A Non-linear Development Dynamics of Capital Accumulation, Distribution and Technological Innovation

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Abstract: It is elaborated a development dynamic model of accumulation, growth and distribution in which endogenous technological innovation plays a significant role. Firms' rate of labour-saving technological innovation is made to depend non-linearly on the distributive (wage and profit) shares, with the latter determining both the incentives to innovate and the availability of funding to carry it out. As it turns out, the direction and the intensity of the effect of a change in distribution on the rates of accumulation and growth depend on the prevailing distribution, with a similar dependence applying – alongside the relative bargaining power of capitalists and workers – to the dynamic stability properties of the system. Hence, the model does not rely on full capacity utilisation being reached for a change in the accumulation and growth regime to take place.

Key words: capital accumulation, distribution, technological innovation

Resumo: Elabora-se um modelo de desenvolvimento dinâmico de acumulação de capital, crescimento e distribuição em que inovação tecnológica endógena desempenha um papel fundamental. A taxa de inovação tecnológica poupadora de mão-de-obra das firmas depende não-linearmente da distribuição entre salários e lucros, com esta determinando tanto os incentivos à inovação quanto a disponibilidade de fundos para financiá-la. Como resultado, a direção e a intensidade do efeito de uma mudança na distribuição sobre as taxas de acumulação e crescimento dependem da distribuição vigente, com uma dependência similar sendo aplicável, juntamente com o poder de barganha relativo de capitalistas e trabalhadores, às propriedades de estabilidade dinâmica do sistema. Assim, o modelo não se baseia no alcance da plena utilização da capacidade para que a mudança no regime de acumulação de capital e crescimento aconteça.

Palavras-chave: acumulação de capital, distribuição, inovação tecnológica

JEL code: O1; O3; O4
ANPEC classification: DESENVOLVIMENTO

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1. Introduction

The Post Keynesian approach to capital accumulation, growth and distribution can be traced back to the models developed independently by N. Kaldor, J. Robinson and L. Pasinetti in the 1950s and 1960s, in which a long-run inverse relation between the real wage and the growth rate obtained. In the newer Post Keynesian models developed independently by authors more closely associated with the Kalecki-Steindl tradition like R. Rowthorn and A. K. Dutt in the 1980s, however, that relation between distribution and growth is usually positive. The reason is that while in the earlier models capacity utilisation cannot be an adjusting variable in the long run, in the newer ones it can be so in whatever run.

Once technological change is brought into the picture, labour-saving innovations taking place will affect distribution by affecting unit labour costs and thus the share of labour in income. This way technological change which raises labour productivity becomes a fundamental determinant of capital accumulation and growth, either directly by requiring the installation of new machines or indirectly by affecting distribution. Indeed, this influence becomes even greater and more complex when technological innovation is made endogenous rather than assumed to drop as manna from heaven. Now, when technological change has been treated as endogenous in the Post Keynesian literature, it is invariably linked to capital accumulation using some variant of either Kaldor's technical progress function or Arrow´s learning-by-doing function (e.g. Rowthorn 1981; Dutt 1990, 1994; You 1994; Watanabe 1997).[1]

This paper contributes to the Post Keynesian literature by elaborating a dynamic model of capital accumulation, growth and distribution in which endogenous technological innovation also plays a significant role, though through a different route. Firms' rate of labour-saving technological innovation is made to depend on the prevailing distribution, with the latter determining both the incentives to innovate and the availability of funding to carry it out. The rate of innovation is determined by distribution in a non-linear way, with the former being lower for both low and high levels of wage share, and higher for intermediate levels of wage share. While at low levels of wage share the availability of funding for innovation is high but the incentives to innovate are low, at high levels of wage share the incentives to innovate are high but the availability of funding is low. As it turns out, firms' desired capital accumulation and hence the growth rate will also be non-linear in distribution, with the same relation applying – along with the relative bargaining power of capitalists and workers – to the dynamic stability properties of the system. Hence, the model does not rely on full capacity utilization being reached for a change in the capital accumulation and growth regime to take place.[2]

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[1] For Kaldor, anything that raises the growth rate also leads to a faster rate of technical change. He formulated this idea in several ways, ranging from the technical progress function of his early work (1957, 1961), where the growth rate of labour productivity is positively related to the growth rate of capital per worker, to the Verdoorn's Law of his later writing (1966), where the growth rate of labour productivity is positively related to the growth rate of the economy. Arrow (1962), in turn, takes productivity as rising with experience in production, with this experience being measured by cumulative investment – which, in the absence of depreciation, is given by the capital stock.

[2] While there is no special connection between the old neoclassical model of growth developed by Solow (1956) and the determination of income distribution, the more recent neoclassical theoretical literature on such connection has expanded enormously. This literature provides an array of very different explanations for a positive correlation between income equality and growth, all of them quite different from the post-keynesian one

The remainder of the paper is organised in the following manner. Section 2 describes the building blocks of the model, whereas Section 3 analyses its behaviour in the short run. The behaviour of the model in the long run is discussed in Section 4, while Section 5 examines one of the possible long-run multiple equilibria dynamics leading to the emergence of cyclical behaviour. The paper closes with a summary of the main conclusions derived along the way.

2. The structure of the model

The economy is assumed to be a closed one and with no government activities, and to produce only one good for both investment and consumption. Only two (homogeneous) factors of production are used, capital and labour, which are combined through a fixed-coefficient technology. This assumption can be justified by reference to an independence of the choice of techniques of factor prices or to technological rigidities in factor substitution, it being amply supported by a reputable literature. As several eminent contributors to the economics of technological change have documented – from David (1975) and Rosenberg (1976) to Nelson & Winter (1982) and Dosi (1984) – technological change is marked by strong cumulative effects – 'learning' in its various forms. As a result, it is typically characterised by 'localised' shifts in some production function, to use David's (1975) term, or by progress along particular 'natural trajectories', to use Nelson & Winter's (1982) concept. This implies that a more rigid, if not (at least in the short run) fixed set of production coefficients will prevail.[3]

Capitalist firms in oligopolistic markets carry out production. They produce (and hire labour) according to demand, it being modelled only the case in which excess capacity prevails.[4] Labour employment is determined by production

$$L = aX \tag{1}$$

where L is the employment level, X is the output level, and a stands for the labour-output ratio. In turn, firms make accumulation plans described by the following function

$$g^d = \alpha + \beta r + \gamma h \tag{2}$$

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suggested in this paper, though. To put it briefly, in this mainstream literature an increase in inequality is argued to lower growth by raising redistributive government expenditure and therefore distortionary taxation, by increasing sociopolitical instability, or by reducing private investment in human capital. Some surveys of this literature can be found in Benabou (1996), Perotti (1996) and Aghion, Caroli & Garcia-Peñalosa (1999).

[3] Freeman and Soete (1987) & Verspagen (1990) also showed that localised technological change strongly diminishes the short-run possibilities for factor substitution. Probably the most quoted formalisation of localised technological change is still the one by Stiglitz & Atkinson (1969). The underlying idea is that for any industrial grouping the range of efficient techniques (in terms of relative factor intensity per unit of output) is often very small, sometimes reaching the limit of one technological system which rules at any point in time. Localised technical change strongly diminishes the short-run possibilities for substitution, and constant improvements of one single production technique usually lead to a Leontief shape similar to the one assumed here.

[4] Steindl (1952) claims that firms plan excess capacity so as to be ready for a sudden expansion of sales. First, the existence of fluctuations in demand means that the producer wants to be in a boom first, and not to leave the sales to new competitors who will press on her market when the boom is over. Second, it is not possible for the producer to expand her capacity step by step as her market grows because of the indivisibility and durability of the plant and equipment. Finally, there is the issue of entry deterrence: if prices are sufficiently high, entry becomes feasible even where capital requirements are great; therefore, the holding of excess capacity allows olipolistic firms to confront new entrants by suddenly raising supply and driving prices down.

where α, β, and γ are all positive parameters, g^d is firms' desired accumulation as a ratio of the capital stock, r is the profit rate, which is the flow of money profits divided by the value of capital stock at output price, and h is the rate of technological innovation, whose labour-saving nature is better specified below. To the extent we are dealing with a one-good economy, the 'production' of technological innovation does not constitute another production process or productive sector. Indeed, it is assumed that the single good that can be used for both investment and consumption may be used for 'innovative' purposes as well.[5]

We follow Rowthorn (1981) and Dutt (1984, 1990), who in turn follow Kalecki (1971) and Robinson (1956, 1962), and make desired accumulation to depend positively on the profit rate. The rationale is that the current profit rate is an index of expected future earnings, on the one hand, and provides internal funding for accumulation plans and make it easier to raise external funding, on the other hand. Firms' desired investment is also made to depend positively on the innovation rate, the point being that the latter results in more investment, at any given level of profit rate, than would otherwise be the case. While Dutt (1994) invokes Kalecki's (1971) claim that the higher the rate of technological change, the more desirable is to install new machinery, other plausible reasons for the above specification can be called up as well. Indeed, it is compatible with the Marxian contention that cost-reducing technical change places continuous pressures on any individual firm to invest, with Schumpeter's (1912, 1942) view that the process of innovation itself opens up new investment opportunities for firms, and with the neo-Schumpeterian notion (e.g. Nelson & Winter 1982) that investment behaviour is influenced by the dynamics of technical change.[6]

At a point in time, the technological parameters are given, having resulted from previous technological change and capital accumulation. Over time, though, labour-augmenting (Harrod-neutral) technological innovation is assumed to take place, which results in the labour-output ratio falling at rate h. In terms of the taxonomy proposed by Freeman (1982), distinguishing between incremental innovation, radical innovation, new technology systems and changes of techno-economic paradigm, we are dealing with the former; that is, innovations which occur more or less continuously, although at differing rates in different industries, and are concerned only with improvements in the existing array of processes of production. Rather than being exogenously given, though, technological innovation depends on the prevailing distribution in the following non-linear way

[5] Indeed, a more inclusive model could as well drop the assumption of homogeneous labour – e.g. by bringing in overhead labour devoted to R&D, who would be remunerated differently from direct labour – and/or of homogeneous capital – e.g. by bringing in human or knowledge capital. However interesting a more inclusive specification along these lines, it will be the subject of a future research – for which we invite the reader to stay tuned.

[6] For Landesmann & Goodwin (1994), the introduction of new technologies may indeed raise the propensity to invest. First, firms may not be willing to lose out vis-à-vis competitors by not having the most updated technology that could allow them to attract a larger share of demand. Second, firms may expect strong learning-by-using effects from introducing the new technologies and they may not want to miss out on them. Third, firms may expect additional secondary innovations complementary to the initial one, and an early introduction might bring further competitive and learning advantages over competitors. Steindl himself admitted later on that his *Maturity and Stagnation* was carried out on the naive assumption that technological change does not affect investment activity (Steindl 1981). Actually, in the introduction to the 1976 reprint of the book he had already recognised that innovations which are sufficiently advanced, and which can be exploited without too much delay and risk, are a powerful inducement to investment.

$$h = \sigma - \sigma^2 \quad (3)$$

where h is the technological innovation rate and σ is the wage share in output. Since this concave-down parabola has two real roots, $h(0) = h(1) = 0$, h is positive throughout its relevant domain given by $\sigma \in (0,1)$. The level of distribution which will yield the maximum rate of technological innovation is given by $\sigma^* = 1/2$, which means that a higher wage share will speed up (slow down) the rate of innovation for levels of distribution below (above) σ^*. This simplified innovation function is intended to capture a plausible non-linearity in the influence of the wage share on firms' propensity and ability to adopt labour-saving innovations, namely, that the rate of innovation is lower for both low and high levels of wage share, it being higher for intermediate ones. While at high levels of profit share the availability of funding for innovation is high but the incentives to innovate are low, at low levels of profit share the incentives to innovate are high but the availability of funding is low.[7]

The economy is inhabited by two classes, capitalists and workers. Following the tradition of Marx, Kalecki (1971), Kaldor (1956), Robinson (1956, 1962) and Pasinetti (1962), we assume that these classes have different saving behaviour. Workers provide labour and earn only wage income, which is all spent in consumption. They are always in excess supply, with the number of potential workers (labour supply) growing at the rate n.[8] Capitalists receive profit income, which is the entire surplus over wages, and save all of it. Division of income is then given by

$$X = (W / P)L + rK \quad (4)$$

where W is the money wage, P is the price level, and K is the capital stock. From eqs. (1) and (4), the share of labour in income, σ, is given by

$$\sigma = Va \quad (5)$$

where $V = (W / P)$ stands for the real wage. The profit rate is then given by

$$r = (1 - \sigma)u \quad (6)$$

where u is the rate of capacity utilisation. Since we assume that the ratio of capacity output to the capital stock remains constant, we can identify capacity utilisation with the output-capital ratio.

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[7] An alternative specification of endogenous technological innovation, rather (neo-)schumpeterian in spirit, can be found in Lima (2000). In the context of a Post Keynesian model of capital accumulation and distribution slightly similar to the one developed here, the rate of technological innovation is made to depend non-linearly on market concentration – which is itself endogenous to the processes of capital accumulation and technological change. By examining one possible long-run multiple equilibria it is as well developed a qualitative analysis of the potential emergence of endogenous, self-sustaining fluctuations in concentration, growth and distribution.

[8] According to Sawyer (1989), the supply of labour to the capitalist economies (and within capitalist economies, supply to the capitalist sectors) can, at least over some range, be readily expanded whenever it is necessary. Within a country, the capitalist economy may cover only a part of the economy, so that the capitalist sector can pull workers from the non-capitalist sector when demand for labour is relatively high and push workers back when demand is low. Other mechanisms include migration of labour from one country to another and changes in the age of entry into and departure from the labour force. Therefore, extra supply of labour can be obtained when demand is strong by pulling people into the labour force. Conversely, when the overall demand for labor is low, unemployment can to some degree be hidden by the re-absorption of workers back into home.

The price level is given at a point in time, but over time it will rise whenever the desired markup of firms exceeds their actual markups. Formally, we have

$$\hat{P} = \tau[\sigma - \sigma_f] \tag{7}$$

where $\hat{P}$ is the proportionate rate of change in price, $(dP/dt)(1/P)$, and τ is the speed of adjustment. As in Dutt (1994), inflation is determined within a framework of conflicting income claims, inflation resulting whenever the income claims of workers and capitalists exceed the available income. The markup over prime costs, à la Kalecki (1971), is given by

$$P = (1+z)Wa \tag{8}$$

where z is the markup. Given labour productivity, the markup is inversely related to the wage share, so that the gap between the desired and the actual markups can be measured by the gap between the actual and the desired wage share. The desired markup by firms depends on the state of the goods market, and higher capacity utilisation, which reflects more buoyant demand conditions, will induce firms to desire a higher profitability. We can express the wage share implied by firms' desired markup as

$$\sigma_f = \varphi - \theta u \tag{9}$$

where φ and θ are positive parameters. Several arguments can be invoked to support procyclicity of the markups. Eichner (1976) argues that during expansions firms may want to invest more by generating higher internal savings and therefore desire a higher markup. Rowthorn (1977) claims that higher capacity utilisation allows firms to raise prices with less fear of being undercut by their competitors, who would gain little by undercutting due to higher capacity constraints. Gordon, Weisskopf & Bowles (1984) argue that marked-up prices are inversely related to the perceived elasticity of demand, which is a negative function of industry concentration and of the fraction of potential competitors that are perceived to be quantity-constrained and thus not engaged in or responsive to price competition. In the downturn, markup will fall because the fall in capacity utilisation gives rise to a smaller share of potential competitors being perceived to be under capacity constraints, and hence to an increase in the perceived elasticity of demand facing the firm. Later on, though, we will examine how sensitive are the long-run stability properties of the system to this assumption that the desired markup rises with capacity utilisation.[9]

At a point in time the money wage is given, and with labour being always in excess supply, employment is determined by labour demand. Over time, though, the money wage will change in line with the gap between the wage share desired by workers, σ_w, and the actual wage share. As in Dutt (1994), this wage adjustment equation can be expressed as

[9] Regarding anticyclicity, Minsky (1975) argues that the fall in sales in a downturn forces firms to raise markups to meet outstanding financial obligations. Kalecki (1954) argues that since the markup depends partially on the ratio of overheads to prime costs, the rise in this ratio in downturns causes markups to rise. On the neoclassical side, Rotemberg & Woodford (1992) claim that it is more difficult for oligopolistic firms to sustain collusive prices during booms, when the incentive for any firm to cut its price rises because it becomes more worthwhile to capture current sales than to maintain collusion in the future. Bils (1989) argues that demand may become less elastic during recessions, allowing firms to increase markups. Chevalier & Scharfstein (1996), in turn, argue that since capital-market imperfections constrain the ability of firms to raise external financing, liquidity-constrained firms will increase (lower) markups during recessions (booms). An excellent survey of most developments in the neoclassical literature on the cyclical behavior of markups can be found in Rotemberg & Woodford (1999).

$$\hat{W} = \mu[\sigma_w - \sigma] \tag{10}$$

where $\hat{W}$ is the proportionate rate of change in money wage, $(dW/dt)(1/W)$, and μ is the speed of adjustment. The wage share desired – and bargained for – by workers is assumed to depend on their bargaining power in the labour market. A higher employment rate, by raising workers´ bargaining power, will stimulate workers to desire a higher wage share, so that

$$\sigma_w = \eta + \lambda e \tag{11}$$

where η and λ are positive parameters and e is the employment rate, L/N, which is linked to the state of the goods market in the following way

$$e = uk \tag{12}$$

where k stands for the ratio of capital stock to labour supply in productivity units, that is, $k = K/(N/a)$, with N being the supply of labour. This formal link between u and e is necessary because the fixed-coefficient nature of the technology implies that an increase in output in the short run will be necessarily accompanied by an increase in employment.

Since firms will produce according to demand and operate below full capacity, the equality between desired investment and saving will be reached by changes in capacity utilisation. Assuming that capital does not depreciate, g, the growth rate of capital stock, which is the growth rate for this one-good economy, is given by

$$g = r \tag{13}$$

which follows from the assumptions that workers do not save and capitalists save all of their income.[10]

3. <u>The behaviour of the model in the short run</u>

The short run is defined as a time period in which the stock of capital, K, the supply of labour, N, the labour-output ratio, a, the price level, P, and the money wage rate, W, can all be taken as given. The existence of excess capacity implies that capacity utilisation will adjust to remove any excess demand or supply in the economy, which implies that in short-run equilibrium, $g = g^d$. Substituting from (2), (3), (6) and (13), we can solve for the short-run equilibrium value of u given σ and the other parameters, to obtain

$$u^* = \frac{\alpha + \gamma\sigma(1-\sigma)}{(1-\beta)(1-\sigma)} \tag{14}$$

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[10] Interestingly, variable capacity utilisation has been claimed by some proponents of the real business cycle approach to be a source of propagation of technology shocks. Burnside & Eichenbaum (1996), for example, models variable capacity utilisation by assuming that technology depends on effective capital services and effective hours of work. Depreciation is a function of capital utilisation, and in equilibrium firms will choose to hoard capital, so they can increase its effective stock at once in response to shocks that raise the marginal product of capital. However, the supply-side nature of this approach makes its determination of capacity utilisation quite distinct from the demand-driven one pursued here. Besides, technological change is endogenously determined in the model of this paper, rather than being guided by exogenous shocks as in the real business cycle approach – which actually assumes that the economy is always at full-employment equilibrium and therefore has generally no intrinsic cyclical movements.

Regarding stability, we assume a Keynesian short-run adjustment mechanism stating that output will change in proportion to the excess demand in the goods market. This means that the equilibrium value for u will be stable provided the denominator of the expression in (14) is positive, which implies that aggregate saving is more responsive than desired investment to changes in capacity utilisation. In turn, to assume that $0 < \beta < 1$, as required for stability of capacity utilisation throughout its relevant domain, ensures a positive value for u^* itself. As it turns out, a change in the wage share will lead to a change in the same direction in capacity utilisation

$$\frac{du^*}{d\sigma} = u^*_\sigma = \frac{\alpha + \gamma(1-\sigma)^2}{(1-\beta)(1-\sigma)^2} \tag{15}$$

Like in the newer Post Keynesian model developed independently by Rowthorn (1981) and Dutt (1984, 1990), an increase in the wage share – by redistributing income from capitalists who do save to workers who do not – raises the level of activity. For any $\sigma < \sigma^*$, a rise in the wage share will also increase the innovation rate, raise firms' desired accumulation and hence increase capacity utilisation even more. Even though a rise in the wage share will lower the rate of innovation – and put a downward pressure on desired accumulation – when $\sigma > \sigma^*$, that will not be strong enough to make for a falling capacity utilisation.

Now, our assumption that workers do not save and capitalists save all of their income implies that the rates of profit and accumulation are identical, as shown by eq. (13). For a given level of capacity utilisation, as shown by (6), an increase in the wage share, by lowering the profit share, will unambiguously exert a downward pressure on the rates of profit and accumulation. However, a higher wage share may generate a rise in capacity utilisation that is strong enough to more than compensate the accompanying fall in the profit share, thus leading to a rise in the rates of profit and accumulation. All this ambiguity is captured by the expressions for $r^* = g^*$ and $r^*_\sigma = g^*_\sigma$, which, using (6), (13) and (14) are

$$r^* = g^* = \frac{\alpha + \gamma\sigma(1-\sigma)}{(1-\beta)} \tag{16}$$

and

$$r^*_\sigma = g^*_\sigma = \frac{\gamma(1-2\sigma)}{(1-\beta)} \tag{17}$$

Hence, a rise in the wage share will raise (lower) the rates of profit and accumulation when $\sigma < \sigma^*$ $(\sigma > \sigma^*)$. For lower levels of profit share, a rise in the wage share, by lowering the rate of innovation, will make for a falling accumulation (and growth) rate.[11] Therefore, the model does not rely on full capacity utilisation being reached for a change in the accumulation and growth regime to take place.[12]

[11] Were capitalists to reduce their saving rate, in turn, it can be checked that the short-run equilibrium values of capacity utilisation and growth would both rise, in line with the paradox of thrift.

[12] However, it should be underlined that it is not the non-linear nature of the innovation function itself that makes for that change in the growth regime, but the dependence of the desired accumulation on the rate of technological innovation, as shown by eq. (2) above. Indeed, it can be checked that assuming away that dependence would imply a different behaviour for the growth rate. While capacity utilisation would keep rising

The relevant distributional domain can therefore be divided into two regions. In the first one (hereafter LW region), in which the wage share is lower $(\sigma < \sigma^*)$, the rates of technological innovation, capacity utilisation and growth are all directly related to the wage share. In the second one (hereafter HW region), in which the wage share is higher $(\sigma > \sigma^*)$, while capacity utilisation is still directly related to the wage share, the rates of innovation and growth become inversely related to it.

4. <u>The behaviour of the model in the long run</u>

In the long run we assume that the short-run equilibrium values of the variables are always attained, with the economy moving over time due to changes in the stock of capital, K, the supply of labour, N, the labour-output ratio, a, the price level, P, and the money wage rate, W. One way of following the behaviour of the system over time is by examining the dynamic behaviour of the short-run state variables σ, the wage share, and k, the ratio of capital stock to labour supply in productivity units. From the definition of these variables, we have the following state transition functions:

$$\hat{\sigma} = \hat{W} - \hat{P} + \hat{a} \tag{18}$$

and

$$\hat{k} = \hat{K} - \hat{N} + \hat{a} \tag{19}$$

Substitution from (3) into (18), from (11) and (12) into (10), and then from the resulting expression into (18), along with substitution from (9) into (7), and then from the ensuing expression into (18), yields

$$\hat{\sigma} = \mu(\lambda uk - \sigma) - \tau(\sigma - \varphi + \theta u) - (\sigma - \sigma^2) \tag{20}$$

where u is given by eq. (14). Substituting from (6) into (13) and then the resulting expression into (19), and from (3) into (19), we obtain

$$\hat{k} = (1 - \sigma)u - (\sigma - \sigma^2) - n \tag{21}$$

where u is again given by eq. (14), while n is the growth rate of labour supply, assumed to be exogenously given. A constant unemployment rate as a long-run characteristic, required to make for a stationary wage share, implies rate of economic growth equal to the rate at which the reserve army is replenished through growth of the labour supply and labour productivity. Since we are dealing with endogenous technological innovation and exogenous labour supply growth, long-run equilibrium is determined by the interaction between the warranted rate (capital accumulation) and the natural rate (labour supply growth plus growth of productivity), with the course of both of them depending on the prevailing distribution.

Eqs. (20) and (21), after using (14), constitute a planar autonomous two-dimensional non-linear system of differential equations in which the rates of change of σ and k depend on the levels of σ and k, and on the parameters of the system. The Jacobian matrix of partial derivatives for this dynamic system is given by

$$J_{11} = \partial\hat{\sigma} / \partial\sigma = \mu(\lambda k u_{\sigma}^* - 1) - \tau(1 + \theta u_{\sigma}^*) - (1 - 2\sigma) \tag{22}$$

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with the wage share, the rate of growth would cease to depend on distribution to become constant. The next section will deal with the implications of such independence for the long-run behaviour of the system.

$$J_{12} = \partial\hat{\sigma} / \partial k = \mu\lambda k u^* > 0 \qquad (23)$$

$$J_{21} = \partial\hat{k} / \partial\sigma = g^*_\sigma - (1-2\sigma) \qquad (24)$$

$$J_{22} = \partial\hat{k} / \partial k = 0 \qquad (25)$$

Indeed, not all of these partial derivatives, evaluated at a stationary point, can be unambiguously signed. Eq. (23) shows that an increase in the ratio of capital to labour supply in productivity units, by raising the employment rate, will raise the wage share desired by workers and thereby raise the rate of increase in money wages. Eq. (25) shows that since an increase in k does not affect either σ or u there is no effect either on the rate of accumulation or the rate of innovation, and hence no effect on the rate of growth of k. Eq. (22) shows that the impact of a change in the wage share on its rate of growth is mediated directly and indirectly by its impact on capacity utilisation. The reason is that both the wage share desired by workers and the wage share implied by firms' desired markup depend on capacity utilisation. While σ_f depends directly on the state of the goods market, σ_w depends directly on the state of the labour market. Now, given the fixed-coefficient nature of the technology, an increase in capacity utilisation in the short run will necessarily be accompanied by an increase in employment. Whether, in turn, a change in the wage share will raise or lower the rate of innovation depends on the level of distribution. As for the sign of J_{21}, it would be given by the relative impact of changes in the wage share on the rates of growth and technological innovation. However, it is seen below that a necessary condition for the occurrence of at least one equilibrium with $\hat{\sigma} = \hat{k} = 0$ within the relevant domain given by $0 < \sigma < 1$ is that the absolute value of $dh / d\sigma$ is greater than the absolute value of $dg^* / d\sigma$. Hence, it follows that J_{21} is positive (negative) in the HW region (LW region).

We now have all the elements for a qualitative phase-diagrammatic analysis of this dynamic system. The way we proceed is by analysing the (local) stability of an equilibrium position in each one of the two regions into which we divided the meaningful subset of the domain. Now, eq. (21) shows that the equation describing the $\hat{k} = 0$ isocline is quadratic in the wage share, which means that there may be up to two values for the wage share in the relevant (σ, k)-space at which a respective vertical $\hat{k} = 0$ isocline would be located – recall that $\hat{k}$ does not depend on $\hat{k} = 0$.[13] Given this geometry, the way we proceed is by analysing the (local) stability properties of the equilibrium solution in each one of the two regions were one of the $\hat{k} = 0$ isoclines and some portion of the $\hat{\sigma} = 0$ isocline to be located there.[14]

In the LW region, a higher wage share exerts an upward pressure on its rate of change by raising capacity utilisation and thus employment, which then raises the wage share desired

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[13] It can be checked that a necessary condition for these two values of the wage share to be contained in the relevant domain given by $(0,1)$ is that $\beta + \gamma < 1$, which we assume – recall that in the preceding section it was assumed that $0 < \beta < 1$ to ensure that the short-run equilibrium value of capacity utilisation is positive and stable. Using eqs. (3) and (17), in turn, it can be easily checked that $\beta + \gamma < 1$ implies that the absolute value of $dh / d\sigma$ is greater than the absolute value of $dg^* / d\sigma$, as mentioned right above.

[14] We do not consider the case given by $\sigma = \sigma^*$ since it implies that $J_{21} = 0$, and hence that $Det(J) = 0$.

by workers. However, this same rise in capacity utilisation also raises the markup desired by firms, which then exerts a downward pressure on the rate of change of the wage share by lowering firms' desired wage share. Besides, a higher wage share exerts a downward pressure on its rate of change by raising the innovation rate. The sign of J_{11} depends on the relative strength of all these effects, with closer inspection of eq. (22) revealing that it is negative unless the impact via workers' desired wage share is very strong. Recalling that the existence of an equilibrium solution in this region implies a negative sign for J_{21}, a positive sign for $Det(J)$, which is a necessary condition for stability, will follow no matter the sign of J_{11}. The latter, in turn, by having the same sign as $Tr(J)$, will ultimately determine whether the long-run equilibrium solution with $\hat{\sigma} = \hat{k} = 0$ is stable or unstable.

In case J_{11} is negative, Fig. 1 shows that the resulting steady-state solution will be stable. Given that J_{12} is positive, the (local) slope of the $\hat{\sigma} = 0$ isocline, which is given by $-(J_{11}/J_{12})$, is positive. Since $\partial\hat{\sigma}/\partial k$ is positive, $\hat{\sigma}$ undergoes a steady rise as k increases, so that the sign of $\hat{\sigma}$ is negative (positive) to the right (left) of the $\hat{\sigma} = 0$ locus, which explains the direction of the horizontal arrows. The slope of the $\hat{k} = 0$ isocline, given by $-(J_{21}/J_{22})$, is equal to zero. Given that $\partial\hat{k}/\partial\sigma < 0$, $\hat{k}$ undergoes a steady fall as σ rises, thus implying that the sign of $\hat{k}$ is negative (positive) to the right (left) of the $\hat{k} = 0$ isocline, which explains the direction of the vertical arrows.

[Figure 1 about here]

In case J_{11} is positive, which means that the rate of change in nominal wages is very responsive to changes in capacity utilisation, Fig. 2 shows that the resulting steady-state solution will be unstable. Given that J_{12} is positive, the (local) slope of the $\hat{\sigma} = 0$ isocline becomes positive, with $\hat{\sigma}$ again undergoing a steady rise as k increases. As before, the slope of the $\hat{k} = 0$ isocline is equal to zero, with $\hat{k}$ undergoing a steady fall as σ increases.

[Figure 2 about here]

In the HW region, a higher wage share also exerts an upward pressure on its rate of change by raising capacity utilisation and thus employment, which then raises the wage share desired by workers. However, this same rise in capacity utilisation raises the markup desired by firms as well, which then exerts a downward pressure on the rate of change of the wage share by lowering firms' desired wage share. Besides, a higher wage share now exerts an upward pressure on its rate of change by lowering the innovation rate, with the sign of J_{11} depending on the relative strength of all these effects. Regarding the sign of J_{21}, recall that the existence of an equilibrium solution in this region implies that it is positive.

In case J_{11} is negative, Fig. 3 shows that the resulting steady-state solution will be saddle-point unstable, given that $Det(J) < 0$. The slope of the $\hat{\sigma} = 0$ isocline is positive, while the direction of the horizontal arrows is given by the positive (negative) sign of $\partial\hat{\sigma}/\partial k$ ($\partial\hat{\sigma}/\partial\sigma$). The slope of the $\hat{k} = 0$ isocline, in turn, is equal to zero, while the direction of the vertical arrows is given by the positive sign of $\partial\hat{k}/\partial\sigma < 0$. Hence, the equilibrium solution with $\hat{\sigma} = \hat{k} = 0$ is saddle-point unstable, with the corresponding separatrix having a negative slope. Indeed, an equilibrium solution located in this region will be saddle-point unstable anyway, the reason being that $Det(J)<0$ no matter the sign of J_{11}. In case the latter is positive, however, both the slope of the $\hat{\sigma}=0$ isocline and the slope of the separatrix of the saddle-point become negative.

[Figure 3 about here]

Now, to evaluate how sensitive are all these stability requirements to the assumption that firms' desired markup is directly related to capacity utilisation, let us briefly examine the general implications of assuming an inverse relationship instead – along the lines briefly recalled on fn. 8. In the LW region, where wage-led growth obtains, recall that equilibrium will be stable (unstable) in case $\partial\hat{\sigma}/\partial\sigma$ is negative (positive). In comparison to the original specification with procyclical markups, therefore, firms' desired markup being anticyclical would lower the chances of stability. The reason is that now a higher wage share, by raising capacity utilisation, would raise the wage share implied by the (lower) markup desired by firms and hence make for a lower inflation rate – and a weaker downward pressure on the rate of change of the wage share – than before. In the HW region, in turn, where a profit-led growth regime obtains, an anticyclical desired markup would not remove the source of saddle-point instability, the reason being that $Det(J)<0$ no matter the sign of J_{11}.[15]

5. Multiple equilibria analysis

The non-linearity embodied in the desired investment function through the innovation function makes for the possibility of double equilibria within the relevant domain. As seen above, there may be up to two real values for the share of wages at which a corresponding vertical $\hat{k}=0$ isocline would be located in the (σ,k)-space. Let us call σ_1 and σ_2 these two levels of wage share located in the LW and HW, respectively, and call $\hat{k}_1=0$ and $\hat{k}_2=0$ the corresponding $\hat{k}=0$ isoclines.

Amongst the possible configurations leading to multiple equilibria, one worthy of a more detailed phase-diagrammatic analysis contains an unstable equilibrium in the LW region, E_1, and a saddle-point unstable equilibrium in the HW region, E_2 – as shown in isolation in Figs. 2 and 3, respectively. This situation is pictured in Fig. 4, where it is revealed an interesting potentially cyclical feature of equilibrium points such as E_1. It can be seen that there is a subset of the phase plane which the economy will never leave in the event it gets into it. We refer to this subset as zone of stability and to its complement as zone of instability. The zone of stability can be found by tracing back the path of the economy which leads into the higher part of the separatrix all the way through the LW region and then back to the HW region.

[Figure 4 about here]

Once inside the zone of stability, the economy will move cyclically. Suppose we begin a trajectory at point A in Fig 4. The direction of motion of the system indicates that it

[15] In turn, recall from fn. 11 that to assume away the positive dependence of desired accumulation on the rate of innovation would make for an independence of the growth rate from distribution. However, the $\hat{k}=0$ isocline would still be quadratic in the wage share, while the sign of J_{21} would remain the same in each one of the two regions. Hence, the same qualitative possibilities in terms of equilibrium solution for these regions would arise.

must flow leftward down until the lower part of the $\hat{k}_1 = 0$ isocline is reached, after which it will flow leftward up. After a while, the system will end up reaching the higher portion of the $\hat{\sigma} = 0$ isocline in the LW region, and will start flowing rightward up until the upper part of the $\hat{k}_1 = 0$ isocline is reached. Given this direction of motion, the system may reach the borderline between the LW and HW regions before reaching the lower portion of the $\hat{\sigma} = 0$ isocline in the LW region. In case the system reaches the HW region – through, say, point B – before reaching the $\hat{\sigma} = 0$ isocline, it will keep flowing rightward down until the lower part of $\hat{\sigma} = 0$ isocline in the HW region is reached, after which it will flow leftward down. If by a fluke the system reaches back the borderline between the LW and HW regions at point A, the cyclical motion just described will then re-start and the system, if left undisturbed, will keep having this cyclical motion. In case the system reaches the LW region through a point above A, say point C, another cyclical motion will re-start. Since trajectories of differential equations with continuous partial derivatives must be unique, they cannot cross each other. Hence, this new cyclical motion will have to be completed either at a point above C in the borderline between the LW and HW regions or in the lower part of the $\hat{\sigma} = 0$ isocline in the LW region.

Now, E_1 being unstable means that there is a neighbourhood of it, say D, within which all trajectories of the system will move away from E_1. Since the system will end up reaching that neighbourhood along the hypothesised trajectory initiated at point A, it will not reach E_1. Indeed, there may eventually be a closed, bounded area encircling the neighbourhood D, and from which no trajectory will exit. Since this area would contain no equilibrium points, the Poincaré-Bendixson theorem would then ensure that it must contain at least one stable limit cycle (Beavis & Dobbs 1990). But whether or not some limit cycle will emerge, the system will move cyclically in this subset of the zone of stability, which shows its propensity to experience endogenous, self-sustaining fluctuations in the capital to efficiency labour supply ratio and the wage share, with technological innovation, capacity utilisation, accumulation and employment fluctuating as well.

Recall that in the LW region the rates of technological innovation, capacity utilisation and accumulation are all directly related to the wage share. In the situation pictured in Fig. 4, a rise in the wage share will raise the rate of change of nominal wage by more than it raises the rate of change in prices and the rate of innovation combined, so that the level and the rate of change of the wage share move in the same direction. On the other hand, the wage share and the rate of change of the capital to efficiency labour supply ratio move in the opposite direction. A rise in the wage share, for instance, will both raise the rate of accumulation and raise the labour supply in productivity units, though the absolute value of the latter effect is greater than the absolute value of the former one. Also, recall that the fixed-proportion nature of the technology implies that capacity utilisation and the rate of employment move in the same direction, with the strength of the connection being given precisely by the capital to efficiency labour supply ratio. The higher k, the higher the change in the employment rate brought about by a change in capacity utilisation. Since the price-setting power of capitalists and workers is endogenous to capital accumulation through a conflict theory of inflation, a fall in k, for instance, given the wage share, leads to a fall in the extent to which the nominal wage change effect is greater than the price change and innovation effects combined. Therefore, in the upper (lower) part of the zone of instability the level and the rate of change of the wage share move in the same (opposite) direction throughout the domain precisely because k does not fall (rise) enough to make for a cyclical reversal. In the zone of stability, on the other hand, the system will move cyclically precisely because k can rise or fall to the extent that it is necessary to make for a cyclical reversal.

Starting from a point in the upper part of the $\hat{\sigma} = 0$ isocline in the LW region, for instance, a rise in the wage share will raise the rates of innovation, capacity utilisation and accumulation rate. At such levels of σ and k, though, k will keep rising until the upper part of the $\hat{k}_1 = 0$ isocline is reached, after which further rises in the wage share will raise σ and lower k, with higher levels of wage share carrying the seeds of their own reversal. Once the lower part of the $\hat{\sigma} = 0$ isocline is reached, σ will have risen, and k have fallen, by enough for the upward motion of the wage share to cease. A lower wage share will, by lowering the rates of innovation, capacity utilisation and accumulation, reduce σ and k even more, until the lower part of the $\hat{k}_1 = 0$ isocline is reached. At that point, σ and k will have fallen by enough for the reversal of the downward motion of σ to occur. Once σ has fallen, and k have risen, by enough, the upper part of the $\hat{\sigma} = 0$ isocline will then be reached back.

Therefore, this model shares with the classic contribution by Goodwin (1967) a cyclical growth dynamics governed by the interaction between capital accumulation, employment and distribution.[16] Unlike the Goodwin model, though, this one allows effective demand to play a role through a variable degree of capacity utilisation, incorporates price and nominal wage dynamics by means of a conflict theory of inflation, and is based on an endogenous mechanism of technological change. If left undisturbed, the Goodwin model will produce conservative cyclical fluctuations in the wage share and in the rate of employment. However, the trajectories will no longer be closed orbits if direct feedbacks from the wage share to its rate of change – or from the level of the employment rate to its rate of change – are introduced. As the partial derivatives (22)-(25) show, this model introduces a complex non-linear feedback from the wage share to its rate of change through variable capacity utilisation, conflict inflation and endogenous technological change. Nonetheless, Fig. 2 shows that convenient restrictions in the parameters of the $\hat{\sigma} = 0$ isocline might lead to the emergence of a cyclical dynamics à la Goodwin for lower levels of wage share.

These general features of the model are shared with Dutt (1994), from which we have drawn a lot of inspiration. Unlike the latter, though, this model does not rely on full capacity utilisation being reached for profit-led accumulation and growth as well as multiple equilibria to obtain within the distributive domain. Given the non-linear investment function used here, the system may well experience self-sustaining fluctuations in the same pair of state variables (σ, k) featured in Dutt (1994) – eventually alternating phases of wage-led accumulation and growth with phases of profit-led accumulation and growth – even below full capacity utilisation. Besides, while technological change is endogeneised in Dutt (1994) by being

[16] In the Goodwin model, capital accumulation taking place increases employment. Once the employment rate has crossed a threshold level in the neighbourhood of full employment, the real wage begins to rise. This lowers the rate of profit and therefore the possible rate of accumulation. When the accumulation rate declines sufficiently, the employment rate falls low enough to cause the real wage to decline. This re-establishes the rate of profit, so that the rate of accumulation can then increase once again.

made to depend linearly on the rate of accumulation, in this model it is endogeneised by being made to depend non-linearly on distribution.[17]

6. Summary

This paper contributes to the Post Keynesian literature on capital accumulation and distribution by developing a macrodynamic model in which technological innovation depends non-linearly on distribution, with the latter determining both the incentives to innovate and the availability of funding to carry it out. In turn, when technological change has been treated as endogenous in the Post Keynesian literature, it is invariably linked to capital accumulation using some variant of either Kaldor's technical progress function or Arrow´s learning-by-doing function. As it turns out, capital accumulation and growth also become non-linear in distribution, with the same dependence applying – alongside the relative bargaining power of capitalists and workers – to the dynamic stability properties of the system.

Inflation is determined within a framework of conflicting income claims, with the relative price-setting power of capitalists and workers being endogenous to accumulation – the power of capitalists (workers) to drive up prices (nominal wages) rising with capacity utilisation (the employment rate). In the short run, the existence of excess capacity implies that capacity utilisation will adjust to remove any excess demand or supply in the economy. As it turns out, a change in the wage share will lead to a change in the same direction in capacity utilisation. For lower levels of profit share, however, a rise in the wage share, by lowering the rate of innovation, will make for a falling accumulation and growth. Hence, the model does not rely on full capacity utilization being reached for a change in the capital accumulation and growth regime to take place.

Regarding long run dynamics, a steady-state equilibrium (in distribution and the ratio of capital stock to labour supply in productivity units) with lower shares of wage will be stable (unstable) in case workers´ wage-setting power is weak (strong) enough. For higher shares of wages, in turn, steady-state equilibrium will be saddle-point unstable no matter the relative bargaining power of capitalists and workers. While these stability properties were derived under the assumption that the markup is procyclical, they mostly survive to a change of assumption to an anticyclical one, *ceteris paribus*. Indeed, while stability of the steady-state with lower shares of wage would now require a weaker wage-setting power by workers, that saddle-point instability with higher shares of wage would not be removed by an anticyclical markup. The paper closed with a qualitative phase-diagrammatic analysis of a possible configuration leading to double equilibria and cyclical behaviour, showing the propensity of the system to experience endogenous, self-sustaining fluctuations.

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[17] Dutt (1994) considers two possibilities in terms of a linear relationship between the rate of technological change leading to higher labour productivity and the rate of capital accumulation. The first one is a variant of Arrow's (1962) learning-by-doing notion that productivity increases with experience in production, an assumption which is also consistent with the idea that increases in productivity can be attained only with the introduction of new machines. The second one is a variant of Schumpeter's (1912) suggestion that when the profit rate falls, so that the accumulation rate falls as well, firms will be pressured to innovate to increase their profits.

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Figure 1. Long-run dynamics: stable equilibrium in the LW region

Figure 2. Long-run dynamics: unstable equilibrium in the LW region

Figure 3. Long-run dynamics: saddle-point unstable equilibrium in the HW region

Figure 4. Long-run dynamics: multiple equilibria, stability zone and cyclical behaviour