 capital mobility, real exchange rate appreciation and asset price bubbles in emerging economies

A post keynesian macroeconomic model for a small open economy *

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Abstract: The objective of this paper is to show that open economies that (i) have (short-term) capital mobility and (ii) a low degree of organization in financial markets may have bubbles in asset prices if there is an exogenous shock to these economies that produces a real exchange rate appreciation. In order to do that, we will present a post keynesian macroeconomic model for small open economies based in Taylor (1983) and Taylor and O’Connell (1985). Our model differs from models developed by these authors in several aspects. First of all, we will consider a broad list of assets (9 assets). Second, we will consider the fulfillment of Marshall-Lerner condition, so that a real exchange appreciation will produce a reduction in net exports. Third, we will suppose, contrary to Taylor and O’Connell, that profits are expected to increase if current rate of interest is bigger than normal or safe rate of interest. Finally, we will suppose that there is imperfect capital mobility – in the sense of Mundell and Fleming – so that interest rate differential is a major factor determining capital inflows to emerging countries.

Key-Words: Asset Price Bubbles, capital mobility and Marshall-Lerner Condition

Resumo: O objetivo do presente artigo é mostrar que economias abertas caracterizadas por (i) mobilidade de capitais de curto termo e (ii) baixo nível de organização dos mercados financeiros podem exibir bolhas nos preços dos ativos financeiros se ocorrer um choque exógeno à essas economias que produza uma apreciação da taxa de câmbio real. Para demonstrar a validade desse ponto, iremos apresentar um modelo macroeconômico pós-keynesiano para uma pequena economia aberta baseado em Taylor (1983) e Taylor e O’Connell (1985). O modelo apresentado neste artigo difere dos modelos desenvolvidos por esses autores em vários aspectos. Em primeiro lugar, considera-se uma lista mais ampla de ativos (9 ativos). Em segundo lugar, iremos supor que a condição de Marshall-Lerner é atendida de forma que uma apreciação da taxa de câmbio irá produzir uma redução das exportações líquidas. Finalmente, iremos supor a existência de mobilidade de capitais imperfeita a la Mundell-Fleming de forma que o diferencial de taxa de juros é um determinante fundamental dos fluxos de capitais de curto prazo para os países emergentes.

Palavras-Chave: Bolhas de preços de ativos, mobilidade de capitais, condição de Marhall-Lerner.

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1. Introduction

One of the most far-reaching economic development of the late decades is the explosive growth of international financial transactions and capital flows. Powerful forces have driven the rapid growth of international capital flows, like revolutionary changes in information and communications technologies in financial services industry world-wide and the trend in both industrial and developing countries toward economic liberalization and the globalization of trade.

The liberalization of capital account has contributed to higher investment in many countries and increases the volume and volatility of international capital flows. So financial liberalization has been associated with costly financial crises in several countries. The arguments about potential risk of open capital markets arising from problems of incomplete information and other distortions.1 They point out that there are information gaps in financial markets. Such imperfections give rise to several problems that have potential to lead to inefficient and unstable financial markets.

Critics of the “efficient markets” view argue that between several problems affect financial markets one of the most important is herding behavior. Herding can make sense when the private return of adopting a particular course of action is an increasing function in the number of agents that adopt the same course of action. Agents may try to follow the lead of someone they believe to be better informed (cf. Banerjee, 1992) or herding can occur when investor lack information about the quality of those who manage their funds (cf. Shaferstein & Stein, 1990). It is not difficult to see that in the presence of asymmetric or incomplete information, investor will quite rationally take actions that can amplify price movements and precipitate sudden crises. In other words, herding behavior can lead to sharp investor reactions, and bubbles in financial markets.

Financial liberalization and herding behavior can promote bubbles in capital markets, and even financial crises with potentially damaging consequences. A consequence of this statement is that financial liberalization unambiguously improves the efficiency of resource when it is not accompanied by policies to limit inefficient and unstability of capital markets, especially herding behavior. This suggests that the theoretical presumption in favor of the liberalization of capital account, like propose the classic case for capital mobility, is not correct by the presence of incomplete information.

Developing countries have adopted policies to increase international capital flows. In particular, restrictions on capital account transactions began to decline in Latin America at the end of the 1980s when highly indebted countries put the worst aspect of the debt crisis behind them and the industrial countries evinced a renewed willingness to undertake

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1 Eichengreen et alii (1998) examines several aspects of capital account liberalization.
lending to developing countries. Nevertheless, preexisting inefficiencies led to the emergence of financial instability unrelated to fundamentals.²

Financial markets of these countries, however, have some particular features that make them more susceptible to the occurrence of bubbles in asset prices than financial markets in developed countries in the face of financial liberalization. In fact, financial markets in emerging countries like Brazil or South Korea have a low degree of organization which produces a great volatility in asset prices when compared to the fluctuations in asset prices observed in the financial markets of developed countries. The low degree of organization that prevails in financial markets of emerging countries tends to increase the possibility of occurrence of herding behaviour, making assets demand more sensible (more elastic) to changes in current conditions. This increased sensibility in assets demand results in greater volatility of asset prices.

The objective of this paper is to show that open economies that (i) have (short-term) capital mobility and (ii) a low degree of organization in financial markets may have bubbles in asset prices – which are defined by Dymski (1998) as cumulative increase in equity prices in relation to supply price of capital goods - if there is an exogenous shock to these economies that produces a real exchange rate appreciation. In fact, almost all emerging countries have experienced a real exchange rate appreciation in the 1990’s (cf. Mishkin, 1999, p.11). This suggests that exchange rate appreciation can be the triggering event of asset price bubbles observed in some of these countries in the last decade paper is to show that open economies that (i) have (short-term) capital mobility and (ii) a low degree of organization in financial markets may have bubbles in asset prices if there is an exogenous shock to these economies that produces a real exchange rate appreciation.

In order to do that, we will present a post Keynesian macroeconomic model for open economies based in Taylor (1983) and Taylor and O’Connell (1985). Our model differs from models developed by these authors in several aspects. First of all, we will consider a broad list of assets (9 assets). Second, we will consider the fulfillment of Marshall-Lerner condition, so a real exchange appreciation will produce a reduction in net exports. Third, we will suppose, contrary to Taylor and O’Connell, that profits are expected to increase if current rate of interest is bigger than normal or safe rate of interest. Finally, we will suppose that there is imperfect capital mobility – in the sense of Mundell and Fleming – so that interest rate differential is a major factor determining capital inflows to emerging countries.

The present article is organized in 6 sections. Section 2, presents the basic structure of the economy under discussion. Section 3 shows the behavior of the economy in the short run, i.e in the interval of (logical) time where asset stocks and expectations are held constant. Section 4 presents the behavior of our model economy in the long run, showing the existence of a saddle-path trajectory to the steady-state position of the model. Section 5, discusses the response of the economy to a particular exogenous shock: a real exchange.

rate appreciation. As a result of this shock, a bubble in asset prices may occur. Section 6 presents the conclusions of the article.

2- The Basic Building Blocks

Let us consider a small open economy that produces a single good with the assistance of labour and an imported input. The technology employed by firms of this economy is *leontieff type*, that is technical coefficients of production – i.e labour-output ratio and imports-output ratio – are constant. We will also suppose that price of the single good produced in this economy is determined by a fixed *mark-up* over unit costs. In this case, price determination can be represented by the following equation:

\[ p = (1 + \tau)[w b + e p^* a_0] \]  

(1)

Where : \( p \) is the money price of the single domestic good, \( w \) is the money wage rate, \( e \) is the nominal exchange rate, \( p^* \) is the price of the imported input in foreign currency, \( b \) is the labour-output ratio and \( a_0 \) is the import input-output ratio.

Since the *mark-up* is constant then all variations in the rate of profit come from variations in the degree of capacity utilization. The rate of profit can be expressed by the following equation:

\[ r = \frac{\tau}{1+\tau} u \]  

(3)

In the demand side of the economy, we will suppose, like Kalecki, that workers spend all their income in consumption goods and that capitalists save a constant fraction of profits. In this case, the consumption in nominal terms can be expressed by the following equation:

\[ pC = wbX + (1-s)r pK \]  

(3)

Where : \( C \) is the real consumption expenditure, \( X \) is the quantity of output produced in the economy, \( s \) is the propensity to save out of profits, \( K \) is the capital stock in real terms.

The goods market will be in equilibrium if and only if the following condition is met:

\[ pC + pl + p (G-T) + p E = p X \]  

(4)

where : \( I \) is the real investment spending, \( G-T \) is the real fiscal deficit and \( E \) is the real net exports.

Using (3) in (4) we found after all algebraical manipulations that:
In equation (5), \( g \) is the growth rate of the capital stock desired by entrepreneurs, \( \gamma \) is the fiscal deficit as a proportion of the economy’s capital stock, \( \varepsilon \) is the net exports as a proportion of the capital stock and \( \phi \) is the fraction of imported input costs in unit costs.

In order to determine the equilibrium loci of the goods market, we will suppose first that the rate of growth of the capital stock which is desired by the entrepreneurs is given by the following equation:

\[
g = g_0 + \frac{\hat{r}}{K} + \gamma - \phi \tau^{-1} r \equiv 0 \tag{5}
\]

where:

\[
g = \frac{\dot{K}}{K}; \quad \gamma = \frac{G - T}{K}; \quad \varepsilon = \frac{E}{K}; \quad \phi = \frac{ep^* a_0}{wb + ep^* a_0}
\]

In equation (5), \( g \) is the growth rate of the capital stock desired by entrepreneurs, \( \gamma \) is the fiscal deficit as a proportion of the economy’s capital stock, \( \varepsilon \) is the net exports as a proportion of the capital stock and \( \phi \) is the fraction of imported input costs in unit costs.

Equation (6) establishes that growth rate of capital stock is a function of the difference between expected rate of profit (\( \rho \)) and the current rate of interest (\( i \)). This is a simple formalization of the theory of investment behavior that Keynes presented in chapter eleven of the General Theory. The state of long term expectations which, according to Keynes, is a major determinant of investment spending, is represented by \( \rho \) in equation (6). The current rate of profit (\( r \)) is also added to the investment equation in order to represent the influence of changes in the level of capacity utilization on investment demand. More precisely, we are supposing that an increase in the degree of capacity utilization, \textit{coeteris paribus}, will increase investment spending.

We will also consider that \( \varepsilon \) is determined by the following equation:

\[
\varepsilon = \varepsilon_0 + \varepsilon_i q \tag{7}
\]

where: \( q = \frac{ep^*}{p} \)

In equation (7), net exports as a fraction of economy’s capital stock is a positive function of real exchange-rate (\( q \)). In other words, we are supposing that Marshall-Lerner condition is being fulfilled. The nominal exchange-rate (\( e \)) is fixed, that is, monetary authorities – the Central Bank – employ the stock of foreign reserves to sustain a fixed parity with foreign currencies. In this setting, all variations in real exchange rate come from changes in domestic price level or in foreign prices.

Using (6) and (7) in (5), we obtain the loci of combinations between the current rate of profit and the current rate of interest for which the goods market is in equilibrium, i.e aggregate demand is equal to aggregate supply. So we have the following equation:
In order to determine the slope of the *loci* of goods market equilibrium – which will be labelled as GG´ *loci* – let us take the total derivative of \((8)\). After all mathematical manipulations we have:

\[
\frac{\partial r}{\partial i} = -\frac{h}{s + \phi \tau^{-1} - h} \quad (8a)
\]

Equation \((8a)\) shows that in order to the GG´ *loci* to be downward sloping is necessary that \((s + \phi \tau^{-1} - h)\) be positive, that is the *sum* of propensity to save out of profits and propensity to import be bigger than the propensity to invest. This condition is, in general, assumed by all macroeconomic models, so there is no loss of generality if it is also assumed by the present model.

The GG´ *loci* is shown in figure 1.

![Figure 1](image_url)

The next step is to consider the asset markets and the financial sector of this economy. We will suppose that there is 9 different assets in the economy: foreign reserves \((eR)\), domestic bonds \((B)\), foreign bonds \((B^*)\), equities \((P,E)\), loans \((L)\), bank reserves \((H)\), demand deposits \((D)\), capital goods \((P, K)\) and securities \((F)\). These assets are hold by four different agentes: Central Bank, comercial banks, firms and rentiers.

Equations \((9)-(12)\) describe the balance sheets of all agents of this economy:

\[
F + eR = H \quad (9)
\]
\[
H + L = D \quad (10)
\]

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\[ P_kK = L + P_eE + N \quad (11) \]
\[ D + eB^* + B + P_eE = W \quad (12) \]

Where : \( P_k \) is the demand price of capital goods, \( P_e \) is the price of equities, \( N \) is firm’s net worth, \( W \) is rentiers’ total wealth.

For simplicity, we will assume that commercial banks are forced by law or convention to hold a constant fraction of demand deposits in the form of bank reserves, that is :

\[ H = \theta D \quad ; \quad 0 < \theta < 1 \quad (13) \]

We will also suppose that rentiers decide about what proportion of their financial wealth that will be hold in demand deposits, domestic and foreign bonds and equities. These assets are supposed to be imperfect substitutes so that demand for each asset is influenced not only by its own rate of return but also by the rate of return of other assets. The same assumption is also employed by Tobin (1969) and Taylor and O’Connell (1985).

In this setting, rentiers’ portfolio decision is described by the following system of equations :

\[ \mu(i, i^* + d^e, r + \rho)W = D \quad (14) \]
\[ \beta(i, i^* + d^e, r + \rho)W = B \quad (15) \]
\[ \beta^*(i, i^* + d^e, r + \rho)W = B^* \bar{e} \quad (16) \]
\[ \epsilon(i, i^* + d^e, r + \rho)W = P_eE \quad (17) \]

\[ W = D + B + B^* \bar{e} + P_eE \quad (18) \]

Equations (14)-(17) describe the demand of each kind of asset by rentiers. As we can see, the fraction of wealth that rentiers wish to hold in each of the four possible assets is a function of three variables : domestic interest rate, foreign interest rate plus expected devaluation of domestic currency and current rate of profit plus expected rate of profit. The first variable can be thought as the rate of return on domestic bonds while the second is clearly the rate of return on foreign bonds calculated in terms of domestic currency. Finally, the third variable is a (poor) proxy for the rate of return on equities.

Equation (18) is the (stock) budget constraint of rentiers. It shows that total demand of assets cannot be larger than total wealth. But it is necessary to observe that, while total wealth is a datum in almost all IS-LM type macroeconomic models, in the present model total wealth is an endogenous variable (cf. Taylor and O’Connell, 1985, p.871). In fact, total wealth is a positive function of equity prices which, in turn, are endogenously determined.

In fact, using (14), (15) and (17) in (18) it can be shown that :
Equation (19) determines the value of total wealth as a function of (i) the stock of foreign bonds evaluated in terms of domestic currency, (ii) the domestic rate of interest; (ii) the foreign rate of interest plus the expected rate of depreciation in domestic currency and (iv) the current and expected rate of profit.

Using (13) and (19) in (14), it can also be shown that:

\[
\mu(i, i^e + d^e, r + \rho) = \theta^{-1} \alpha \left( 1 - \mu(i, i^e + d^e, r + \rho) - \beta(i, i^e + d^e, r + \rho) - \varepsilon(i, i^e + d^e, r + \rho) \right)
\]  

Equation (20) can be thought as the equilibrium loci of financial markets. Such loci will be labelled as loci \( FF' \). In order to determine the slope of \( FF' \) loci, it is necessary to make some assumptions about the sign of partial derivatives in the system presented by equations (14)-(17). More specifically, we will suppose that:

\[\begin{align*}
\mu_1 &< 0, \mu_2 < 0, \mu_3 > 0 \quad (21a) \\
\beta_1 &> 0, \beta_2 < 0, \beta_3 < 0 \quad (21b) \\
\varepsilon_1 &< 0, \varepsilon_2 < 0, \varepsilon_3 > 0 \quad (21c)
\end{align*}\]

Given the conditions stated in (21a) to (21c), we can be shown that:

\[
\frac{\partial i}{\partial r} = -\left\{ \frac{(1+\alpha)\mu_1 + \alpha^* \beta_1 + \alpha^* \varepsilon_1}{(1+\alpha)\mu_1 + \alpha^* \beta_1 + \alpha^* \varepsilon_1} \right\} (22) \quad ; \quad \alpha^* = \alpha \theta^{-1}
\]

In equation (22), both numerator and denominator have ambiguous sign. In order to solve this ambiguity, it is necessary to impose additional restrictions to the parameters of (22). For such, we will first calculate the effect of an increase in international rate of interest over domestic rate of interest. It is a stylized fact about macroeconomic performance of small open economies under fixed exchange-rate regime that an increase in foreign rate of interest will produce a sharp increase in domestic rates. So we can analyse what restrictions should be imposed to the parameters of the model in order to reproduce this stylized fact.

Thus we obtain the following expression:

\[
\frac{\partial i}{\partial (i^e + d^e)} = -\left\{ \frac{(1+\alpha)\mu_1 + \alpha^* \beta_3 + \alpha^* \varepsilon_3}{(1+\alpha)\mu_1 + \alpha^* \beta_3 + \alpha^* \varepsilon_3} \right\} (23)
\]

In order to the sign of (23) be positive, denominator must have a positive sign. But the denominator in (23) is equal to the denominator in (22), so this one has also a positive sign.

But what about the sign of numerator in (22) ? In order to determine it’s sign, we have to impose additional restrictions to the relation between \( \mu_3, \beta_3 \) and \( \varepsilon_3 \). It is a stylized fact about financial markets in emerging economies like Brazil or South Korea that stock markets have a low degree of organization, i.e they have low liquidity and show wide
fluctuations in equity prices. These special features of stock markets in emerging economies implies that demand elasticity of equities in relation to its own rate of return is very high. On the other hand, it can be easily demonstrated that a high elasticity of demand for equities can result from a positive sign in the numerator of (22). Under these assumptions, the FF' loci will be downward sloping (see figure 2).

Figure 2

3 – Short Run Equilibrium and Comparative Statics

We are now able to analyse the short run equilibrium of this economy, that is the simultaneous equilibrium of all markets when asset stocks and expectations about future profitability of capital goods are kept constant.

From (8) and (20), we know that:

\[
\begin{align*}
  r &= r(i, q, \gamma, \rho) \quad ; \quad r_1 < 0, r_2 > 0, r_3 > 0, r_4 > 0 \quad (24) \\
  i &= i(r, d^\gamma, \rho, \alpha) \quad ; \quad i_r < 0, i_2 > 0, i_3 > 0, i_4 > 0 \quad (25)
\end{align*}
\]

Equations (24) and (25) are simple representations of the GG' and FF' loci, respectively.

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3 Carvalho (1992) defines an organized market as one in “(...) which avoids excessive potentially disruptive fluctuations in the price of assets, avoiding thereby solvency crisis that could be threaten the performance of that market. To contain the fluctuations in asset prices is the function of market makers (...) residual buyers or sellers that absorb excess supplies or demands when they exceed some acceptable margin” (p.87).

4 See appendix 1 for a formal proof of this statement.
General equilibrium of goods and financial markets requires a pair of values for $r$ and $i$ such that equations (24) and (25) are solved simultaneously. This will happen in the point where GG´curve intercept FF´curve, as shown by figures 3$^a$ and 3$^b$.

Figures 3$^a$ and 3$^b$ show two possible short run equilibrium configurations. In the first one – shown by figure 3$^a$ – FF´ curve is less inclined than GG´curve. In the second – represented by figure 3$^b$ – GG´ curve is less inclined than FF´curve.

For short run analysis we will not consider the issue of equilibrium stability; in other words, we will suppose that the economy is always in short run equilibrium. In this case, the relative inclinations of GG´and FF´curves is important only for comparative statics.
For analysis of comparative statics, we will suppose that \( GG' \) is steeper than \( FF' \) curve. So, using \((25)\) in \((24)\) and taking total derivative of the resulting equation, we can easily demonstrate that:

Expressions \((26)\) and \((27)\) summarizes the effects of changes in:

(i) expected devaluation of domestic currency, (ii) expected rate of profit, (iii) real exchange rate, (iv) fiscal deficit as a fraction of economy’s capital stock and (v) ratio of bank reserves and foreign bonds evaluated in terms of domestic currency over equilibrium levels of current rates of interest and profit. These results allow us to write the following equations:

\[
\begin{align*}
\frac{\partial r}{\partial d^e} &= \frac{r_i}{1 - i, r_i} < 0; \quad \frac{\partial r}{\partial \rho} = \frac{r_i + r_4}{1 - i, r_i} > 0; \quad \frac{\partial r}{\partial \alpha} = \frac{r_i i_i}{1 - i, r_i} < 0; \quad \frac{\partial r}{\partial q} = \frac{r_2}{1 - i, r_i} > 0; \quad \frac{\partial r}{\partial \gamma} = \frac{r_3}{1 - i, r_i} > 0 \quad (26) \\
\frac{\partial i}{\partial d^e} &= i_2 \left(1 + \frac{i, r_i}{1 - i, r_i}\right) > 0; \quad \frac{\partial i}{\partial \rho} = \frac{i_3 + i_i (r_i i_i + r_i)}{1 - i, r_i} < 0; \quad \frac{\partial i}{\partial q} = \frac{i, r_2}{1 - i, r_i} < 0; \quad \frac{\partial i}{\partial \alpha} = i_4 \left(1 + \frac{i, r_i}{1 - i, r_i}\right) > 0 \\
\frac{\partial i}{\partial \gamma} &= \frac{i_5 i_i}{1 - i, r_i} < 0 \quad (27)
\end{align*}
\]

devaluation of domestic currency, (ii) expected rate of profit, (iii) real exchange rate, (iv) fiscal deficit as a fraction of economy’s capital stock and (v) ratio of bank reserves and foreign bonds evaluated in terms of domestic currency over equilibrium levels of current rates of interest and profit. These results allow us to write the following equations:

\[
\begin{align*}
r &= r(d^e, \rho, \alpha, q, \gamma) \quad (28) \\
i &= i(d^e, \rho, \alpha, q, \gamma) \quad (29)
\end{align*}
\]

### 4 – Long Run Equilibrium and Dynamics

Now we must turn our attention to the behavior of the system in the long run. As already been told, in the long run asset stocks and expectations will be changing as a result of the endogenous operation of the economy. In particular, monetary base – which, in the present model, is equal to bank reserves – may change as a result of balance of payments deficits or surplus, and expected rate of profit may change as a result of changes in the current situation of the economy.

Taking the first derivative of \( \alpha \) in relation to time, we obtain the following expression:

\[
\dot{\alpha} = \frac{\alpha}{H} = \frac{H}{H - \dot{B}^*} \quad (30)
\]

Equation (30) shows that rate of growth of the ratio between monetary base and foreign bonds is equal to the difference between the rate of growth of monetary base and the rate of growth of the stock of foreign bonds. For simplicity, we will assume that stock of foreign bonds is kept constant through time, so all changes in \( \alpha \) comes from changes in monetary base.

In the absence of sterlization, changes in the stock of high powered money will be equal to changes in foreign reserves (cf. McCallum, 1996, p.138). Since there is a fixed exchange rate regime, all variations in the stock of foreign reserves come from surplus or deficits in the balance of payments.
We will suppose that balance of payments deficits or surpluses are determined by the following equation:

\[ BPA = [\varepsilon_0 + \varepsilon_1 q - \phi \tau^{-1} r - \phi (i - i^* - d^*)]K : 0 < \phi < \infty \]  (31)

The first term in (31) is the current account surplus which is a positive function of real exchange rate – i.e Marshall-Lerner condition is satisfied – and a negative function of current rate of profit. The second term is the capital account surplus which is a positive function of the difference between domestic rate of interest and international rate of interest plus expected rate of devaluation in domestic currency. The coefficient \( \phi \) represents the sensibility of capital account surplusus to the difference between domestic and international rate of interest. Since \( \phi < \infty \), this economy is not under a system of perfect capital mobility. More precisely, we are supposing, like Mundell (1962, 1963) and Fleming (1962), that there is imperfect capital mobility between countries (cf. McCallun, 1996, p.145). This imperfection on mobility of capital can, in turn, be the result of governmental restrictions on capital movements, like those that prevailed in most countries during the post war period.

Dividing both sides of (31) by \( K \), and remembering that BPA is equal to time derivative of monetary base, we obtain the following expression:

\[ \frac{\dot{H}}{H} = h^{-1} \left[ \varepsilon_0 + \varepsilon_1 q - \phi \tau^{-1} r + \phi (i - i^* - d^*) \right] \ ; \ h = \frac{H}{K} \]  (32)

Using (32) in (30), we have the following differencial equation:

\[ \dot{\alpha} = h^{-1} \left[ \varepsilon_0 + \varepsilon_1 q - \phi \tau^{-1} r(\rho, \alpha) + \phi(\rho(\rho, \alpha) - i^* - d^*) \right] \]  (33)

From (33) we can obtain the loci of all combinations in \( \rho \) and \( \alpha \) for which the ratio between monetary base and the stock of foreign bonds is kept constant through time. The slope of this loci is given by:

\[ \frac{\partial \rho}{\partial \alpha} = \frac{\phi \tau^{-1} r_{\alpha} - \phi i_{\alpha}}{\phi \tau^{-1} r_{\rho} - \phi i_{\rho}} > 0 \]  (34)

where:

\[ r_{\alpha} = \frac{\partial r}{\partial \alpha} ; \quad r_{\rho} = \frac{\partial r}{\partial \rho} ; \quad i_{\rho} = \frac{\partial i}{\partial \rho} ; \quad i_{\alpha} = \frac{\partial i}{\partial \alpha} \]

Now we must turn our attention to the evolution of expected rate of profit through time. We will suppose, like Taylor and O’Connell (1985), that:

\[ \dot{\rho} = \psi(i(\rho, \alpha) - \bar{i}) \ ; \ \psi > 0 \]  (35)

where:

\[ \bar{i} \] is the "safe" rate of interest

Equation (35) establishes that if current rate of interest is bigger than safe rate of interest than there will be a continuous increase in expected rate of profit. The racionale for this assumption is that, in this case, entrepreneurs will expect a future reduction in the
domestic rate of interest and thus a future increase in aggregate demand and in the degree of capacity utilization\(^5\).

From (35) we can obtain the loci of all combinations in \(\rho\) and \(\alpha\) for which expected rate of profit is constant through time. The slope of this loci is given by:

\[
\frac{\partial \rho}{\partial \alpha} = -\frac{i_\alpha}{i_\rho} > 0 \quad (36)
\]

This economy will be in steady-state when values of \(\rho\) and \(\alpha\) are such that (33) and (34) are both set equal to zero. Grafically, this will occur in the point where curve \(\alpha = 0\) intercepts the curve \(\rho = 0\). A possible steady-state configuration of this economy is represented in Figure 4.

![Figure 4](image)

Where: \(\rho^*\) and \(\alpha^*\) are the steady-state values of \(\rho\) and \(\alpha\).

Now we have to analyse the stability of steady-state equilibrium. In order to do that, is necessary to calculate the matrix M of partial derivatives of the dynamic system composed by equations (33) and (35) [ cf. Lima, 1999, p. 1644]. The elements of this matrix are given by:

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\(^5\) In Taylor and O’Connell (1985), expected profits are supposed to fall when current rate of interest is bigger than safe or normal rate of interest. However, the rationale for this assumption is not clear. If current rate of interest is bigger than “normal” rate of interest than is reasonable to assume that entrepreneurs will expect a future deduction in the rate of interest. Since aggregate demand and the degree of capacity utilization will increase if interest rate falls, so entrepreneurs should expect a future increase in rate of profit. In this case, there is no clear reason to assume that expected rate of profit will fall.
The steady-state equilibrium of this economy will be unstable if and only if the trace of Matrix M is negative (cf. Takayama, 1993, p. 408). This will occur, in turn, if the following condition is satisfied:

\[ \varphi < \frac{\phi \tau^{-1} \rho - i_\rho}{i_\alpha} = \varphi^c \]  

Expression (37) states that steady-state equilibrium will be unstable if \( \varphi \) is less than a critical value \( \varphi^c \). In other words, for steady-state equilibrium to be unstable, capital mobility cannot be very high.

This economy will have a saddle-path if and only if the determinant of matrix M is also negative (Ibid, p. 408). This will occur if and only if the following condition is satisfied:

\[ r_\alpha i_\rho > r_\rho i_\alpha \]  

It can be easily demonstrated that (38) implies that \( \alpha = 0 \) curve is steeper than \( \rho = 0 \) curve, as it is shown in figure 4.

5 – Real exchange rate appreciation and dynamics of growth and equity prices: booms and bubbles.

Now, we will turn our attention to the effects of real exchange rate appreciation over a set of variables, in particular: (i) the current rate of profit, (ii) the current rate of interest and (iii) equity prices. In the framework developed in sections 2-4, we will be able to show that a real exchange rate appreciation may cause a temporary increase in growth rate of capital stock and a bubble in equity prices, i.e. a cumulative increase in equity prices during a finite length of time.

First of all, we will analyse the effect of real exchange rate appreciation over \( \rho = 0 \) curve. Setting (35) equal to zero, and taking the total derivative of the resulting equation, we obtain the following expression:

\[ \frac{\partial \rho}{\partial q} = -\frac{i_\alpha}{i_\rho} < 0 \]  

Expression (39) states that a reduction in real exchange-rate – i.e. a real exchange rate appreciation – will produce an increase in expected rate of profit. Grafically, this effect is represented by a rightward shift in \( \rho = 0 \) curve, as it is shown in figure 5.
In order to determine the effect of a real exchange rate appreciation over $\alpha = 0$ curve, we have to set (33) equal to zero and take total derivative of the resulting equation. After all necessary algebraical manipulations, we arrive at the following expression:

$$\frac{\partial \rho}{\partial q} = \frac{\xi_1 - \phi \tau^{-1} r_q + \phi i_q}{\phi \tau^{-1} r_{\rho} - \phi i_{\rho}}$$  \hspace{1cm} (40)$$

Expression (40) has an ambiguous sign because it is not possible to determine the sign of numerator. Clearly, ambiguity is caused by the presence of $\xi_1$ – i.e the sensitivity of net exports as a fraction of economy’s capital stock to changes in real exchange rate. However, it is a stylized fact about semi-industrialized economies that net exports are not very sensitive to changes in real exchange rates. So, we can assume that $\xi_1$ is near zero. In this case, the sign of (39) will be negative, meaning that real exchange rate appreciation will produce an increase in expected rate of profit. This effect is represented graphically by a rightward shift in $\alpha = 0$ curve, as shown in figure 6.
The final effect of a real exchange rate appreciation over steady-state values of $\alpha$ and $\rho$ will depend on the relative shift of both curves. If the rightward shift in $\rho = 0$ curve is sufficient strong, so there will be a reduction in steady-state value of $\alpha$ and an increase in steady-state value of $\rho$ (see Figure 7). Since equilibrium rate of profit is a positive function of $\rho$ and a negative function of $\alpha$, than current rate of profit will certainly increase. On the other hand, equilibrium rate of interest is a negative function of $\rho$ and a positive function of $\alpha$, so there will be a clear reduction in the rate of interest.

An increase in equilibrium rate of profit and a reduction in equilibrium rate of interest will induce rentiers to substitute domestic bonds for equities in their portfolios. Thus there will be a sharp increase in demand for equities which will produce an increase in equity prices.
Table I summarizes the effects of a real exchange rate appreciation over steady-state values of $\alpha$, $\rho$, $r$, $i$ and $P_e$.

Table I

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>$\alpha$</th>
<th>$R$</th>
<th>$i$</th>
<th>$P_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Real exchange rate appreciation</strong></td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

So far we have only analysed the effects of real exchange rate appreciation over steady-state values of $\alpha$ and $\rho$. However, out of steady-state dynamics is much more interesting.

In fact, let us consider the situation displayed in Figure 8. Point A represents the initial steady-state position of the economy, i.e before the occurrence of a shock over real exchange-rate. After the appreciation in real exchange-rate, economy starts to move in $DD$ path. As we can see, from point A to point B there is (i) a continuous reduction in the value of $\alpha$; and (ii) a continuous increase in $\rho$. From point B on, expected rate of profit continue to increase, but the ratio between bank reserves and foreign bonds begins to rise.
The behaviour of $\alpha$ and $\rho$ from A to B produces the following effects:

1. First of all, since $r$ is a positive function of $\rho$ and a negative function of $\alpha$ there will be a cumulative increase in the current rate of profit. This increase is accompanied by a continuous reduction in the rate of interest, since it is a negative function of $\rho$ and a positive function of $\alpha$. The combined effect of (i) increase in current and expected rate of profit; and (ii) reduction in the rate of interest will result in an increase in the desired rate of capital accumulation, i.e. an increase in the rate of growth of capital stock.

2. Since economy is under a fixed (nominal) exchange-rate regime, a continuous reduction in the ratio between bank reserves and foreign bonds is followed, under no-sterelization assumption, by a continuous reduction in the stock of foreign reserves.

3. As a result of (i) a continuous increase in current and expected rates of profit; and (ii) a continuous reduction in the rate of interest, there is a cumulative increase in price of equities as soon as *rentiers* substitute domestic bonds for equities in their portfolios. In other words, a *bubble* in equity prices – defined as a cumulative increase in asset prices relative to prices of capital goods - is produced as a result of the dynamic path of $\alpha$ and $\rho$.

4. Since the degree of capacity utilization is increasing over time, and domestic rate of interest is being reduced; there will be a continuous increase in imports of raw materials and an outflow of capital. In other words, this economy will have a current account and a capital account deficits, which will be financed by a continuous loss of foreign reserves.

From point B on, the behaviour of main macroeconomic variables is ambiguous. For example, if the continuous increase in the expected rate of profit has the effect of increasing current rate of profit; the continuous increase in the ratio between bank reserves and stock of foreign bonds will have just the opposite effect. Since the behaviour of $r$ is undetermined, we can’t say anything about the behavior of equity prices and the rate of growth of capital stock. The only statement that can be made is about the behaviour of foreign reserves. >From point B on, there is a clear increase in bank reserves and, under no-sterelization assumption, in the stock of foreign reserves.

The time path of current rate of profit, current rate of interest, stock of foreign reserves and equity prices are displayed in figure 9.
6 – Conclusion

Throughout this article, we have shown that an asset price bubble – defined as a cumulative increase in asset prices relative to prices of capital goods – can result from a real exchange rate appreciation in economies that have (i) imperfect capital mobility and (ii) a low degree of organization in financial markets. These are common features of emerging countries like Brazil or South Korea, so that the possibility of occurrence of asset price bubbles in these economies is not small.

Appendix 1: The Slope of FF locus and demand elasticity of equities.

If the numerator in (22) is positive then:
\[ \beta_3 > -\frac{(1+\alpha')}{\alpha'} \mu_3 - \epsilon_3 \] (A1)

Multiplying both sides of (A1) by \[(r+\rho)/\epsilon W\] , we have :

\[ \frac{\beta_3 (r + \rho)}{\epsilon W} > -\frac{(1+\alpha')}{\alpha'} \frac{\mu_3 (r + \rho)}{\epsilon W} - \frac{\epsilon_3 (r + \rho)}{\epsilon W} \] (A2)

But the last term in (A2) is the elasticity of equities demand in relation to the expected rate of profit, which we will define as \( \eta_{e,r+\rho} \). It is easy to show that :

\[ \frac{\beta}{\epsilon} \eta_{\beta,r+\rho} > -\left(1+\alpha'\right) \eta_{\mu,r+\rho} \frac{\mu}{\epsilon} - \eta_{e,r+\rho} \] (A3)

Where : \( \eta_{\beta,r+\rho} \) is the demand elasticity of domestic bonds in relation to expected rate of profit and \( \eta_{\mu,r+\rho} \) is the demand elasticity of “money” in relation to expected rate of profit.

Manipulating (A3) we arrive at the following expression :

\[ \eta_{e,r+\rho} > -\left[ \frac{\beta}{\epsilon} \eta_{\beta,r+\rho} + \left(1+\alpha'\right) \eta_{\mu,r+\rho} \right] = \eta_{e,r+\rho}^* \] (A4)

Expression (A4) shows that numerator in (22) will be positive if and only if demand elasticity of equities in relation to expected rate of profit is bigger than some critical value \( \eta_{e,r+\rho}^* \).

References


