and non-capital expenditures determine income growth and distribution. The basic idea is to build a simple dynamical accounting model for the growth rate of the capital stock, the ratio of non-capital expenditures to the capital stock, and the labor share of income. By inserting some stylized behavioral functions in the identities, the paper analyzes the implications of alternative theoretical closures of income determination (effective demand) and distribution (social conflict). On the demand side, two behavioral functions define the growth rates of capital and non-capital expenditures as functions of capacity utilization (measured by the output-capital ratio) and income distribution (measured by the labor share of income). On the distribution side, another two behavioral functions describe the growth rates of the real wage and labor productivity also as functions of capacity utilization and income distribution. The growth rates of total factor productivity and employment follow residually from the accounting identities and, in this way, the demand-led model can encompass supply-driven models as a special case.

Sumário: este artigo apresenta um modelo de um setor onde os gastos correntes e de capital determinam o crescimento e distribuição da renda. A idéia básica é construir um modelo contábil simples e dinâmico para a taxa de crescimento do estoque de capital, a razão entre gastos correntes e estoque de capital e a parcela salarial da renda. Mediante a inserção de algumas funções comportamentais estilizadas nas identidades contábeis, o artigo analisa as implicações de fechamentos teóricos alternativos para a determinação (demanda efetiva) e distribuição (conflito social) da renda. Do lado da demanda, duas equações definem as taxas de crescimento das despesas correntes e de capital como funções do grau de utilização da capacidade produtiva (medida pela relação renda-capital) e distribuição de renda (medida pela parcela salarial da renda). Do lado da distribuição, outras duas equações descrevem as taxas de crescimento do salário real e da produtividade do trabalho também como funções do grau de utilização da capacidade produtiva e da distribuição de renda. A taxa de crescimento do emprego e da produtividade total dos fatores são obtidas residualmente das identidades contábeis e, desta forma, o modelo liderado pela demanda pode incluir modelos liderados pela oferta como casos especiais.

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> "One of the major weakness in the core of macroeconomics as I represented it is the lack of real coupling between the short-run picture and the long-run picture." Robert Solow (1997, p.231).

1 – Introduction

Modern macroeconomic theory has a strange way to deal with economic growth. When analyzing short-run issues, most economists tend to explain income variations in terms of changes in aggregate demand. When dealing with long-run issues, the focus changes to aggregate supply and the analysis shifts to the determinants of potential output in some sort of growth accounting based on the Solow-Swan model. Exactly how effective and potential income levels converge in the long run is not usually stated clearly in supply-driven growth models. Instead, it is usually assumed that, either because of government intervention or because of the self-adjusting nature of market forces, capitalist economies tend to operate at their potential income level in the long run. If so, one can then understand growth just from the supply side and effective demand vanishes from long-run macroeconomic theory.

Independently of the importance of supply issues, the emphasis of modern growth theory on potential output tends to ignore the fact that capitalist economies may stay below their maximum output for long periods of time. Even if one accepts Say's law and assume that effective demand does converge to potential output in the long run, the adjusting period may be long enough to make a demand-led growth theory worthy for medium-run macroeconomics. In the words of Solow (1997, p.230): "(…) what about those fluctuations around the trend of potential output? (…) In my picture of the usable core of macroeconomics, those fluctuations are predominantly driven by aggregate demand impulses and the appropriate vehicle for analyzing them is some model of the various sources of expenditures." If one rejects Say's law and assumes instead that it is potential output that converges to effective demand in the long run, the need for a demand-led growth theory becomes even more obvious.

The demand determination of income is a point usually emphasized by post Keynesian and structuralist economists. Building upon the works of Keynes (1936) and Kalecki (1971), these economists tend to analyze growth in terms of the dynamics of autonomous expenditures under the assumption that potential output itself may be demand-driven. The basic idea is that effective demand may determine the growth rate of potential output through its effects on the capital stock and multifactor productivity.[1] If income growth is mainly demand-driven, the focus of the analysis shifts to the determinants of effective demand. In the post Keynesian and structuralist literature, the usual suspects are income distribution, macroeconomic policy, and the autonomous demand coming from the private or the foreign sectors.[2] The structure of the

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[1] Effective demand is also assumed to influence the labor supply through changes in the labor-force participation rate. However, because this rate cannot obviously be higher than 100%, this transmission mechanism from demand to labor supply is limited without migration.

[2] For methodological discussion of the role of aggregate demand and supply in growth theory, see Leon-Ledesma and Thirlwall (2000), Panico (2003), and Solow (1997 and 2000). For a survey of Keynesian demand-led growth models, see, for instance, Commendatore *et alli* (2003).

models vary according to which source of demand is supposed to drive income and this tends to be an obstacle for the wider use of such models in applied macroeconomics. Unlike supply-driven models, demand-led models are not usually defined in terms of a common growth-accounting expression. The result is an apparent inconsistency between the alternative models even though the theories behind them share a common view about the importance of effective demand.

The objective of this paper is to present a simple dynamical-accounting model that summarizes most of the topics emphasized by demand-led growth theory. More formally, the objective is to expand the 2x2 dynamical-accounting model proposed by Barbosa-Filho (2003) to include the functional income distribution between wages and profits as an endogenous variable. The result is a 3x3 dynamical model for the growth rate of the capital stock, the ratio of non-capital expenditures to the capital stock, and the labor share of income. Following the structuralist approach of Taylor (1991 and 2004), the dynamics of these variables are assumed to depend on effective demand, technology and the social conflict between workers and capitalists. The result is a simple and flexible model that can be closed in many different ways depending on how the global rate of capacity utilization responds to income distribution and vice versa.

The text is organized in six sections in addition to this introduction. Section two outlines the basic structure of the model in continuous time. Section three discusses the possible assumptions about the partial derivatives of the model. Based on a structuralist set of assumptions, section four analyzes the stability of the steady state of the model and section five discusses the impact of exogenous shocks to such a steady state. Section six presents the model in discrete time and simulates the impact of an exogenous increase in the growth rate of non-capital expenditures on its endogenous variables. Section seven concludes the analysis with a summary of the main results of the model.

2 - The model in continuous time

Consider a one-sector economy and let Q represent its real GDP. By definition:

$$Q = F + A, \tag{1}$$

where F represents capital expenditures (investment in fixed capital) and A all other non-capital expenditures (private and government consumption plus net exports). Barbosa-Filho (2003) divided GDP in three demand categories: investment, consumption induced by income, and all other expenditures. However, because potential output is usually assumed to be proportional to the capital stock in post Keynesian and structuralist models, it is better to work with just two categories to obtain a more parsimonious representation of demand-led growth.

To keep the model as simple as possible, assume that there is no capital depreciation and divide (1) by the capital stock K, that is:

$$u = k + z, \tag{2}$$

where u is the output-capital ratio, k the growth rate of the capital stock and z the ratio of non-capital expenditures to the capital stock.[3] The change in k and z are given by

$$\dot{k} = k(f - k) \tag{3}$$

and

[3] A constant rate of capital depreciation can be introduced in the model without major changes in its theoretical interpretation.

$$\dot{z} = z(a - k) ; \tag{4}$$

where f and a represent respectively the exponential growth rates of capital and non-capital expenditures.

Next, assume that the growth rates of capital and non-capital expenditures can be modeled as functions of capacity utilization (measured by the output-capital ratio u) and income distribution (measured by the labor share of national income l). As we will see in the next section, the basic idea is that effective demand depends on the level of economic activity and on the social conflict between capital and labor.[4] For the moment let

$$f = f(u,l) \tag{5}$$

and

$$a = a(u,l) . \tag{6}$$

Given the labor share l and since u=k+z, by substituting (5) in (3) and (6) in (4) we obtain a 2x2 dynamical system that represents demand-led growth on the $k{\times}z$ plane.[5] To see this, let q be the exponential growth rate of GDP, by definition

$$q = \left(\frac{k}{k+z}\right) f + \left(\frac{z}{k+z}\right) a . \tag{7}$$

In words, the growth rate of income is a weighted average of the growth rates of capital and non-capital expenditures.

What if the labor share changes? To introduce the dynamics of income distribution into the analysis, assume that national income can be expressed as a constant proportion of real GDP.[6] Then, from the national income and product accounts we have

$$\phi Q = WN + RK , \tag{8}$$

where ϕ is the ratio of national income to GDP, W the real wage, N the employment index associated with W, and R the real rental price (or user cost) of capital. Since $l=WN/\phi Q$ and based on the assumption that ϕ is constant we have

$$q = l(w+n) + (1-l)(r+k) , \tag{9}$$

where naturally w, n and r are respectively the exponential growth rates of W, N and R.

From the assumption that ϕ is constant we can also define the change in the labor share simply as

$$\dot{l} = l(w-b) , \tag{10}$$

where b is the exponential growth rate of labor productivity.

[4] In post Keynesian and structuralist models the economy does not necessarily operates at full capacity or full employment because of imperfect competition and the social conflict between workers and capitalists. The basic assumption is that changes in excess capacity are an important instrument for large firms to deter the entry of new firms into their markets and, what is most important, to discipline workers' real-wage claims. For a survey of structuralist and post Keynesian economics, see, respectively, Taylor (1991) and Lavoie (1996).

[5] See Barbosa-Filho (2003) for the possible closures of this 2x2 model.

[6] Recall that the gross national income equals the GDP minus net indirect taxes plus net income received from abroad. For simplicity I assume that the latter two variables are a constant component of the GDP, so that we can concentrate the analysis on the conflict between capital and labor.

By analogy with our previous assumptions about effective demand, assume that the growth rates of the real wage and labor productivity can also be modeled as functions of capacity utilization and income distribution, that is,

$$w = w(u,l) \tag{11}$$

and

$$b = b(u,l)\,. \tag{12}$$

Then, to obtain the joint dynamics of k, z, and l, just substitute (11) and (12) into (10) and combine the resulting differential equation with (3) and (4). The result is a 3x3 dynamical system of demand-led growth and income distribution, that is:

$$\begin{cases} \dot{k} = k[f(k,z,l) - k] \\ \dot{z} = z[a(k,z,l) - k] \\ \dot{l} = l[w(k,z,l) - b(k,z,l)] \end{cases} .$$

In economic terms the intuition is that the solution of this dynamical system determines the pace of capital accumulation (k), the composition of aggregate demand (z) and the distribution of income (l) as a function of time and some initial conditions. From this solution we can then obtain the output-capital ratio (u) and the growth rates of capital expenditures (f), non-capital expenditures (a), income (q), real wage (w), and labor productivity (b). The growth rate of employment (n) follows residually from

$$n = y - b\,, \tag{13}$$

and, in a similar way, the growth rate of the rental price of capital (r) follows residually from (9).

Before we proceed to the structuralist theory behind the dynamical system it is worthy to stop and link demand-led growth with supply-side growth accounting. Because of the supply-driven nature of mainstream growth theory, it would be useful if the demand-led system could also be translated in terms of multifactor productivity. To do so let m be the exponential growth rate of the latter. From (9)

$$m = lw + (1-l)r\,, \tag{14}$$

that is, the "Solow" residual can also be derived from the demand-led system.[7]

In summary, the model of demand-led growth and income distribution consists of three differential equations (equations 3, 4 and 10), four behavioral functions (equations 5, 6, 11 and 12), and five accounting identities (equations 2, 7, 9, 13 and 14). Altogether we have twelve equations that, in principle, can be solved for twelve variables (k, z, l, f, a, w, b, u, q, n, r, and m). In fact, if we focus on the non-trivial solution of (3), (4) and (10), the three differential equations give us three equilibrium conditions: $f=k$, $a=k$ and $w=b$. These conditions can then be combined with the remaining nine equations to form a non-linear system of simultaneous equations for the twelve variables involved. We will return to this point after we analyze the stability of the system.

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[7] Note that the growth rate of multifactor productivity is also a weighted average of the growth rates of the labor and capital average products, that is, $m=l(q-n)+(1-l)(q-k)$. For an analysis of the economic theory and accounting identities behind growth accounting, see, for instance Felipe and Fisher (2003).

For the moment it should be noted that the behavioral functions are the theoretical and analytical core of the model. These functions can be "closed" in many different ways as proposed, for instance, by Sen (1963), Marglin (1984), Dutt (1990), Taylor (1991) and Foley and Michl (1999). Given the choice of theoretical closure, the three differential equations dictate the demand-led dynamics of the three state variables, and the behavioral functions and accounting identities translate these dynamics in terms of the remaining nine variables.[8]

It is also important to point out that, for the system to be completely demand-led, it is obviously necessary for output to be below its maximum value. In the labor market the implicit assumption is that the growth rate of employment is not limited from the supply side because of, for instance, disguised unemployment in a non-capitalist sector of the economy or migration.[9] In the same vein, in the capital market the implicit assumption is that the output-capital ratio is below its "full-capacity" level. Altogether these two assumptions represent the old classical idea that capital is the scarce factor in capitalist economies.[10]

In relation to the mainstream and non-mainstream literature on the topic, the demand-led system presented above is a simple, flexible and parsimonious way to emphasize the central role of effective demand and income distribution in the dynamics of capitalist economies. Moreover, by determining the growth rate of multifactor productivity, the demand-led system can also encompass supply-side models without ignoring demand dynamics. In fact, because the system is built around accounting identities, it can be easily expanded to include other factors, provided that we include the candidate variables as inputs to the behavioral functions.[11] The price is that complexity increases geometrically with the addition of new variables and equations. Fortunately we do not have to expand the system much to obtain interesting results, the simplified version outlined above already gives us a wide range of results.

3 - The alternative closures of growth and distribution

We have to specify the partial derivatives of the four behavioral functions at the center of the investigation to analyze the impact of alternative economic hypotheses on growth and distribution. The simplest approach is to assume that the 3x3 dynamical system has at least

[8] Following Dutt (1990), the neoclassical closure corresponds to the case where u and m are given, so that we have to drop the demand functions f and a. In the Marxian closure u and w are given and, therefore, we also have to drop the demand functions. In the neo-Keynesian closure u is given and we have to drop one of the demand functions (usually a). Dutt's "Kalecki-Steindl" closure corresponds to the post Keynesian structuralist closure analyzed in the next sections.

[9] The basic idea comes from Lewis's (1954) model, where employment in the non-capitalist sector of the economy varies according to the demand for labor from the capitalist sector. For a more modern version applicable to developing countries see, for instance, Taylor (1979).

[10] Let $umax$ be the maximum output-capital ratio. The fact that $umax \geq k+z$ imposes a constraint on the possible values of k and z. As long as the system remains below such an upper bound, growth and income distribution can be completely determined by the four behavioral functions outlined in the text. It should be noted that the very own fluctuations of capacity utilization and income distribution may lead to changes in the maximum capital productivity. For an analysis of the correlation between capital and labor productivities, see Foley and Marquetti (1997).

[11] For instance, to separate the dynamics of consumption, exports and imports, we can divide non-capital expenditures into these three components and work with three new behavioral functions (one for each new demand category) instead of just one. Formally, the differential equation for z would have to be replaced by three differential equations, one for each new demand category normalized by the capital stock. The causality would still be the same, from effective demand to income, but the complexity would be obviously much higher.

one nontrivial equilibrium point and take a linear approximation of the behavioral functions about such a point.[12] Formally, let

$$f(u,l) = f_0 + f_u(k+z) + f_l l\,, \tag{15}$$

$$a(u,l) = a_0 + a_u(k+z) + a_l l\,, \tag{16}$$

$$w(u,l) = w_0 + w_u(k+z) + w_l l\,, \tag{17}$$

and

$$b(u,l) = b_0 + b_u(k+z) + b_l l\,. \tag{18}$$

In each function the intercept coefficient is meant to represent the fixed effects of other variables than capacity utilization and the labor share, that is, the intercept coefficients are the shift parameters through which exogenous shocks enter in the analysis.

The usual "accelerator" assumption implies that investment is a positive function of capacity utilization because, given the labor share of income, an increase in the output-capital ratio leads to a higher rate of profit ($f_u > 0$).[13] In post Keynesian and structuralist models the transmission mechanism usually involves the positive impact of current profits on expected profits, as well as the reduction of the liquidity constraint on investment brought by higher profits. By analogy the labor share is assumed to have a negative impact on investment because, given the output-capital ratio, an increase in the labor share reduces the rate of profit ($f_l < 0$).

The response of non capital expenditures to capacity utilization is not as straightforward as the accelerator hypothesis about investment. On the one hand, an increase in capacity utilization is usually accompanied by a reduction of net exports. On the other hand, an increase capacity utilization may lead to an increase in consumption because of the possible reduction in the unemployment rate associated with it. Depending on what effect is higher, the growth rate of non-capital expenditures may be either pro or counter-cyclical. Given that government expenditures also enter in non-capital expenditures and fiscal policy tends to be an automatic stabilizer, let us assume that the positive impact capacity utilization may have on the growth rate of private consumption is more than compensated by its negative impact on the growth rates of net exports and government consumption ($a_u < 0$).

The response of non-capital expenditures to income distribution is also not clear a priori. On the one hand, an increase in the labor share tends to reduce the international competitiveness of the economy and, therefore, to reduce its net exports. On the other hand, an increase in the labor share tends to increase consumption if the propensity to consume out of wages is greater than the propensity to consume out of profits. The response of government expenditures to changes in the labor share is not clear and, to simplify the analysis, let us follow the post Keynesian and structuralist tradition and assume that, because of a large difference between the propensities to consume out of wages and profits, the positive effect predominates over the negative effect, so that an increase in the labor share accelerates the growth of non-capital expenditures ($a_l > 0$).

[12] By non-trivial I mean a point where the three state variables are positive.

[13] Let ρ be the rate of profit, by definition $\rho=[1-(l/\phi)]u$.

The "reserve army" or "wage-curve" assumption implies that the growth rate of the real wage is a positive function of the level of economic activity because workers' bargaining power varies pro-cyclically.[14] The basic idea is that an increase in the income-capital ratio is accompanied by a reduction in the rate of unemployment and, through this, it allows workers to demand and obtain higher real wages ($w_u > 0$). In contrast, the impact of the labor share on the real wage is not clear because workers usually state their claims in terms of a real-wage target instead of a labor-share target. However, if we assume that the workers' real-wage target is a positive function of labor productivity, then a low labor share means that the effective real wage is too low in relation to such a target, which in its turn leads to an increase in the workers' claims on income. By analogy the opposite happens when the labor share is high and, therefore, the growth rate of the real wage tends to be a negative function of the labor share ($w_l < 0$).

The impact of capacity utilization on the growth rate of labor productivity is a controversial topic in mainstream and non-mainstream growth models. Restricting our analysis to post Keynesian and structuralist models, the main issue in debate is the impact of labor hoarding and scale economies through the business cycle. In general labor- productivity growth accelerates at the beginning of an upswing as firms increase output without hiring new employees. Then, as the expansion proceeds and new workers are hired, labor-productivity growth slows down and, when the economy moves into a recession, it may even become negative because firms do not immediately adjust their labor demand to the reduction in output. Thus, depending on the phase the cycle, the growth rate of labor productivity can be either pro or counter-cyclical. The impact of scale economies is similar to labor hoarding, that is, scale economies are usually more intense in the beginning of an upswing, when there is plenty of idle capacity to be used.[15] As the economy grows the intensity of scale economies tends to diminish, which in its turn slows down labor-productivity growth. We will return to this point when analyzing the alternative hypotheses about the impact of effective demand on income distribution.

The last parameter to be considered is the impact of income distribution on the growth rate of labor productivity. By analogy with our previous assumption about the labor share and the workers' real-wage target, let us assume that firms adjust their investment in labor-augmenting innovations according to the discrepancy between the real wage and labor productivity. A high labor share means a high labor cost and, therefore, an incentive for firms to increase labor productivity. The growth rate of labor productivity tends therefore to be a positive function of the labor share ($b_l > 0$).

In order to translate the above assumptions in terms of the dynamics of capacity utilization and income distribution, consider the linear approximation of the 3x3 dynamical system of the previous section about the nontrivial equilibrium point (k^*, z^*, l^*).[16] The Jacobian matrix of the system about this point is

[14] For a modern and new Keynesian version of the reserve-army assumption see Blanchflower and Oswald (1995).
[15] In other words, the unit cost is usually a concave-up function on the output×unit cost plane.
[16] Note that the linear behavioral functions imply that there are eight possible stationary points in the $k \times z \times l$ space, but only in one them the three state variables can be different than zero.

$$J = \begin{bmatrix} k^*(f_u - 1) & k^* f_u & k^* f_l \\ z^*(a_u - 1) & z^* a_u & z^* a_l \\ l^*(w_u - b_u) & l^*(w_u - b_u) & l^*(w_l - b_l) \end{bmatrix}. \tag{19}$$

Next, to analyze the impact of the labor share on capacity utilization, note that about the equilibrium point we have

$$\frac{\partial q^*}{\partial l^*} = \frac{k^* f_l + z^* a_l}{u^*}.$$

The economy is considered "wage-led" when this derivative is positive, that is, when the positive impact of the labor share on the growth rate of non-capital expenditures more than offsets its negative impact on growth rate of investment. In other words, in a wage-led economy an increase in the labor share leads to an increase in the growth rate of income so that, given the growth rate of the capital stock, the increase in the labor share also results in an increase in the growth rate of capacity utilization. In contrast, the economy is considered "profit-led" when the opposite happens.[17]

The off-diagonal elements of the third column of J determine whether the system is wage or profit led. Not surprisingly, the off-diagonal entries of the third line of J determine the impact of capacity utilization on the labor share. Following the taxonomy proposed by Barbosa-Filho (2001), the economy is considered "Marxian" when the impact of capacity utilization on the growth rate of the real wage more than offsets its impact on the growth rate of labor productivity. The result is that an increase in capacity utilization leads to an increase in the growth rate of the labor share of income. In contrast, the economy is "Kaldorian" when the opposite happens, that is, when the growth rate of the labor share of income decelerates during an upswing and accelerates during a downswing.

Note that, from the assumptions that real-wage growth is pro-cyclical, we necessarily have a Marxian economy when labor-productivity growth is counter-cyclical because $w_u > 0 > b_u$. In contrast, when labor-productivity growth is pro-cyclical, we can have either a Marxian ($w_u > b_u > 0$) or a Kaldorian economy ($b_u > w_u > 0$).

Finally, note that the dynamics of the 3x3 dynamical system represented by J can be projected on the $u \times l$ plane by simply summing its first two differential equations. The result is the 2x2 dynamical system analyzed by Barbosa-Filho and Taylor (2003), where the labor share and capacity utilization exhibit a predator-prey pattern, with the labor share being the "predator", along the lines originally proposed by Goodwin (1967) for employment and the real wage.[18]

4 – Stability conditions

In the previous section we analyzed the qualitative structure of demand-led growth and income distribution about an equilibrium point. The next natural question is whether or not the system is stable about such a point. Adapting the mathematical analysis of Gandolfo

[17] The wage-led and profit-led terms comes from Taylor (1991) and they correspond respectively to the "stagnationist" and "exhilarationist" terms used by Marglin and Bhaduri (1990).

[18] The "predator-prey" pattern means that, when capacity utilization (the "prey") is above its equilibrium value, the labor share (the "predator") rises. The opposite happens when capacity utilization is below its equilibrium value. If the system is locally stable, temporary and small shocks lead to a temporary predator-prey cycles while economy converges to its steady state.

(1997) to our case, the 3x3 dynamical system is locally stable if all of the following conditions are satisfied:

$$trace(J) = k^*(f_u - 1) + z^* a_u + l^*(w_l - b_l) < 0, \tag{20}$$

$$|J| = k^* z^* l^* [(w_l - b_l)(f_u - a_u) + (w_u - b_u)(a_l - f_l)] < 0 \tag{21}$$

and

$$|\tilde{J}| = \begin{vmatrix} k^*(f_u - 1) + z^* a_u & z^* a_l & -k^* f_l \\ l^*(w_u - b_u) & k^*(f_u - 1) + l^*(w_l - b_l) & k^* f_u \\ -l^*(w_u - b_u) & z^*(a_u - 1) & z^* a_u + l^*(w_l - b_l) \end{vmatrix} < 0; \tag{22}$$

where $\tilde{J}$ is a matrix obtained from a linear transformation of *J*.[19]

Starting with (20), recall that we are considering an equilibrium point where all state variables are positive. From the previous section we have $a_u < 0$ and $w_l < 0 < b_l$, so that (20) is valid as long as the accelerator effect of capacity utilization on investment f_u is not too strong. The intuitive meaning is that the economy is stable as long as investment does not exhibit a strong response to capacity utilization, that is, as long as we do not have "knife-edge" demand dynamics *à la* Harrod (1939). It should also be noted that, even if we relax the assumption that the growth rate of non-capital expenditures is counter-cyclical, (20) can still be valid from the assumption that $w_l < 0 < b_l$. The intuition is that when the labor share quickly converges to its equilibrium value, it may offset explosive demand dynamics.

The second stability condition can be simplified in terms of the impact of capacity utilization on income distribution. To see this, recall that we assumed that capacity utilization has a positive impact on the growth rate of capital expenditures ($f_u > 0$) and a negative impact on the growth rate of non-capital expenditures ($a_u < 0$). Since we also assumed that the labor share is stable in isolation ($w_l < 0 < b_l$), we have $(w_l - b_l)(f_u - a_u) < 0$. Then, recall that we assumed that the labor share has a negative impact on the growth rate of capital expenditures ($f_l < 0$) and a positive impact on the growth rate of non-capital expenditures ($a_l > 0$). In a Kaldorian economy we have therefore $(w_u - b_u)(z^* a_l - k^* f_l) < 0$ and the second stability condition is valid. In a Marxian economy we have $(w_u - b_u)(z^* a_l - k^* f_l) > 0$ and second stability condition may or may not hold. The intuitive meaning of this result is that the economy can be stable provided that the labor share is not strongly pro-cyclical.

The third stability condition is difficult to simplify or translate into economic terms. Even under the restrictive assumptions made so far we cannot easily state it in intuitive terms. To avoid cluttering the analysis with more mathematics, let us leave this issue for the appendix and just state that (22) is consistent with the assumptions we made so far, but these assumptions are not sufficient for (22) to hold. We have therefore to add the extra assumption that (22) holds for the 3x3 dynamical system to be locally stable.

Before we move to the comparative static analysis of equilibrium points, it should be noted that, when capacity utilization has no impact on the labor share ($w_u = b_u$) or when the

[19] In mathematical terms the three conditions imply that the eigenvalues of the coefficient matrix *J* have negative real parts.

labor share has no impact on capacity utilization ($f_l = a_l = 0$), the third stability condition is reduced to

$$[k^*(f_u - 1) + z^* a_u)]\left\{[k^*(f_u - 1) + l^*(w_l - b_l)][z^* a_u + l^*(w_l - b_l)] - z^*(a_u - 1)k^* f_u\right\} < 0 , \qquad (23)$$

which despite the many parameters involved is valid under the assumptions made in the previous section and the auxiliary condition that the accelerator is not too strong. In economic terms, the intuitive meaning of this result is that demand-led growth is stable when effective demand has no impact on income distribution or when income distribution has no impact on effective demand.

The case with not transmission mechanism from effective demand to income distribution corresponds to the closure where the real wage and labor productivity are completely independent from the level of economic activity. This closure is usually associated with classical or neo Ricardian models where the prices of production do not depend on the level of output.[20] It should be noted that in this case there can still be a transmission mechanism in the opposite direction, that is, the 3x3 dynamical system continues to be locally stable if changes in income distribution lead to changes in effective demand because of, for instance, the difference between the propensities to consume out of wages and profits. In fact, the neo Ricardian case represents a situation where income distribution is given by technology and by the social conflict, which in their turn are independent of effective demand but can impact on effective demand. In the jargon of the structuralist models presented by Taylor (1991 and 2004), this is the case with constant coefficients of production and an exogenous markup rate.

The case with no transmission mechanism from income distribution to effective demand corresponds to a closure where, for instance, there is no difference between the propensities to consume out of wages and profits. As in the previous case, this does not imply that there is no transmission mechanism in the opposite direction, that is, changes in effective demand may still lead to changes in income distribution. In terms of the literature on the demand-led growth, this closure resembles the Neo-Keynesian macroeconomics of Hicks (1965) because it allows changes in effective demand to alter the real wage and the labor productivity along the lines of, for instance, the marginal-productivity theory of income distribution.[21]

5 – Comparative statics

To complete the analysis we have to check the impact of exogenous shocks on the equilibrium values of the model. Given the linear behavioral functions outlined in section three, the nontrivial equilibrium conditions can be written as

$$\begin{bmatrix} f_u - 1 & f_u & f_l \\ a_u - 1 & a_u & a_l \\ w_u - b_u & w_u - b_u & w_l - b_l \end{bmatrix} \begin{bmatrix} k^* \\ z^* \\ l^* \end{bmatrix} = -\begin{bmatrix} f_0 \\ a_0 \\ w_0 - b_0 \end{bmatrix}. \qquad (24)$$

To simplify the notation, let **H** be the coefficient matrix in (24). From the previous section we know that one of the stability conditions is

[20] See, for instance, Kurz and Salvadori (1995) for an outline of the neo Ricardian theory of production.

[21] For a detailed comparison of the neo-Keynesian closure with the neoclassical, Marxist, and structuralist or post Keynesian alternatives, see Marglin (1984) and Dutt (1990).

$$|\mathbf{H}| = (w_l - b_l)(f_u - a_u) + (w_u - b_u)(a_l - f_l) < 0$$

so that the coefficient matrix can be inverted under the assumption of local stability. The solution of (24) is given by

$$\begin{bmatrix} k^* \\ z^* \\ l^* \end{bmatrix} = -\frac{1}{|\mathbf{H}|} Adj(\mathbf{H}) \begin{bmatrix} f_0 \\ a_0 \\ w_0 - b_0 \end{bmatrix}, \tag{25}$$

where $Adj(\mathbf{H})$ represents the adjoint matrix of $\mathbf{H}$.

Since $-1/|\mathbf{H}|>0$, the signs of the entries of $Adj(\mathbf{H})$ give us the signs of the partial derivatives of k^*, z^* and l^* in relation to the intercept coefficients of the behavioral functions. In other words, the signs of the entries of $Adj(\mathbf{H})$ tell us how the economy responds to exogenous shocks to the growth rates of effective demand, the real wage and labor productivity. More formally:

$$\frac{\partial k^*}{\partial f_0} = -\frac{1}{|\mathbf{H}|}[a_u(w_l - b_l) - a_l(w_u - b_u)]; \tag{26}$$

$$\frac{\partial k^*}{\partial a_0} = -\frac{1}{|\mathbf{H}|}[f_l(w_u - b_u) - f_u(w_l - b_l)]; \tag{27}$$

$$\frac{\partial k^*}{\partial w_0} = -\frac{\partial k^*}{\partial b_0} = -\frac{1}{|\mathbf{H}|}(f_u a_l - f_l a_u); \tag{28}$$

$$\frac{\partial z^*}{\partial f_0} = -\frac{1}{|\mathbf{H}|}[a_l(w_u - b_u) - (a_u - 1)(w_l - b_l)]; \tag{29}$$

$$\frac{\partial z^*}{\partial a_0} = -\frac{1}{|\mathbf{H}|}[(f_u - 1)(w_l - b_l) - f_l(w_u - b_u)]; \tag{30}$$

$$\frac{\partial z^*}{\partial w_0} = -\frac{\partial z^*}{\partial b_0} = -\frac{1}{|\mathbf{H}|}[f_l(a_u - 1) - a_l(f_u - 1)]; \tag{31}$$

$$\frac{\partial l^*}{\partial f_0} = -\frac{1}{|\mathbf{H}|}(b_u - w_u) \tag{32}$$

$$\frac{\partial l^*}{\partial a_0} = -\frac{1}{|\mathbf{H}|}(w_u - b_u) \tag{33}$$

$$\frac{\partial l^*}{\partial w_0} = -\frac{\partial l^*}{\partial b_0} = -\frac{1}{|\mathbf{H}|}(f_u - a_u) \tag{34}$$

As usual in structuralist models, we cannot determine the impact of most exogenous shocks a priori. Even under the assumptions we made so far the economy can still respond in different ways. For instance, take (26). An exogenous increase in the growth rate of capital expenditures may result in a higher growth rate of the capital stock when, for instance, the growth rate of the labor share of income is not strongly pro-cyclical (say, $w_u \cong b_u$). The intuition is that when the demand expansion induced by an increase in capital expenditures does not have a large impact on the labor share, the resulting increase in the rate of profit leads to higher growth rate of the capital stock. In the terminology adopted by Barbosa-Filho and Taylor (2003), an exogenous increase in the growth rate capital expenditures leads to an increase in the growth rate of the capital stock when the economy is not strongly "Marxian".

The same reasoning can be applied to the other partial derivatives outlined above and, in most of them, the response of the equilibrium values to exogenous shocks cannot be determine a priori. It all depends on the structure of the economy, which is summarized by the coefficients of the behavioral functions in each partial derivative. It should be noted that the flexibility of the model is one of its great advantages from a structuralist perspective. Since capitalist economies do not exhibit the same structure through time or across countries, the 3x3 model offers us one possible way to organize the analysis in terms of just a few parameters. Even though we are not able to specify the response of the economy without further investigation about the size of the parameters, we can still obtain some general results about the dynamics of demand-led growth and income distribution.

First, consider the response of the growth rate of the capital stock to exogenous shocks. On the one hand the impact of changes in the growth rate of effective demand, be it capital or non-capital expenditures, depends basically on the cyclicality of the labor share. If the growth rate of the labor share is not strongly pro-cyclical, an increase in the growth rate of effective demand ends up increasing the growth rate of the capital stock. The basic idea is that an increase in effective demand drives capacity utilization up and, given the small change in the labor share, it ends up increasing the rate of profit and, therefore, the growth rate of the capital stock. In contrast, when the labor share is strongly pro-cyclical, the change in income distribution induced by the increase in effective demand may end up reducing the growth rate of the capital stock.

Still on the growth rate of the capital stock, given the response of effective demand to capacity utilization, the higher the difference between a_l and f_l, the more "wage-led" the economy and, therefore, the higher the probability that an exogenous increase in the growth rate of the labor share ends up increasing the growth rate of the capital stock. By analogy the opposite holds for a strongly profit-led economy.

Next, consider the ratio of non-capital expenditures to the capital stock. The impact of exogenous changes in effective demand depends again on the cyclicality of the labor share. If the growth rate of the labor share is not strongly pro-cyclical, an exogenous increase in the growth rate of capital expenditures ends up reducing non-capital expenditures in relation to the capital stock. The intuition is that investment grows faster than other expenditures while the economy moves to its new equilibrium. In contrast, an exogenous increase in the growth rate non-capital expenditures tends to increase these expenditures in relation to the capital stock. The intuition is that non-capital expenditures grow faster than the capital stock while the economy moves to its new equilibrium.

On the side of income distribution, an exogenous increase in the growth rate of the labor share tends to increase non-capital expenditures in relation to the capital stock when the accelerator effect is small. The intuition is that the increase in the labor share makes consumption grow faster than the capital stock while the economy moves to its new equilibrium.

Finally, consider the labor share. The impact of an exogenous increase in the growth rate of effective demand varies according to the source of the shock. In a Marxian economy an increase in the growth rate of investment reduces the labor share, whereas an increase in the growth rate of non-capital expenditures increases the labor share. The intuition is that the "capacity-building" effect of investment predominates over its demand effect, so that an increase in investment reduces capacity utilization and, through this, it reduces the labor share when the growth rate of the labor share is pro-cyclical. Since an increase in non-capital expenditures has only a demand effect, it increases capacity utilization and, therefore, the labor share of income. In a Kaldorian economy the roles are reversed because the growth rate of the labor share is counter-cyclical.

As for exogenous shocks to the distribution of income, from the assumption that the growth rates of capital and non-capital expenditures are respectively pro and counter-cyclical we can conclude from (34) that an exogenous increase in real-wage growth (or decrease in labor-productivity growth) leads to an increase in the labor share.

6 - The model in discrete time

For simulation and empirical purposes it is easier to work in discrete time. Because our previous analysis was built around accounting identities, this poses not great problem. To see why let K_t be the capital stock at the end of period t. Without capital depreciation the growth rate of the capital stock is simply the ratio of investment to the initial capital stock, that is, I_t / K_{t-1}. After some simple algebraic substitutions we arrive at

$$k_t = \left(\frac{1 + f_t}{1 + k_{t-1}} \right) k_{t-1}, \tag{35}$$

where all variables have the same qualitative meaning of the previous section.[22] In the same vein, the ratio of non-capital expenditures to the capital stock and the labor share of income are given by.

$$z_t = \left(\frac{1 + a_t}{1 + k_{t-1}} \right) z_{t-1}, \tag{36}$$

and

$$l_t = \left(\frac{1 + w_t}{1 + b_t} \right) l_{t-1}. \tag{37}$$

Altogether, (35), (36), and (37) form a 3x3 non-linear system of difference equations that is the discrete-time equivalent to the 3x3 system of differential equations analyzed in the previous sections. The accounting identities are basically the same as in the continuous case, that is

$$u_t = k_t + z_t; \tag{38}$$

$$y_t = \left(\frac{k_{t-1}}{k_{t-1} + z_{t-1}} \right) f_t + \left(\frac{z_{t-1}}{k_{t-1} + z_{t-1}} \right) a_t; \tag{39}$$

[22] Note that (35) is the discrete-time equivalent of (3). The continuous and discrete-time formulations of capital accumulation have been proposed respectively by Barbosa-Filho (2000) and Freitas (2002).

$$y_t = l_{t-1}(w_t + n_t + w_t n_t) + (1 - l_{t-1})(r_t + k_t + r_t k_t); \tag{40}$$

$$n_t = \frac{q_t - b_t}{1 + b_t}; \tag{41}$$

and

$$m_t = l_{t-1}(1 + n_t)w_t + (1 - l_{t-1})(1 + k_t)r_t. \tag{42}$$

So, if we add four behavioral functions (a_t, f_t, w_t and b_t) we obtain again a nonlinear system of twelve equations and twelve variables for some given initial conditions (k_{t-1}, z_{t-1} and l_{t-1}). As we did in the continuous-time case, the simplest way to specify the model is to define the behavioral functions as linear functions of the state variables. To keep the analysis simple let us restrict these functions to just one lag, that is

$$f(u_{t-1}, l_{t-1}) = f_0 + f_u(k_{t-1} + z_{t-1}) + f_l l_{t-1}; \tag{43}$$

$$a(u_{t-1}, l_{t-1}) = a_0 + a_u(k_{t-1} + z_{t-1}) + a_l l_{t-1}; \tag{44}$$

$$w(u_{t-1}, l_{t-1}) = w_0 + w_u(k_{t-1} + z_{t-1}) + w_l l_{t-1}; \tag{45}$$

and

$$b(u_{t-1}, l_{t-1}) = b_0 + b_u(k_{t-1} + z_{t-1}) + b_l l_{t-1}. \tag{46}$$

After substituting the above functions in (35), (36) and (37) we obtain a non-linear dynamical system in discrete time that, in principle, can be calibrated or estimated to reproduce the dynamics of real-world capitalist economies. To illustrate this point, figures 1 through 4 show the response of an artificial profit-led Marxian economy to an exogenous increase in the growth rate of autonomous expenditures. The parameters of the model were chosen to obtain a steady state where the labor share of income is 0.55 and the income-capital ratio is 0.4, of which 0.03 correspond to the growth rate of the capital stock and 0.37 to non-capital expenditures.[23] The implicit period is one year and, starting from the equilibrium point, the economy is subject to a permanent one-percentage point increase in the growth rate of its non-capital expenditures. Figures 1 and 2 show the response of the four behavioral functions to the shock, whereas figures 2 and 3 show how the state variables move to their new equilibrium values.

FIGURES 1 THROUGH 4 HERE

On the demand side, the growth rate of non-capital expenditures slows down immediately after the shock and then it oscillates while converging to its new equilibrium value. In contrast, the growth rate of capital expenditures accelerates substantially after the shock and then it also oscillates while converging to its new equilibrium value. On the income side, the growth rate of the real wage accelerates after the shock and the growth rate of labor productivity follows it after one period. Both variables oscillate while converging to their common and higher new equilibrium value. On the $z \times k$ plane the adjustment happens through counterclockwise fluctuations around the new equilibrium point. On the $l \times k$ plane the pattern is the same and, altogether, the exogenous increase in the growth rate of non-capital expenditures drives the economy to a new steady state with a faster growth rate, a higher income-capital ratio, and a higher labor share.

[23] Appendix 2 presents the values of the parameters used in the simulation.

7 – Conclusion

In general terms the main results of the previous sections can be summarized as follows:

- Income growth can be demand-led and stable under some plausible assumptions about aggregate demand, technology and income distribution.
- Demand-led growth can be represented by a small dynamical system in either continuous or discrete time. In both cases the steady states and the dynamics around the steady states depends crucially on the intensity of the accelerator effect of income on investment; on the response of effective demand to changes in income distribution; and on the response of income distribution to changes in effective demand.
- Demand-led growth may be stable under alternative assumptions about the cyclical behavior of the labor share (a profit-led or wage-led economy) or the response of effective demand to changes in income distribution (a Marxian or a Kaldorian economy).
- As long as the economy remain below its potential output, exogenous changes in effective demand may alter the growth rate and the functional distribution of income in both the short and the long run. In other words, the economy may be locked in a "slow-growth" or "fast-growth" steady state because of demand factors.
- Given a shock and assuming that demand-led growth and income distribution are jointly stable, the convergence to the steady state may involve fluctuations of capacity utilization and the labor share of income.
- Given the structure of the economy, the impact of exogenous changes in effective demand on growth and distribution may vary according to whether the source of the shock is a change in capital or non-capital expenditures.

Because of we have many parameters in the behavioral functions that describe effective demand, real wages and labor productivity, we have a long list of possible results even in the linear case analyzed in the previous sections. If we allow for nonlinear relations the list of possible results gets longer and the complexity much higher. Fortunately the linear behavioral functions already give us a flexible structure that can be adjusted to describe the evolution of real-world economies in terms of waves of demand expansion.

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Appendix 1: stability conditions

To simplify the notation the third stability condition can be rewritten as

$$|J| = \text{J1}(\text{J2} - \text{J3}) - \text{J4}(\text{J5} + \text{J6}) < 0,$$

where:

$$\begin{cases} \text{J1} = k^*(f_u - 1) + z^* a_u \\ \text{J2} = k^*(f_u - 1) + l^*(w_l - b_l) \\ \text{J3} = z^*(a_u - 1)k^* f_u \\ \text{J4} = l^*(w_u - b_u) \\ \text{J5} = z^* a_l[k^* f_u + z^* a_u + l^*(w_l - b_l)] \\ \text{J6} = k^* f_l[z^*(a_u - 1) + k^*(f_u - 1) + l^*(w_l - b_l)] \end{cases}.$$

From the assumptions made in section three we have J1<0, J2>0 and J3<0, so that J1(J2-J3)<0 as stated in (23). Assuming that the accelerator is not strong, the assumptions made in section three also imply that J5<0 and J6>0, so that we cannot know the sign of J5+J6 a priori. If the economy is strongly wage-led, we tend to have a_l substantially higher than f_l and, therefore, J5+J6 is likely to be negative. If the economy is strongly profit-led the opposite happens. Even if we could determine which is the case, the sign of J4(J5+J6) is still indeterminate a priori if we don't know whether the economy is Marxian (J4>0) or Kaldorian (J4<0). Putting all issues together, we can only say that, when the economy is Marxian (J4>0) and strongly wage-led (J5+J6>0), the third stability condition is likely to hold. By analogy, the third stability condition is also likely to hold if the economy is Marxian (J4<0) and strongly profit-led (J5+J6<0).

Appendix 2: simulation

The values of the intercept coefficients were chosen to obtain an equilibrium point where $k^*=f^*=a^*=w^*=b^*=0.03$, $z^*=0.37$, and $l^*=0.55$. The shock consists of a permanent 0.01 increase in the intercept coefficient of the "*a*" function. The simulation used the following values for the parameters of the behavioral functions:

	Intercept	*z*	*k*	*l*
a	0.120	-0.5	-0.5	0.2
f	-0.350	1.5	1.5	-0.4
w	-0.205	1	1	-0.3
b	-0.100	-0.5	-0.5	0.6

Figure 1: the growth rates of capital (*f*) and non-capital expenditures (*a*).

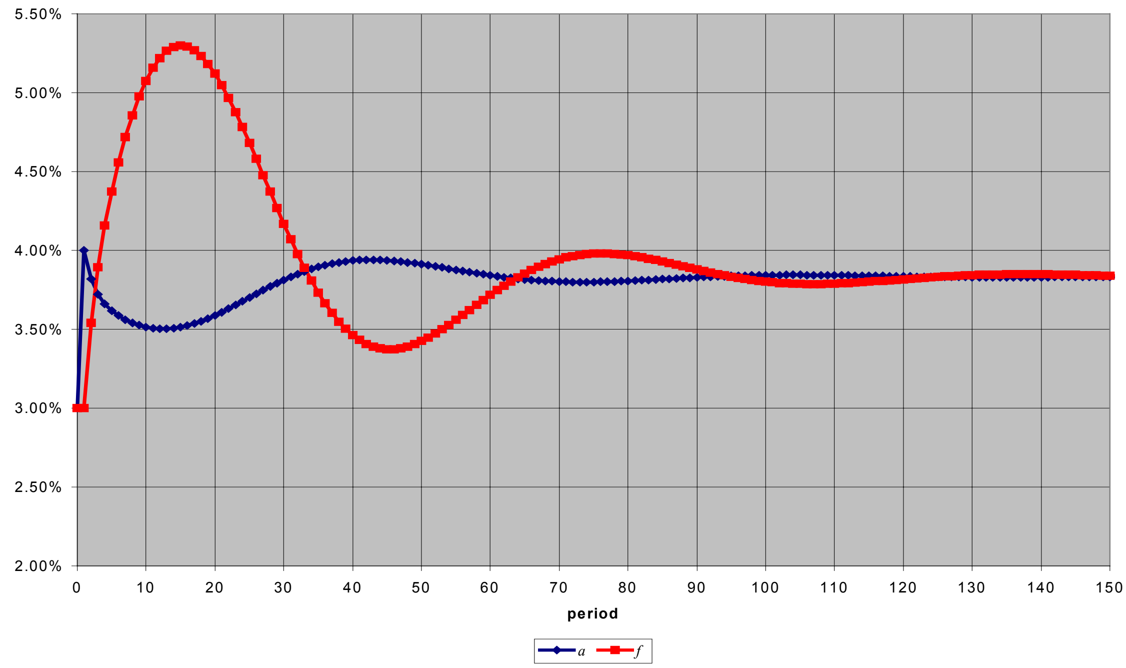

Figure 2: the growth rates of the real wage (*w*) and labor productivity (*b*).

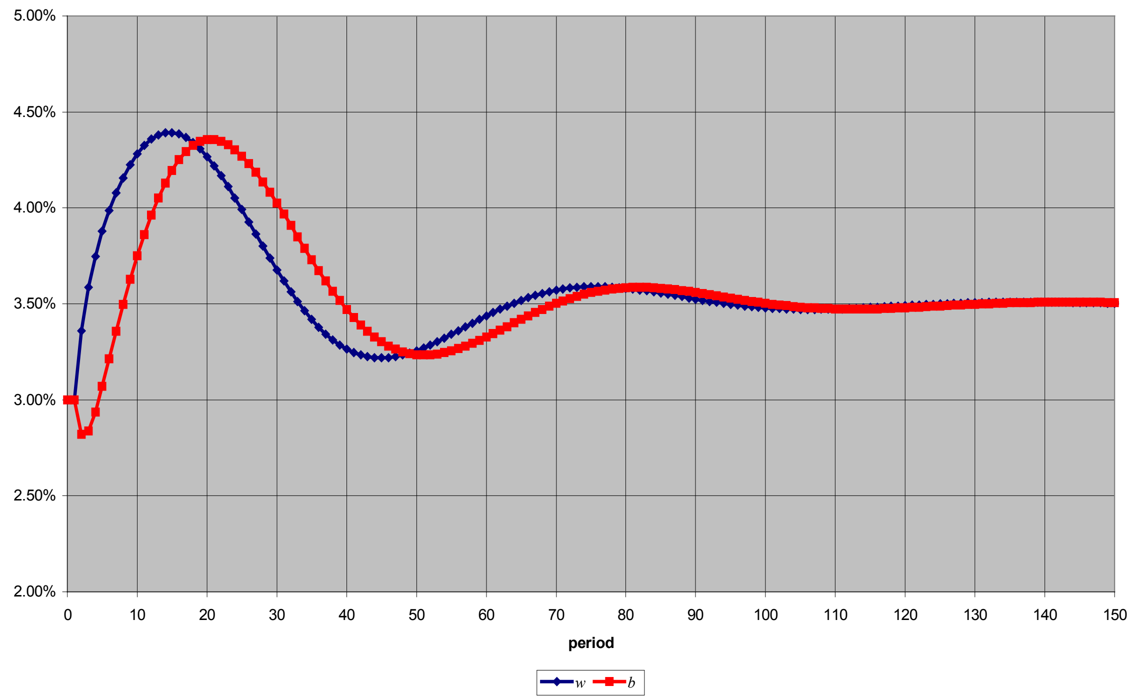

Figure 3: the growth rate of the capital stock (k) and the ratio of non-capital expenditures to the capital stock (z).

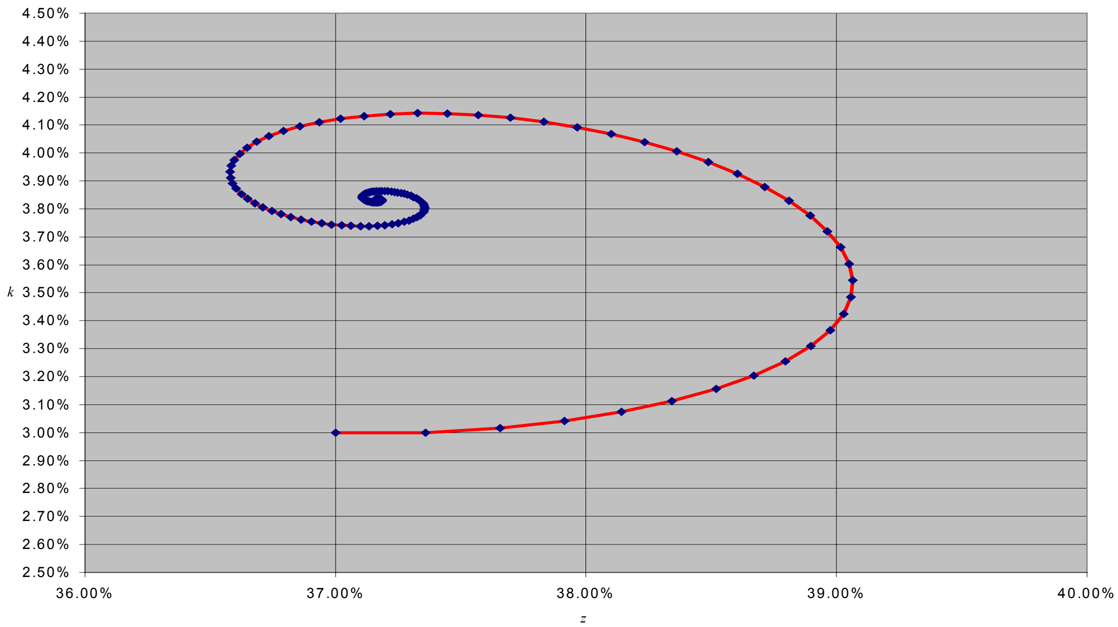

Figure 4: the growth rate of the capital stock (k) and the labor share of income (l).